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ABSTRACT

An empirical study concerning development of the Swedish educational system from a labor market point of view, and a theoretical study on human capital models are discussed. In "Education and Labour Market; The Swedish Experience 1900-1975," attention is directed to the following concerns: the official educational policy regarding education and the labor market; the objectives, means, and general perception of the relations between education and the economy; and organizational development of the public educational system. The quantitative development of the educational system and the effects of education on allocation and growth are reviewed. In addition, the distribution of educational resources is considered, along with the distributional effects of education due to its effect on such factors as earnings. International comparisons are also examined. In "Human Capital Models and Education," the models are developed mathematically and analyzed. All of the models address the same concerns: the educational choices individuals make, factors that explain educational choices, and relationships between education and earnings. For the Schooling Models the only choice the individual makes is the length of initial period of schooling. For Life Cycle Models, additional concerns are the level of training in the form of on-the-job training and recurrent education later in life. In Life Cycle Models Including Leisure Activities, the perspective is further extended, while Intergenerational Models of Individualistic Type and Household Models are possible alternatives or complements to the earlier Life Cycle Models. Bibliographies are appended. (SW)

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Swedish Experiences and Theoretical Analyses

by

ÅSA SOHLMAN

Stockholm 1981

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Preface

Why did, and still do, people believe that education would promote economic growth and that a certain type of educational policy would reduce differences in earnings between individuals? Or the other way round, if such beliefs were held, what kind of view of education and of the economic system would they imply?

Long ago, these types of questions provoked my interest. However, it was not until 1974-1976, due to financial support from the National Board of Universities and Colleges, that I had the opportunity to study these questions more seriously. They also turned out to be more complicated than I initially envisaged and only part of the ambitious research program formulated in 1974 has so far been carried out. I am very grateful to my thesis advisor at that stage, Bengt-Christer Ysander, who inspired and greatly helped me not to take a restricted approach to the subject. I am also very much obliged to Lars Werin who encouraged me to select a few issues and concentrate on them for my thesis and leave the remaining unsettled questions for later research.

Thanks are also due to the members of seminars of Bengt-Christer Ysander and Lars Werin, not least to Emil Ems who also participated in both series of seminars and substantially reduced the number of logical errors. Sören Blomquist read one version of the whole study and gave constructive advice as did Lars Svensson on certain mathematical parts of the study.

Alf Carling has not contributed directly to my work on this thesis. As a colleague in the Energy Systems Research Group (FFE) and a friendly advisor in other research work he has, however, encouraged me and provided inspiration in seeking high academic and ethical standards.

I would also like to thank Clotye Larsson for correcting the English and Kerstin Grundberg who typed endless intermediate drafts. Maj Asplund typed the final version with great care and enthusiasm.

During this entire period my family - husband and two children - remained unflinching supporters in face of the impenetrable laws of the academic world.

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Introduction

Part I Education and the Labour Market: Swedish Experience 1900-1975

Part II Human Capital Models

Introduction

In this volume two studies emanating from the research project "Education and the Labour Market" are presented. The first one, entitled "Education and the Labour Market: The Swedish Experience 1900-1975," is an empirical investigation of the parallel development of the Swedish educational system and the labour market with special emphasis on the last thirty years. The purpose of this study is to achieve a summary description and a rather wide perspective on the relations between education and the labour market out of which various interpretational schemes can be discussed.

The study takes as a point of departure the objectives of official educational policy as far as they have been concerned with education and the labour market. These objectives, the means to achieve them and the general perception of the relations between education and the economy are indicated in the first section. Of course, labor market considerations have not been the sole determinants of educational policy. However, it seems fair to say that they have played a significant role during the period we have studied. The weight attached to them has varied over time and for different educational levels. At present they seem to be as important as ever.

Our empirical material is organised in two sets corresponding to the educational goals. The first one contains data mainly relevant for discussing the allocation of resources and economic growth and the second those mainly relevant from a distributional point of view.

The descriptive material is used to discuss two types of questions: To what extent have the objectives of official educational policy been reached? How can these data be interpreted theoretically? In the latter case we are interested in the extent to which the conception of relations between education and the labour market on which the official policy was based can be said to be born out by the data and what alternative approaches seem plausible in the case of Sweden. The last section of the study contains a discussion of these matters and a certain number of - not necessarily irreconcilable - interpretational schemes are proposed.

In the second study, "Human Capital Models and Ecuation," one of the approaches suggested as relevant in the first study is taken up for further consideration from a theoretical point of view. One reason why, in the first study, we could not arrive at any firm conclusions as to the effects of education and educational policy was, of course, that in order to do so the alternative explanatory models have to be clearly specified theoretically before they can seriously be confronted with empirical data. The purpose of the second study is to set the theoretical stage for further empirical work at least for one type of approach - "the human capital approach." On the theoretical side we would later like to do the same thing for the other types of models and/or combinations of them parallel with further empirical work.

That we initially concentrated our theoretical studies on human capital models was motivated by the fact that policy makers have been inspired by this kind of approach and that according to our preliminary investigations it certainly is of relevance for Sweden. Moreover, it has played a dominant role in the economics of education. The latter fact does not mean, however, that "the human capital approach" is built on a common, consistent theoretical basis. Often even the vaguest reasoning in terms of resemblances between investments in physical capital and in individuals and in terms of present resource allocation in favour of education securing the nation's economic future is labelled human capital theory. This is, of course, rather meaningless as such statements cannot be empirically tested. As for more stringently working human capital theorists - though they share certain basic ideas - they have worked out a number of specific but rather different human capital models. Before continuing empirical work with "the human capital approach" we therefore wanted to analyse these models and investigate to what extent it was possible to give them a unified presentation. Thereby empirical work would be facilitated and suitable testing strategies could be identified. Certain theoretical holes could also be filled in and further theoretical developments suggested.

In the first section of the study we follow the historical development of certain trends in economic theory that are concerned with investments in individuals or individuals investing in themselves. We also indicate how we have defined human capital theory, that is, the common basis for the models analysed in the next sections.

Each specific model addresses the same type of questions - namely, what educational choices do individuals make, what factors explain educational choices, and what relations emerge between education and earnings? The complexity of the models varies, however. In the study, first a rather simple type of models is presented and then successively different aspects are added making the models more complex. Thus, in the first set of models the only choice the individual makes concerns the length of an initiall schooling period. In the next set of models, not only the length of such an initial period of specialisation but also the level of training troughout the life cycle has to be decided. Then the perspective is further extended, educational choices have to be made with regard both to their effects on earning capacity and to their effects on other types of activities. Finally,an intergenerational perspective is added.

In our combined survey and synthesis of human capital models we are especially interested in clarifying:

- the assumptions used;
- the explanatory variables that emerge with respect to educational choices and such related variables as earnings;
- the relations, qualitative as well as more exact, that result between explanatory variables, education, and earnings.

Our reasons for concentrating on these aspects is rather obvious. We want to know what these human capital models explain, how they do so, and ultimately how well they work out compared with other models in the case of Sweden.

We try to state rather carefully the assumptions made in each case which is essential for an understanding of the area of applicability of the models. In the last section we also return to this question as we make a general characterisation of the results obtainable from the models and suggest further empirical and theoretical research possibilities.

I

EDUCATION AND THE LABOUR MARKET; THE SWEDISH EXPERIENCE 1900–1975

Reprint from M. J. Bowman, Å. Sohlman, B-C. Ysander, Learning and Earning. Stockholm 1978.

and Some Implications", in Journal of Political Economy, Supplement, October 1962. Many other publications have followed, by these and other economists. An analytical assessment of this work through 1973 is given in my "Learning and Earning in the Post-School Years", in Fred N Kerlinger and John B Carroll, eds. Review of Research in Education, 2, Itaca, Illinois, F E Peacock Publisher, 1974, pp 202-44.

26. On this see Mary Jean Bowman, "From Guilds to Infant Training Industries", in C A Anderson and M J Bowman (eds.) Education and Economic Development, Chicago, Aldine Press, 1965.

27. Op cit p 49.

28. ibid pp 211-12, p 217

29. Gösta Rehn's well-known work is an example of just such independent and creative thinking. One of his recent papers, which I have seen only in mimeographed form, bears the appropriate title "Towards a Society of Free Choice".

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Introduction

In many fundamental respects the present Swedish educational system differs from that of 1900. Especially the last 30 years have been a period of great changes and important reforms. Even if Sweden can not be said to have entered the era of mass higher education it has successively progressed from mass primary to mass secondary education.

However, this development has not always been a smooth one. Different opinions have been confronted as to the appropriate tasks for the formal educational system, its organisation and the potential effects of educational policy. New objectives have been added to old ones. New problems have arisen as old ones have disappeared or sometimes because of the way in which the old ones were solved.

In this paper empirical data will be presented with the purpose of achieving a summary description of the development of the Swedish educational system from a labour market point of view. Our interest is with the educational system as a whole interacting with the labour market, the development of the educational system and interrelated changes at different educational levels. This approach can give a wider perspective on some of the problems the educational system is and will be confronting.

The descriptive material will be used in an attempt to answer and discuss two types of questions:

1. To what extent have the objectives of the official educational policy with respect to the labour market been reached? What has the development looked like within the different fields of interest?
2. How can these data be interpreted theoretically? What are the relations between educational and economic variables and how are they to be understood? What are the effects of educational policy? To what extent can the conception of relations between education and the labour market on which the official policy is based be said to be born out by data? What theoretical models do seem appropriate in the case of Sweden considering the problems at hand? Those familiar with the theoretical literature on education will recognise that these questions bring us close to the on-going debate on education as a human capital investment versus education as a sorting devise. We will not be able to settle this dispute but we do suggest how work in this field could be persued in accordance with the Swedish experience.

For the empirical survey we have not undertaken any original research work to obtain new statistical data. The survey is based on currently produced statistics and special investigations made by other research workers. We have tried to bring together what we have judged to be the best relevant information. However, being restricted to this data base the picture of the parallel development of the educational system and the labour market will be painted with rather conventional brushes. We will have more to say about the development in quantitative terms than in qualitative terms, more on an aggregated level than on a disaggregated level. Broad definitions are used. Education is measured by years or by a number of educational levels. The content of any such box can be most varying. - Even though it would be desirable to make more detailed descriptions in the future the present survey can serve the purpose of giving a first rough illustration of the development and thus point out areas for further research.

The education-labour market data have been organised in two sets. The first one contains data mainly relevant for discussing allocation and economic growth and the second those mainly relevant from a distributional point of view. Of course, certain types of data can illustrate effects of both types and will be used for both purposes in the summary discussions of the last section. Within these two areas we will use as a point of departure for our description the types of effects aimed at by official educational policy. Although some cost figures will be mentioned this study is not aimed at systematically analysing the costs involved.

As for the definition of "educational system" and "educational policy" we proceeded in a rather pragmatic way. The educational system was identified with those types of education that have been more or less continuously covered by statistics, that is the formal public educational system. We can not say exactly what share of the total educational activities in society they cover. The public educational system in Sweden has during its development taken over more and more educational tasks from the families, private educational institutions and business firms. The share of the Swedish population engaged on a full time basis either as teachers or students in the public educational system in 1970 was estimated at 20%.1) It is thus possible that the share of educational activities we are describing has increased over time. Educational activities outside the formal educational system may, however, also have expanded. In some instances an attempt will be made at indicating at least the order of magnitude of certain types of education outside the formal public educational system. In describing educational policy we will only consider policy measures which had the primary aim of directly influencing educational activities.

Of course, labour market considerations have not been the sole determinant of educational policy. However, it seems fair to say that they have, at all times, influenced the formulation of the Swedish educational policy. The weight attached to them has varied over time as well as the extent to which they have been controversial. At present they seem to be as important and perhaps as controversial as ever.

The plan of this paper is the following:

In the first section the official educational policy will be briefly described as far as it has been concerned with education and the labour market. The objectives, means and general perception of the relations between education and the economy will be indicated. We will also briefly summarise the organisational development of the public educational system. This will facilitate the reading of the statistical material that will be presented later as well as further elucidate the educational objectives as expressed by organisational reforms.

The quantitative development of the educational system is surveyed in section 2. Against this background the effects of education on allocation and growth are investigated in section 3. The description will be based both on disaggregated labour market data and data at the macro level.

Section 4 illustrates the functioning of the educational system as a distributional devise. First the distribution of educational resources is considered. Thereafter the distributional effects of education due to its effects upon such things as earnings are studied.

To get a further perspective on the Swedish development some international comparisons are made in sections 2 through 4.

Finally, section 5 contains a summary discussion of the results of this empirical investigation.

1 Educational objectives with respect to the labour market and organizational reforms[2)]

Generally speaking, educational policy can influence educational activities in society via the supply side and via the demand side. Both supply and demand are, however, determined by a host of factors many of which are outside the direct reach of educational policy. Sometimes individual demand is, however, overruled as when certain types of education are made compulsory. To a considerable extent the Swedish educational expansion originates directly or indirectly in the prolongation of compulsory schooling.

As to the optional levels of the public educational system it can be noted that in Sweden, as in many other countries, the price mechanism, equilibrating demand and supply, has been suspended. In such a situation either the demand or the supply will determine the quantitative development of education. Normally, the number of class-room seats has been a factor which directly restricts the volume of education. Of course, this does not mean that the development of demand did not have a bearing upon the supply of places.

The secondary schools used to attract more applicants than places available. This is still so for upper secondary education, though certain changes have been noted lately. The number of applicants has at least in some instances and for some branches become smaller than the number of places available. As for higher education a numerus clauses has traditionally been applied to an important number of study--courses. These restrictions did, with a few exceptions lately, always hold enrolment back. The main exceptions to this policy have been the "free" faculties, the faculties of theology, law, liberal arts, social sciences and to a certain extent natural sciences, where all qualified applicants were admitted. Because of the qualification requirements, the scope for individual demand at this educational level was, however, rather limited.[3)]

Considering the size of the public educational system and this rather tangible control of its quantitative development it may appear all the more interesting to investigate the motives behind the educational policy.

Of course, the educational policy is not formulated in a vacuum. The political decisions are the outcome of a complicated process. We do not intend to describe this often dramatic process. We concentrate upon the final decisions. In Sweden they were often the result of political compromise. It is therefore no wonder that official documents sometimes lack precision. The basis for the educational policy, the assumed relations between education and the economy and between educational policy measures and goals are not always clearly stated.

1.1 Policy objectives

The main policy objectives pertaining to education and the labour market will be briefly described under the three headings social change, distribution, and allocation and growth.

Social change

One aim of the more important educational reforms has been to achieve a gradual change in society. Educational policy has been considered as a potential force to transform the social framework within which both the allocative and distributional mechanisms have to work as well as these mechanisms themselves. Thereby of course the conditions regulating the labour market would also change.

The development of society has been regarded as demanding a restructuring of the educational system. At the same time changes in the educational system have been considered a prerequisite for further social change. A democratic society needs a democratic educational system and a democratic educational system can permit a deepening of democracy in society as well as on the labour market.

These considerations have motivated the introduction of a public school system that is compulsory and tuition free, a prolongation of compulsory schooling from 6 to 9 years and changes in curricula and teaching methods.[4)]

Already in the 19th century it was suggested that compulsory education was not enough to achieve these objectives of developing a democratic system. It was argued that a basic education common to all was necessary in order to develop a democratic spirit and to strengthen democratic values such as solidarity, tolerance and co-operativeness. All citizens ought to have a common core of learning, a common reference-basis for their participation in political, social, economic and cultural life. During the 20th century these ideas were realised as will be described below with the transfer of all basic education into public comprehensive schools, the organisational unification of these schools and the standardisation of their programmes.

As even those who had the lowest education on the labour market became equipped with a solid basic education and a sufficient amount of self-esteem they could be expected to demand better pay and a restructuring of jobs and employers to accept such changes. It might seem all the more plausible to look forward to these indirect effects of education upon the labour market, that work themselves out via attitudinal change, as a basic amount of education became common to all.

At a later stage the opening up of certain new types of educational activities within the public educational system such as adult education, recurrent education and vocational technical programmes at the university level have been achieved and partly motivated by the same kind of concerns.

As a recent example of an optimistic view of the possibilities to change the labour market through educational policy let us cite a 1975 government Bill:

Higher education "must support the efforts in society to create equality. It should among other things be able to change the division of labour and the recruitment patterns in a more democratic direction".

Distribution

Besides having these long run effects upon society, educational policy has been considered capable of more directly influencing variables that are important from a distributional point of view, that is educational resources and working conditions (earnings, unemployment etc). The aim of educational policy with respect to the distribution of these variables seems to have been:

- given an unequal distribution of educational resources, earnings etc to secure for every-one (without regard to social origin, sex, place of residence, age etc) an equal chance of attaining high educational and income levels.5)
- to equalise the distribution of educational resources (e.g. within and between generations) and reduce the differences in earnings and other conditions of work.6)

The fact that all groups did not utilise the educational system to the same extent was assumed to be due to barriers of different sorts. And, the removal of the barriers would reduce the differences. Policy measures in this area have been aimed at removing first financial and formal barriers and later what could be called emotional barriers. That is, the policies have been directed towards:7)

- increasing the financial aid to students
- widening the geographical spread of educational facilities
- reforming basic education making it comprehensive and less differentiated
- changing the qualification requirements

- changing curricula
- introducing new types of education
- applying a more active recruitment policy.

Education has apparently also been seen as influencing the individual's possibilities on the labour market. This belief seems to be based mainly on a human capital approach. Education develops the capacities of the individual. The educational system does not solely sort out individuals according to "innate" ability or any other feature not influenced by education.

The fact that people with higher education are better paid could within this frame-work be explained by a demand for the capacities developed by higher education that is high relative to supply. Given the assumptions:

- the ability to pursue studies at higher educational levels does not vary between relevant groups
- higher education develops the individual's capacities
- these capacities are demanded by the labour market

a change in e.g. the social composition of the student body, equalising educational opportunity, would also lead to a social equalisation of the chances of obtaining high incomes provided the demand and supply relations remain the same.

The policy measures to achieve this income equalisation would thus be first to remove the barriers to education as discussed above and second to prevent the demand-supply conditions from changing. It is possible that the initiatives toward a greater control of student numbers, for example in the reform of higher education decided upon in 1975 (see below) should be understood from this perspective.

However, an equalisation of chances has not been the sole objective. It has been declared that the distribution of income and welfare should become more even. Whether educational policy would be capable of achieving this directly or only via more general changes in society has not been explicitely indicated. It has been suggested, however, that an increase in the level of education in the population as well as a more equal distribution of individuals over educational levels would promote these aims but not why this should be so. With a perception of education as described above it would of course be possible to achieve income equalisation directly by means of educational policy. What kind of educational policy this requires depends for one thing on what the demand conditions on the labour market look like and how they are developing. With demand conditions producing constant differences in earnings as between educational groups complete income equalisation could only be obtained if everyone had the same education. If income differences decrease when the supply of persons with higher education is increased, income equalisation does not presuppose the same amount of education for everyone. It is difficult to say what stand educational policy makers have taken on these matters.[8)]

Allocation and growth

The allocation of resources to and within the educational sector to secure, from a labour market point of view, an efficient utilisation of resources and economic growth has been discussed at two different levels that we may call the micro and macro levels. We shall indicate how the objectives have been formulated at these two different levels. Let us just first point out that even with a labour market perspective growth does not have to be interpreted in a narrow way to mean only growth in the production of goods and services. It can also include the growth of good working conditions, occupational pride, satisfaction etc.

At the micro level the educational system can be regarded as a link between "raw" labour and the labour market. The individuals must be provided with knowledge and skills demanded on the labour market and get informed about what is demanded on the labour market. These individuals must also be knowledgable about their interests and the jobs they are suited for. The educational system participates in all these processes. It is also supposed to give information in the opposite direction, that is, from the educational system to the labour market so that the labour force can be effectively used.

The discussion at a micro or more disaggregate level is of old standing. It has, however, mostly turned around the problem of how to adapt the output of the educational system to the development of the labour market (technical changes, capital accumulation, structural changes etc).[9)] The weight attached to manpower requirements has varied as for levels and over time. In the government Bill 1975:9 it is said that education at the secondary level is more vocationally oriented than at the primary level and that higher education is to an even still higher degree preparing for working life.[10)] The introduction and prolongation of compulsory education was also mainly motivated by its importance for democracy even though it was at the same time considered healthy for economic development.

What labour market needs the public education system should carter for has also been debated. In the 19th century a not unusual opinion was that the public system should not provide vocational or even technical or natural sciences education. However, these types of education, first technical and natural sciences education and later vocational, have gradually been included in the public educational system.

In the 20th century when education at the secondary level expanded the rate of expansion was not so much directly motivated by labour market considerations. The number of places was increased as fast as possible, given the financial and other restrictions that existed, to correspond to the rising demand for such places. Thus at the end of the 1960s the upper secondary schools had a capacity to receive 90 % of an age-group leaving the compulsory lower secondary schools. The labour market considerations were mainly present in the attempts to direct the flow of students as between more or less theoretical branches and within branches as between different fields of study.[11] Generally more vocationally and less theoretically oriented programmes were favoured.

At the level of higher education the development of the branches with restricted entry has for a long time been discussed in terms of the "need" for persons with these qualifications. As for the "free" faculties the discussion in the 1930s as well as in the 60s centered around the problem of how to maintain an appropriate level and composition of educational output without resorting to a direct regulation of student numbers. Especially it was thought necessary to increase the supply of persons with a natural sciences, technical, social and economic education at the expense of those with a liberal arts education. Vocational guidence activities, both at the secondary schools and at the universities, have gradually been increased. More strictly designed educational programmes were also introduced at the "free" faculties to facilitate the transition from education to work. However, in 1975 Parliament decided to secure a rational development of the number of places with a more fixed planning framework beeing applied to the whole system of higher education. For planning purposes higher education was further devided into five vocational education sectors.[12]

Apparently the substitution possibilities on the labour market and the capacity within the educational system to adapt in a correct way have been regarded as too low at least within the existing institutional framework. Therefore some central planning of the educational output, adapting it to perceived manpower requirements, is needed.

However, the problems involved in planning the educational output in accordance with the "demand" on the labour market have been noticed. Moreover, even if the total number of people with different types of education should in some sense be correct it can make a difference who gets the education. The educational system should therefore permit the individuals to discover their comparative advantages and adapt as they get more information. Adaptability is, however, not only desirable for individuals during their education. Educational content should also be such that once out on the labour market the individual can adapt to new situations and be able to go back to the educational system when more education is needed.

Therefore, while sticking to the planning approach, there has been a tendency towards easing the rigidity of the educational system. On the one hand an increased mobility within the educational system and between education and work has been aimed at. On the other hand it has been stressed, especially recently, that the educational system must become more flexible in the sense that the types of education offered adapt to the changing conditions on the labour market. To achieve these goals increased information from the labour market to the educational system and from the educational system to the individuals is needed.

Let us give a few examples of measures facilitating mobility within the educational system, on the labour market and between education and work:

- later differentiation in basic education
- increased possibilities to transfer from one educational level to another and to change branch at a given educational level
- expanded adult education and recurrent education (a richer and more varied supply of single courses and short cycle programmes at the level of higher education)
- changed qualification requirements and selection processes in higher education
- curricula changes to achieve broad instead of too specialised education, problem-oriented education, interdisciplinary education
- increased educational and vocational guidence at all levels

and of measures aimed at creating an adaptable educational system:

- increased scope for local initiatives to renew education
- introduction of representatives of public interest and working life on educational boards at different levels.13)

That there could exist a need for adaption and increased adaptability on the opposite side, that is in the labour market has not been discussed. Lately it has, however, been recognised that perhaps the educational system should better inform employers about the content of different types of education to permit a more efficient utilisation of labour. It has been proposed that new kinds of educational certificates be introduced for higher education and this would serve that purpose. As well, it has been proposed that higher education be divided into five vocational education sectors and that there be increased contacts with representatives of working life, as for example on the educational boards.14)

The discussion at the macro level is of more recent date. The idea to regard education at a macro level as a growth factor in itself and not merely as a complement to the general economic development only reached Sweden in the 60s via the OECD.[15] Thus even if there are no signs on the labour market that more education is needed increased education can nonetheless lead to a higher economic growth. This way of looking at education increases the scope of action for educational policy but of course it does not quite square with the micro approach in which the substitution possibilities on the labour market were considered to be small. At different times one or the other of these two aspects has been stressed. Perhaps the arguments for education as an independent factor of growth have lost some of their actuality. The 1975 Bill of higher education emphasises the complementarity between education and the development in society and on the labour market.

In policy documents the discussion at the macro level neither became very precise nor were any concrete policy measures ever directly motivated by these types of arguments. They only made policy makers favourably disposed towards educational expansion in general. What was expected in terms of growth was not indicated. Sometimes it has been said that both a generally increased level of education and a more equal distribution of education would promote economic growth but not precisely why this should be the case.

That there could exist a relation in the opposite direction, from economic growth and increased incomes to education, has not often been discussed. This can be due to the generally very negative view of education for consumption purposes.[16]

In the 1950s another type of discussion was going on that could also be related to growth considerations at a macro level. I am referring to the discussion about the "pool of talent" and the educational system's incapacity to bring forth and develop these latent resources. This waste of talent ought to be abolished. A less selective and more flexible school system was expected to achieve this.[17]

Even though it was earlier said that growth does not have to be defined narrowly it is a fact that when policy makers discuss education and growth or education and the labour market from an allocational point of view this is generally done in a narrow perspective. (Discussing education and distribution the perspective has normally been wider bearing upon both earnings and other elements of welfare.) The measures described above could, however, be interpreted as intended to increase not only growth narrowly defined but also the growth of welfare. Increased flexibility and greater possibilities of choice can be important from a welfare point of view.

The educational goals, as described above, generally express an optimistic view of the possibilities to directly and indirectly achieve desired effects by educational means. However, it has been admitted that reaching the goals can be more difficult than initially thought. More differentiated types of policy actions have therefore been undertaken. A more pronounced need to coordinate educational policy with other types of policy, e.g. labour market policy, has also been felt.[18] This is moreover due to the fact that with the expansion of the educational sector educational policy must both as to objectives and means be integrated into general policy making.

The objectives concerning long run structural changes in society have been present along with the reform activities from the introduction of compulsory primary education to the recent(1975) reform of higher education.

The distributional goals aiming at creating equal opportunity are also old. The ambitions in this area have, however, increased. Not only equal opportunity is aimed at but also an equalisation of the opportunity structure. During the 60s when educational expansion was at its height these goals stood a little less in the forefront. The growth effects of education were highlighted and the educational expansion was perhaps expected to solve the distributional problems.[19] However, in the 1975 Bill they once more occupy a central position. Recently a new dimension has also been added to the distributional objectives as a greater attention has been given to educational differences between generations.

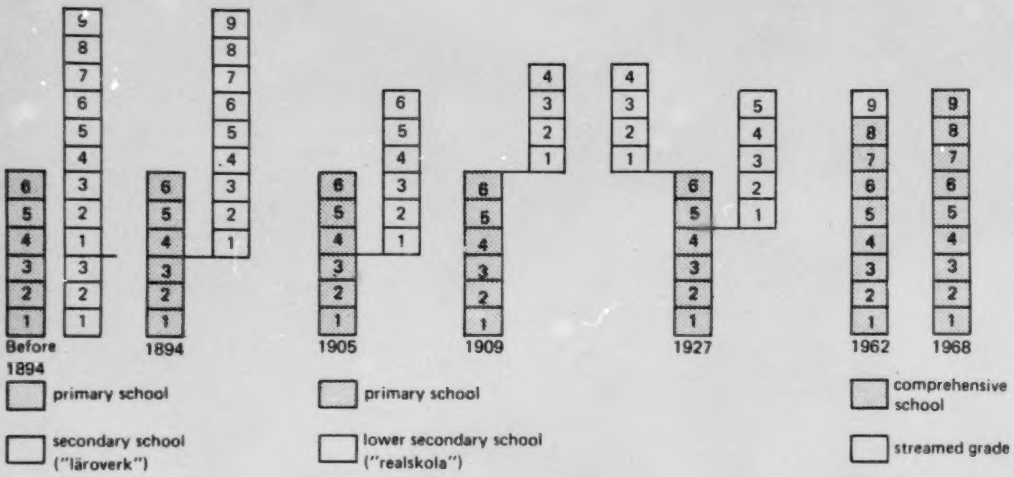
The allocative and growth objects, especially in their micro version, are also of old standing. In this area a broadening of goals has also come about. Practically all sorts of education (except perhaps for consumption purposes) are now considered the responsibility of the public educational system.

1.2 Organizational reforms

As a point of departure let us briefly indicate what the public school system looked like in 1900. It can be devided into three parts:

1. Primary schools ("folkskola"). Compulsory education was introduced in 1842. All children of "school age" had to go to the "folkskola" (with the exception of those who went to public secondary schools, approved private schools or were given the permission to be educated at home). The "school age" normally began the year the child was 7 years old. The first educational plan for these schools was not issued until 1878. It indicated as the main alternative 6 years of schooling. Local variations were, however, considerable and many children received an education much shorter than 6 years. The teachers often lacked teacher training and the equipment was in many cases miserable. The more well-to-do parents therefore normally gave their children private education at least until they could begin secondary school. In 1894 it was established that the entrance qualifications for secondary schools should be knowledge corresponding to 3 grades of primary school. Prior to

Figure 1. Development of the lower educational system



that there existed no link between the "folkskola" and secondary schools as the latter had their own primary schools as indicated in figure 1.

2. Secondary schools ("läroverk"). They prepared for university studies and consisted of 9 grades. What might be called three streams had developed: classical, semi-classical and natural sciences. Since 1862 the secondary schools themselves instead of the universities arranged the examination that qualified for university studies. However, not until 1891 were those who studied natural sciences entitled to go to the universities.

3. Higher education. The two general universities, Uppsala and Lund, together with the Karolinska Medical Institute had a common board of directors. The faculties were four: the faculty of theology, law, medicine, and philosophy (with the liberal arts and natural sciences sections). In addition to this, university education was provided in Stockholm and Gothenburg though not at general state universities. Specialised university colleges provided professional education for veterinaries, dentists, and technicians.

All vocational education at lower levels was privately arranged. The same was, to a large extent, true for the education of girls beyond primary schooling. In 1860, however, a public school for girls the "State Normal Girls' School" had been founded and the public supervision of and financial interest in the private girls' schools gradually grew.

In 1905 the secondary schools were divided into a lower secondary school ("realskola") and a upper secondary school ("gymnasium"). As much of the debate has centered around the connection between the primary and lower secondary schools and they were finally integrated let us first describe this development.

From a parallel to a comprehensive lower educational system

The new "realskola" that was established in 1905 had 6 grades and lead to the "realexamination" (3 + 6 years). It should give a general education but without latin. The transfer to the "gymnasium" took place after grade five of the "realskola".

In communities lacking secondary schools a new type of lower secondary schools was introduced in 1909. It had 4 grades and was based on 6 years of attendence at primary school. With this combination (6 + 4 grades) the way to the "realexamination" was thus one year longer. This reform was, however, important as in this way another small step away from the parallel school system had been taken at least in some communities (see figure 1).

In 1927 the lower secondary schools of 1905 were reformed. Two connections with the primary schools were introduced, one after 4 and one after 6 years of primary schooling. The respective lower secondary education leading to the "realexamination" was 5 and 4 years long as illustrated by figure 1. In 1927 all the secondary schools were moreover opened to girls.

In many communities, but mainly the urban districts, the primary schools were extended by one optional or compulsory year. Not until 1936 was it, however, decided by Parliament that a seventh compulsory year of education should be added throughout the country. This reform was introduced over a twelve year period.

In 1950 Parliament decided that compulsory education should be extended to 9 years and that a number of 9-year comprehensive schools should be established on an experimental basis. The latter part of the decision gave rise to different interpretations as to whether the experiments with comprehensive schools should lead to the introduction of such schools or their introduction would be conditional upon the results of the experiments. In 1962, however, Parliament decided that a comprehensive, compulsory 9-year school should be introduced. By 1972/73 it was established in all communities. The comprehensive school consists of three departments with three grades each: lower, middle and upper department. In the final grade there were nine streams. In 1968 the streaming in grade 9 was abolished.

From grade 7 and on, however, there is a certain differentiation as pupils in addition to a common nucleus of subjects choose other subjects from special elective groups. These choices do to a certain extent influence the possibilities of further studies.

Upper secondary schools

The "gymnasium" introduced in 1905 had 4 grades and 2 streams: the classical and the sciences stream. In 1953 a third stream was added: the modern and general stream. The three streams offered at least two branches of study. Besides these, general academic "gymnasia", technical and commercial gymnasia had been founded.

The 1964 reform of the "gymnasia" meant an integration of these different types of gymnasia. The new "gymnasium" had three grades (with the exception of the 4-year technical branch) and was based on the completion of the comprehensive school. As from grade 2 there were 5 different streams: liberal arts, social sciences, economics, natural sciences and technical. In 1964 a new type of upper secondary school was also introduced: the 2-year specialised professional schools ("fackskola"). They were also based on the completion of the comprehensive school and had three different courses of study: social sciences, economics and technical.

In 1968 Parliament decided that a new integrated upper secondary school ("gymnasieskola") should be introduced as from July 1971. The new upper secondary school was to incorporate not only the new "gymnasium" and the specialised professional schools of 1964 but also the vocational schools. Vocational education had existed in various forms over a long period of time. The vocational school system had also expanded considerably. Generally the programmes had, however, been worked out locally. The character, length and starting point of these courses therefore varied substantially. These schools were mainly sponsored by the communities and the State though schools financed by County Councils and industry also existed. The new unified vocational school that was decided upon in 1968 had two grades, was based on 9 years of comprehensive school and gave not only vocational training but also general education.

The new upper secondary school which is still in use has 22 streams. 5 corresponds to the 3- and 4-year courses of the new "gymnasium" of 1964 and 3 to the 2-year courses of the "fackskola".

It can be added that as from 1968 the adult education corresponding to the level of the upper department of the comprehensive school and the upper secondary school (both vocational and general courses) was more firmly organised as the "municipal adult education" was introduced.[20]

Higher education

Over time new general universities and specialised university colleges have been founded as well as new types of institutions of higher education. Among the latter can be mentioned institutes of journalism, of social work, of economics, of sports and gymnastics etc. Beginning in the autumn of 1975 an additional type of specialised university college with vocational technical programmes was introduced on an experimental level. It is intended to serve the need of further study for those active in trade and industry and the education will be based on their vocational experiences.

In 1975 there were 6 general universities (Gothenburg, Lund, Linköping, Stockholm, Uppsala and Umeå) and 3 branch universities affiliate to the general universities.

The division of academic education into faculties has changed. In 1956 the faculty of philosophy was divided into the faculties of liberal arts and of natural sciences. Out of the former later grew the faculty of social sciences. In the 1970s academic education is thus divided into nine faculties (the faculties of theology, law, medicine, natural sciences, liberal arts, social sciences, technology, pharmacy, and dentistry).

In 1975 Parliament decided that all post secondary education should be integrated into a uniform system of higher education, the "högskola".

If we now look behind the organisational framework at the rules governing higher education they had remained unchanged for a long time. The educational expansion, especially at the "free" faculties, however, caused certain problems. As for educational resources the situation was somewhat alleviated by the decision in 1958 to adjust government appropriations for teachers' salaries automatically to the number of students. Moreover, in 1965 Parliament decided that fixed curricula should be introduced also at the "free" faculties, that the number of places in restricted higher education should be expanded and vocational guidance opened up at the universities.

Certain changes were also introduced as to the qualifications required to ask admission to institutions of higher education. Above all it had to be settled what rights the completion of secondary education in the new upper secondary school should bestow on the pupils. In 1972 Parliament decided that completion of studies in at least a two-year line of upper secondary school together with knowledge of Swedish and English corresponding to at least two grades of the social line would give general qualification for higher education. Special qualifications above the general qualifications are, however, stipulated for various courses and educational programmes.

The possibilities for those who have not finished secondary school to go to institutions of higher education have increased. In 1969 people who were over 25 years and had been gainfully employed for at least 5 years were on an experimental level given the right to follow certain courses at the universities. The reform of higher education decided upon in 1975 implies that those who are over 25 years and have been gainfully employed for 4 years will have general qualifications for higher studies. Command of the Swedish and English language will normally also be required.

The selection to restricted education has normally been based on secondary school merits. The 1975 reform of higher education provides for a more fixed planning framework of the development of student numbers at the "högskola" as well as the number of students following different educational programmes and single courses. A new selection process will be introduced. All applications for admission will be received at a central office. The applications are divided into 4 groups:
1. applicants with three years' upper secondary schooling,
2. applicants with two years' upper secondary schooling,
3. other applicants except those with qualifications from abroad
and 4. applicants with qualifications from abroad. Group 1-3 are allotted places in proportion to their share of the total number of applicants. Group 4 is allotted a number of places not surpassing 10%. The selection in groups 1 and 2 is to be based on school merits and working life experience. Admission to a certain proportion of the places available to group 1 and 2 should be determined on the strength of school merits only. Selection in group 3 is to be based on working life experience and the results of a voluntary test of aptitude for studies.

To summarise the development of the public educational system it can be said that it has been characterised by two interdependent tendencies namely towards integration and diversification. At one level after the other the public educational system has assumed the responsability for new types of education. In many cases the need for new types of training was first met by private institutions, then by special public institutions and finally these were integrated into the regular public school system.

The objective as to a common basic education was finally reached with the establishment of the comprehensive school. The parallel public school system has been abolished and local variations have disappeared. The new upper secondary school integrates the old gymnasium, the commercial and technical gymnasia, the specialised professional schools and the vocational schools. The status of vocational education has thereby been increased. The risk of ending up in educational blind alleys at an early age have diminished.

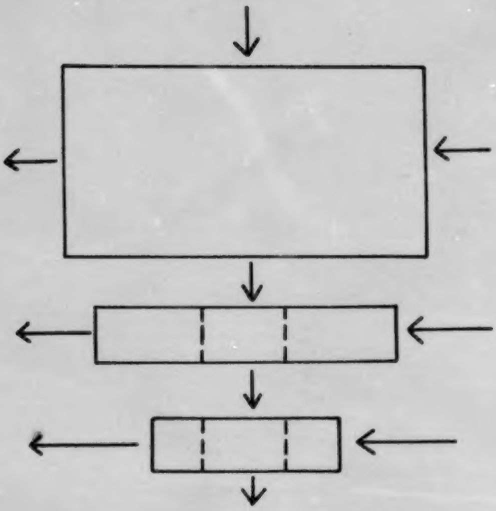
As for higher education the reform decided upon in 1975 implies that all public education even at this level will be organisationally integrated.

The tendency towards organisational integration has been coupled with a tendency towards greater differentiation. On the one hand the supply of different types of educations within the public school system has increased (with the exception of education at primary and lower secondary levels). To the classical education (with hebrew, greek and latin) has been added technical, natural sciences and social sciences education. Practical vocational education has also been increased quantitatively and integrated organisationally both at secondary and higher educational levels. On the other hand there has also been a differentiation as to the groups that can utilise the educational system. As for higher education it is now possible to talk about a double connection: one to upper secondary schools and one to working life.

An organisationally uniform educational system with differentiated educational possibilities has been considered appropriate to reach the distributional objectives at the same time as it permits a growth-promoting interaction with a more differentiated and rapidly changing labour market. "Education has been considered an important instrument to strengthen democracy and to equalise the distribution of welfare, influence and possibilities of cultural experiences. A common basic frame of references among citizens promotes understanding and cooperation. This has been and is an important argument in favour of striving for a unified organisation within the educational system ..."21)

2 Educational expansion

This section describes the quantitative development of the Swedish educational system. The quantitative development of an educational system can of course be described in many ways. To give a first general overview we will in part 1 start by presenting some figures on public educational expenditures. As for student numbers, whether measured in absolute or relative terms, there are as indicated by the figure below:



- the flows entering the educational system at various levels
- the stocks studying at various levels
- the flows leaving the educiational system

These stocks and flows can also be differentiated as to what type of educational programme the students follow, where they live and where they go after leaving the educational system and whether they leave with or without a certificate. In part 2 we will indicate the development of enrolment rates in primary, secondary and post secondary education for age-group 7-29 years as well as the proportion of relevant age-groups passing certain examinations. Regional differences in educational facilities and educational participation will also be briefly considered.

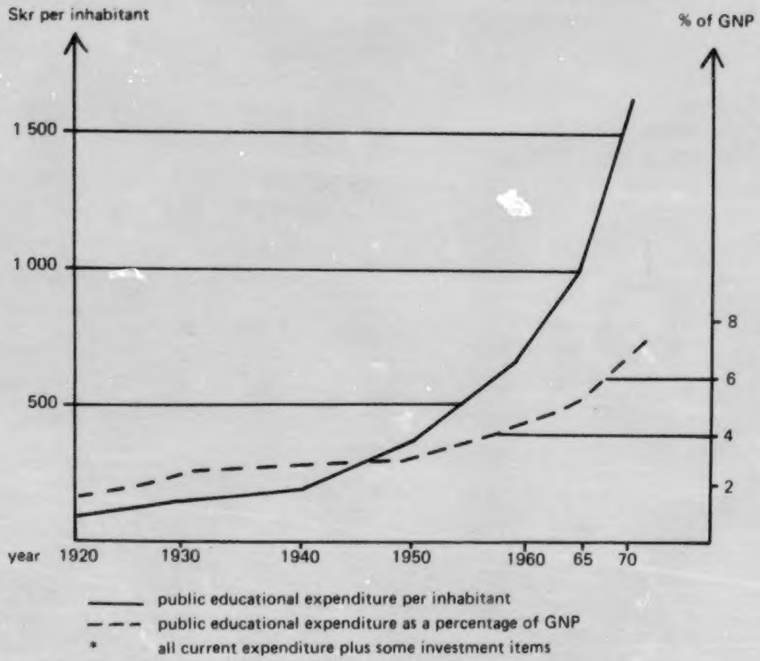
In part 3 absolute numbers will be reported as to total stocks of students at different levels and flows leaving the system. The educational orientation of those entering, leaving and studying at different levels will be dealt with in part 4. In part 5 the drop-out phenomenon is described as well as the amount of educational reorientation that occurs.

Those who leave the educational system with a certain type of education constitute an increment to those already in the population with such an education. Permanent changes in the output of the educational system only slowly affect the educational composition of the population. In part 6 we describe what has happened so far in this respect and what can be expected. The last part of this section will be devoted to international comparisons.

2.1 Public educational expenditure

The increase in public expenditure on education is illustrated in graph 1. Its share of GNP is also indicated. After the Second World War an accelerated growth has taken place and in 1970 the share of GNP was 7.2%. During that period the proportion of public expenditure devoted to education also showed an increasing trend which had not been the case earlier.[22)]

Graph 1. Public educational expenditure on "current account"* per inhabitant and as a percentage of GNP, 1920–1970



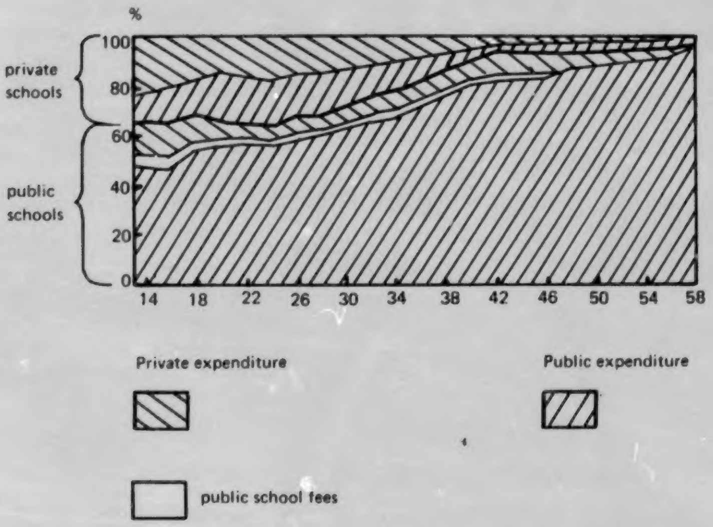
Source: Levnadsförhållanden. Årsbok 1975. SOS. Sthlm 1975 p 139

Given the state of statistics it is not possible to analyse the development of public expenditure on education in any great detail. Let us, however, cite some results from an investigation covering the period 1913-1958. Four types of education can be distinguished: compulsory, vocational, secondary and higher education.

The absolute increase in public consumption expenditure on education mainly stems from the expansion of compulsory education. Next in importance is secondary education followed by vocational and higher education. The relative increase in expenditure was greatest on vocational training while the growth rate of the other three types was about the same. At the compulsory level expenditure grew mainly due to increased costs/pupil, caused by a successive reduction in class-size. During the 50s, however, the effect of increased numbers of pupils was also of some importance for the development of costs at this level. The increased expenditure on secondary and higher education can mainly be explained by an increased number of students.

As for the secondary school the development of public and private financing is illustrated in graph 2.

Graph 2. The financing of running expenses at secondary schools



Source: E. Höök: Den offentliga sektorns expansion. Uppsala 1962 p 170

In 1913 a good 60% of the running expenses were covered by public expenditure. In 1958 the figure was almost 100%. If private expenditure had remained at the same absolute or relative level as in 1913 public expenditure would have been respectively 1% and 36% lower in 1958. This growing importance of public expenditure was caused by an increased number of public schools, a reduction of private spending on public schools and an increased public financing of the dwindling number of private schools.[22)]

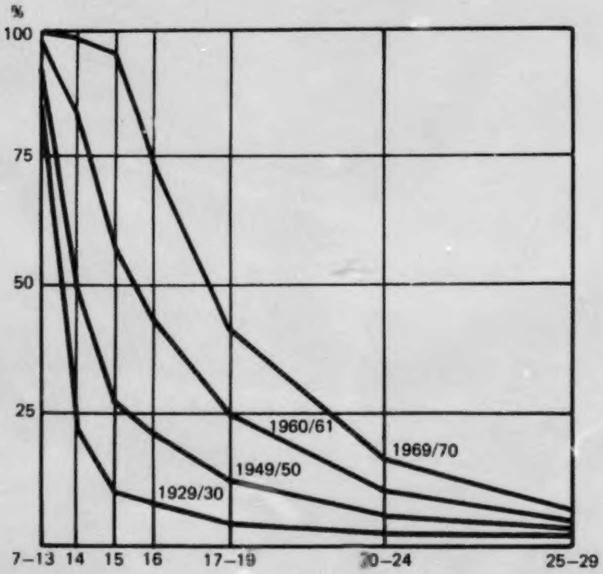
In 1970 another investigation[23)] shows the following relative distribution of public (consumption and investment) expenditure on education:

compulsory education	57
upper secondary education	25
higher education	16
adult education	2
	100 %

2.2 Enrolment rates

The development of enrolment rates is shown in graph 3. During the period 1930-1950 the increases were relatively moderate. However, in the 7-24 age group the enrolment rates doubled. 1950-1970 the enrolment rates in the 15-29 age group trebled. The proportion of 14-year old persons in education was already so high in 1950 that more than the actual doubling was not possible. If we compare the 50s and the 60s enrolment rates grew more during the 50s for the 14-year old persons and more during the 60s for all other age-groups. During the early 70s enrolment rates have continued to grow in age-groups up to 17 years, decreased in age-group 18-23 and remained constant in age--group 24-29.[24)]

Graph 3. Proportion of ages 7–29 in education



Source: SCB. Trender och prognoser. IFP 1974:8 p. 78, 269, IFP 1972:10 p 29

Judged by the enrolment rates of 14-year old persons the seventh compulsory year was not realised until after 1960. In the 7-13 age group the rate of participation approached, however, 100% already in 1950. In 1972 almost all 15-year old and 75% of the 16-year old persons were in school. In higher age-groups the propensity to study declines rapidly. By 18 it is under 50% and by 19 under 25%.

The types of education not covered by the above statistics are:

1. education in the family
2. pre-primary education
3. education of youth in publicly not supported institutions
4. vocational and non-vocational education of adults by ordinary or special adult educational institutions and firms.

The quantitative volume of the educational activities mentioned under 1. and 3. is not possible to estimate. The schools referred to under 3. are, however, relatively few.[25] As for pre-primary education it was estimated that in 1970 30% of the children 3-6 years old went to day-nurseries or nursery schools.[26]

A rough estimation of adult education provided by firms and special educational institutions for adults has been made.[27] The approximative enrolment rates (relating the full-time equivalents of these adult students to persons 30 years or more) were:

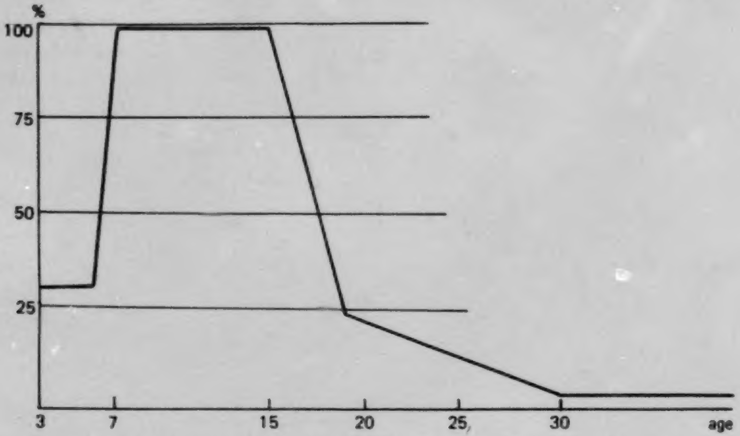
1967	2-3%
1970	3-4%

Adding the adults (30 years old and more) in secondary schools and at universities and specialised university colleges changes these figures by less than one percentage point.[28] The break through for these types of adult education seems to have been the 1960s even through still only a very small fraction of the potential recruitment base is affected.

Figure 2 summarises our information on enrolment rates and indicates the political options as to further educational expansion: more pre-primary education, more education for youth 16 years and older or more adult education.

To further illustrate the development of educational frequences graph 4 has been added. It shows the proportion of an age-group taking university degrees, passing the Higher Leaving Certificate or similar (general academic, commercial and technical gymnasia and corresponding lines in the integrated upper secondary school) and taking the "realexamination" or similar (lower secondary school).

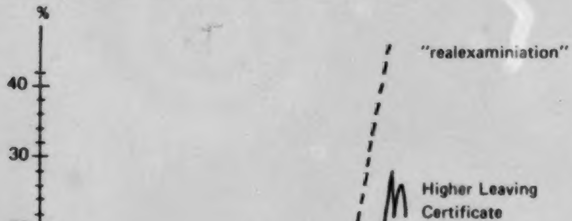
Figure 2. Approximative enrolment rates in the 1970s



In a historical perspective the rise in these proportions stands out as overwhelming and very concentrated in time. The expansion starts in the 20s with the "realexaminations", in the 30s with the Higher Leaving Certificates and in the 40s with the university degrees. Once the expansion started it accelerated up to the late 60s. Thus 10% were taking a university degree by the late 60s. Although this means a great increase compared with the 1% at the beginning of the century one can hardly talk about any mass higher education yet.

For the Higher Leaving Certificate (or similar) the graph contains information up to 1973. There it can be seen how the acceleration is broken and the proportion gaining this certificate starts to fluctuate. - It can be added that an increasing share of the holders of the Higher Leaving Certificate and of university degrees are women even though they still do not make up 50%. 29)

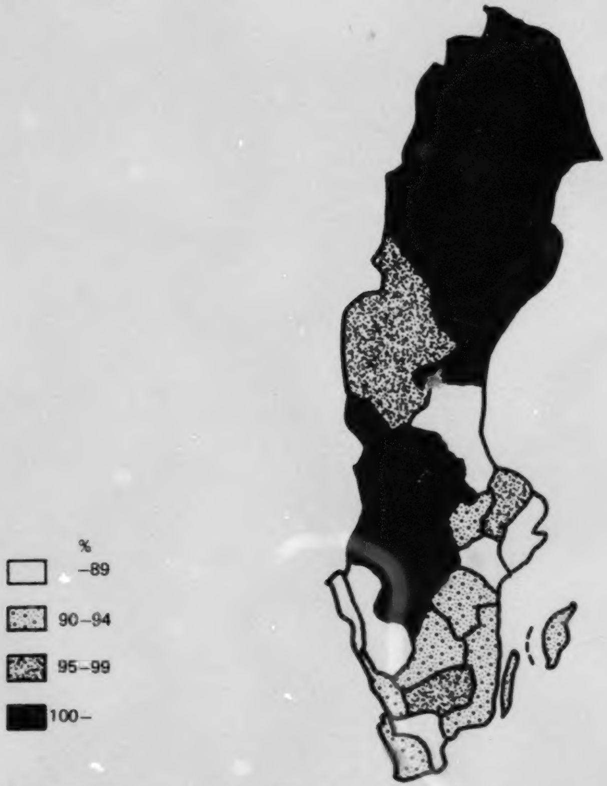
Graph 4. Percentage taking university degrees (in relation to 25-year old persons), passing the Higher Leaving Certificate or similar (in relation to 20-year old persons), taking the "real-examination" or similar (in relation to 16-year old persons)



Soruces: IFP 1974:8 op cit p 54. Levnadsförhållanden. Årsbok 1975 op cit p 134

Finally a few words about the regional supply of education (figure 3). In 1973/74 there were on the average 93 places for beginners in the upper secondary schools per 100 16-year old persons. One fourth of the 16-year old persons lived in counties with a frequency of places lower than 85 % and one fourth in counties with a frequency over 102 %. It may be surprising to learn that the highest frequencies are found in northern Sweden and western middle Sweden. These parts of Sweden are less industrialised and characterised by high unemployment. The migration out of these regions may to a certain extent be responsible for this relative educational abundance. It can, however, also be interpreted as part of a labour market policy creating "employment" in the short run and possibly growth in the long run. It is above all the relatively high supply of places in vocational lines that causes the high frequencies. Places in lines corresponding to the old gymnasia are most frequent in the counties of Stockholm and Uppsala. However, even though the number of places in northern Sweden is relatively high this does not prevent the distance to these educational places from being longer there than elsewhere.30)

Figure 3. Regional supply of places in upper secondary schools[1] 1973/74



[1] Places for beginners as a percentage of 16-year old persons

Sources: IFP 1974:8 op cit p 89

As to the regional propensity to utilise educational facilities (either within or without the region) it can be noted that in the beginning of the 70s it was, as far as university education goes, higher in regions with a university or a branch university than in regions without. These regional differences in the propensity to go to university disappeared, however, when only those who had just completed 3- and 4-year courses in upper secondary school were compared.[31)]

2.3 Development in absolute numbers

In 1973 1.4 million students were enrolled of which 1 million at the "pre-upper secondary" level, 230,000 at the upper secondary level and 150,000 at the post-secondary level.[32)]

As for the development of compulsory education it can be noted that there were fewer pupils in such education in 1940 than in 1930. This was due to a decreasing size of the relevant age-groups. The size of the age-groups beginning primary school when the seventh compulsory year was introduced (1936-1948) first continued to decline and later began to increase but remained relatively small. Therefore, in 1950 there were only 65,000 pupils more than in 1940. During the 50s the size of the age-groups continued to rise and stayed large at the same time as the length of compulsory education was extended to nine years. Consequently during the 50s the number of pupils at what now might be called the "pre-upper secondary" level expanded very substantially. During the 60s and the beginning of the 70s the number of pupils remained at a high but somewhat fluctuating level.[32)]

The wave of cohorts of increasing numbers reached the upper secondary schools during the 50s and the beginning of the 60s. They were on an average larger in the 60s than in the 50s. The increase in the recruitment base but above all the increased propensity to study at this level made for an expansion of the number of students during the 50s and especially during the 60s. Also for higher education the expansion was greater in the 60s than in the 50s due both to a greater propensity to study and increased age-groups. Declining propensity to study combined with smaller groups of relevant age led to a reduction of student numbers during the beginning of the 70s.[32)]

The growth in student numbers after 1950 looks as follows:[32)]

	1950-60	1960-70	1970-73
"pre-upper secondary"	285,000	-50,000	40,000
upper secondary level	70,000	110,000	± 0
other vocational education	15,000	10,000	-5,000
university level	20,000	85,000	-15,000
Total	390,000	155,000	20,000

On the whole the expansion was strongest during the 50s and above all due to the expansion of the "pre-upper secondary" level. The expansion during the 60s was caused by increased numbers in upper secondary and post-secondary education. 1970-1973 only the "pre-upper secondary" level continued to grow.

To this can be added the numbers in pre-primary education which were:26)

1960	49,000
1970	111,000

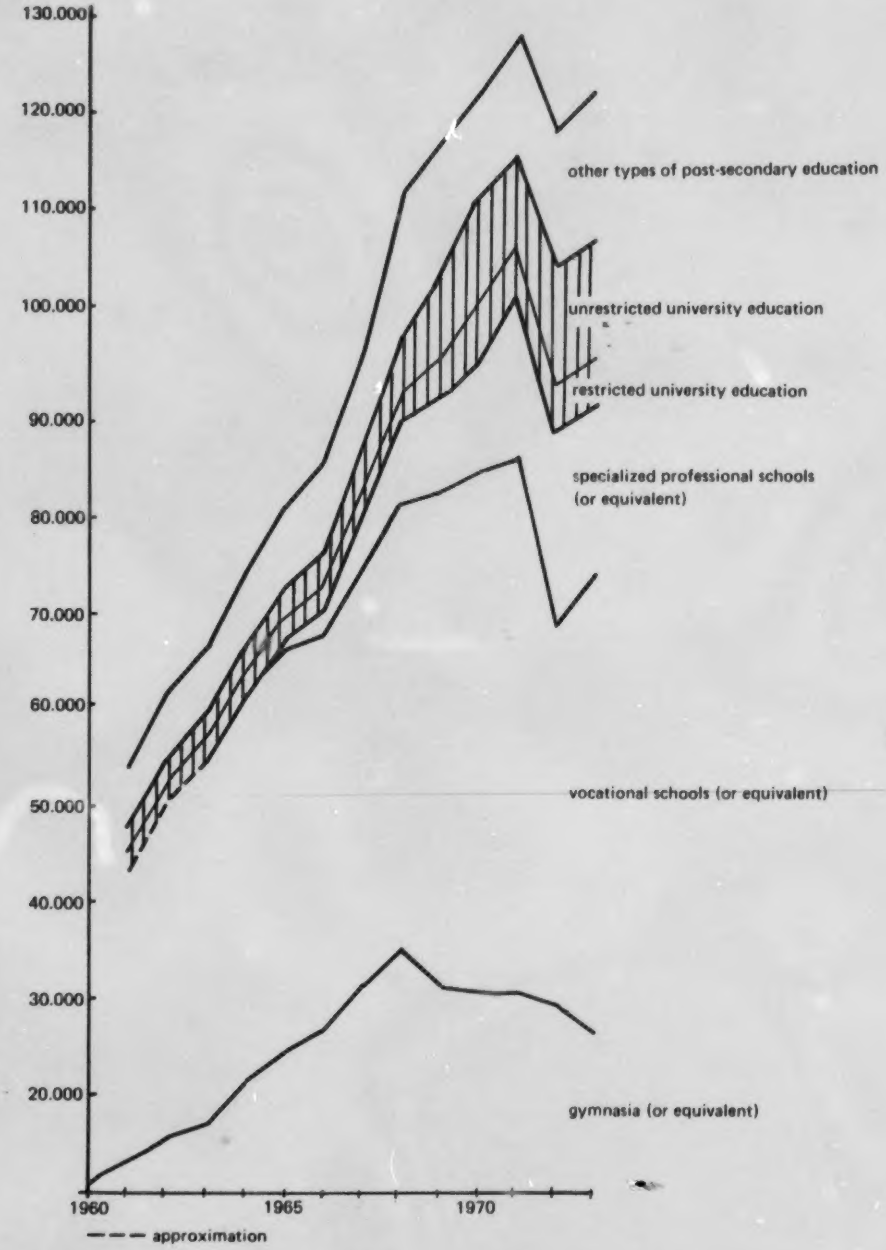
The number in adult education was estimated in 1970 to have been equivalent to 150,000 persons on a full-time basis.27) This figure does not include adults in educational institutions normally providing education for youth. In 1972 the number of persons in upper secondary schools older than 30 years was 3,300.28) 24% of the students at universities and specialised university colleges were over 30 years in 1973 which corresponds to about 26,000. In 1962 they were only 7,000. Moreover 16% of the new entrants to university and specialised university colleges in 1971 lacked the normal secondary school qualifications. Some adult education thus also takes place at secondary schools and universities.28)

As the number of persons obtaining degrees and certificates might reasonably be inferred from the development of enrolments we will restrict ourselves to presenting some details about the examination at levels above the comprehensive school level 1960-1973.

As can be seen in graph 5 the number of pupils completing their secondary education and students taking examinations and degrees at higher educational levels reached a maximum in 1971 when 128,000 persons passed final examinations (or equivalent) at these levels. For the gymnasia (general academic, commercial and technical) the number of Higher Leaving Certificates awarded was at its height in 1968 whereas the expansion in vocational schools continued up to 1971 and in other types of educations up to 1972, with the exception of restricted university education which expanded albeit slowly throughout the whole period.

The number of persons leaving the educational system with certificates, degrees or similar was twice as high in the beginning of the 70s as in 1960. Out of the increment of 60,000 the upper secondary school stood for 40,000, university and specialised university colleges for 11,000 and other post-secondary education for 9,000. Relatively speaking the expansion was greater at the post-secondary levels than at the upper secondary level.

Graph 5. Number of pupils completing their upper secondary education, examinations and degrees taken at hihgher educational levels



Source: IFP 1974:8 op cit pp 278–289

40

2.4 Orientation of studies

The upper secondary school integrates the gymnasia, the specialised professional schools and the vocational schools. The former are more theoretically and the latter more practically oriented. The distribution of students over these three types of education (later lines of the integrated upper secondary school) therefore has something to say about the structure of the educational expansion. It looks as follows:[33]

	1950	1960	1970	1973
Gymnasia	71	57	48	42
Specialised professional schools	-	-	17	17
Vocational schools	29	43	35	41
	100%	100%	100%	100%
Total number	43,000	116,000	224,000	226,000

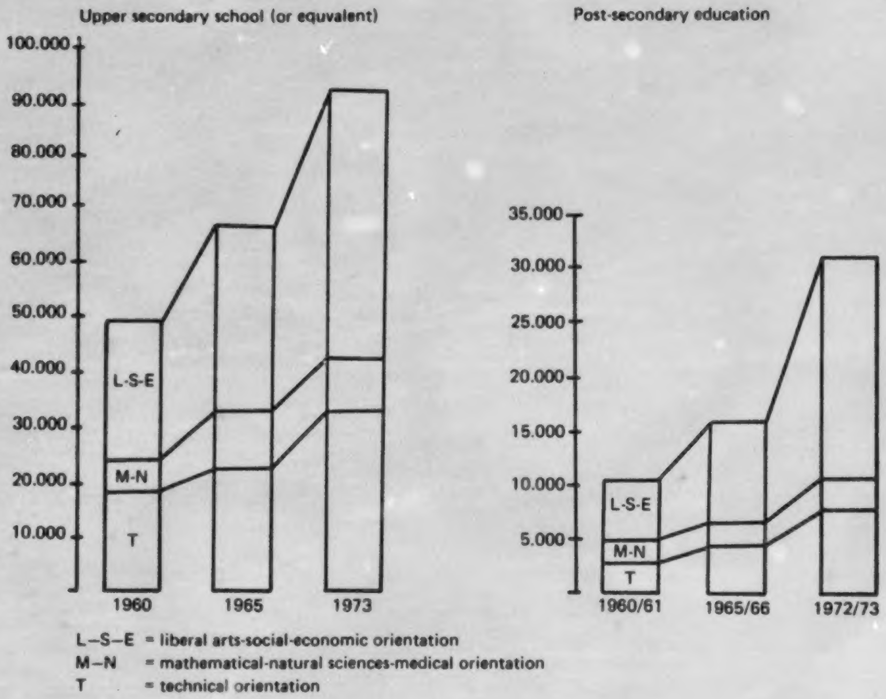
While the gymnasia had 70% of the pupils in 1950 by 1973 the situation had changed so that the lines in the upper secondary schools corresponding to the specialised professional schools and the vocational schools had 60% of the pupils. Thus, in 1973 a greater proportion of students at this level gets a practical vocational education.

If we again consider the different types of education at the post-secondary level that were distinguished earlier, that is restricted and unrestricted university education and other post-secondary education, we can characterise them in the following way. Restricted university education gives longer vocationally oriented education. Other post-secondary education gives shorter vocational training while the unrestricted university education is less directly vocationally oriented. In the beginning of the 70s the two former types of higher education stood for 2/3 of the degrees (or equivalent) and the latter for 1/3. These proportions were about the same as in 1960.

In graph 6 the pupils completing their upper secondary education and examinations and degrees taken at higher educational levels have been classified after their subject-matter orientation into three sectors: the liberal arts-social-economic-(L-S-E-), the mathematical-natural sciences-medical-(M-N-) and the technical-(T-) sectors.

For the upper secondary level it is noted that of these sectors the L-S-E-sector expanded most in absolute numbers but this did not lead to more than an increase from 51% to 54% of the secondary school leaving certificates (or equivalent). The share of the T-sector remained stable while that of the M-N-sector declined slightly. Among the new

Graph 6. Pupils completing their upper secondary education and examinations and degrees taken at higher educational levels by educational orientation



Source: IFP 1974:8 op cit pp 278–289

entrants to this level the longer and more theoretical lines have recently become less popular. The L-S-E- and the T-sector but not the M-N-sector are, however, represented on the shorter lines. It is thus possible that even in the future these two sectors will increase their share. The decline in students gaining secondary shool leaving certificates on natural sciences lines have already caused cries of alarm. Soon the technical university colleges may only have applicants for half of the places and the natural sciences faculty at general universities none! 34)

At the post-secondary level the L-S-E-sector has grown very rapidly both in absolute and relative terms. While it stood at 50% of the degrees (or equivalent) in 1960 the share was 64% in 1972/73. The L-S-E-sector is thus much more important at the post-secondary than at the upper secondary level. The vocational school with mainly technically oriented education serves as a counterbalance for the latter level. At the post-secondary level the relative increase of the L-S-E-sector has taken place at the expense of both the T- and the M-N-sectors. The share of the former declined from 17% to 11% and that of the latter from 33% to 25%. Judging from information about the orientation of new entrants it is possible that the share of the T-sector will expand and that of the L-S-E- and M-N-sectors decline in the near future.[35]

It can be added that adults enrolling at the "free" faculties choose the L-S-E-sector to a higher and the M-N-sector to a lower extent than younger students. In 1971/72 80% of the adults enrolling at the faculties of liberal arts, social and natural sciences choose the former two faculties.[36] Women enrolling at universities are strongly overrepresented at the liberal arts faculty and underrepresented at the other faculties.[37]

2.5 Flexibility within the educational system, drop-out and educational reorientation

Politicians have come to realize that a flexible educational system is desirable. It should permit the students a certain freedom of movement and leave possibilities open for educational reorientation. Dropping out, however, was for a long time viewed very unfavourably. This negative view of dropping out of school still prevails as far as the comprehensive school is concerned. As from the school year 1971/72 the educational code further restricts the possibilities to leave the comprehensive school prematurely. Dropping out of school at this level is in conflict with the goal of providing everyone with a common basic education. At higher educational levels the view of dropping out has changed. It has been recognised that dropping out is not always equivalent to a waste of resources. A flexible educational system may necessarily be characterised by what statistically shows up as high drop-out rates. Partial studies can form a link in a process of recurrent education or be part of an information and decision making process. Moreover, in incomplete statistics educational reorientation will, to a certain extent, be labelled drop-out.[38] "Drop-outs" and partial studies caused by recurrent education and even to a certain extent by vocational and educational reorientation is by now politically accepted and even stimulated at higher educational levels.

Let us now indicate the magnitude of the drop-out and reorientation phenomena.

The drop-out rate in grade 9 in the comprehensive school around 1970 has been estimated at 1-4%.[39] This is much less than the figures for the old lower secondary school. An investigation from the mid-fifties showed that 25% of these pupils never did take the "realexamination" and that another 25% only passed with a certain delay.[40] Quantitatively all but a small minority nowadays receive 9 years of education. What they qualitatively get out of this education is of course more difficult to evaluate. As one piece of information it can be mentioned that in one study it was estimated that about 15% of the pupils leave grade 9 without being able to read, write or calculate better than the average pupil in grade 6.[41]

At the upper secondary school a longitudinal study found that at the beginning of the 70s 15%, 7% and 5% of those who turned up in grade 1, 2 and 3 respectively dropped out during the year. The drop-outrate was lowest for the 3- and 4-year theoretical lines. These latter drop-out rates are also probably lower than those of the old gymnasium.42)

Of those who enrolled at universities and specialised university colleges 1956/57-1961/62 about 70% took a degree within 11 years. Of these students 10% obtained their degrees at faculties other than that of enrolment. 5% were still studying after 11 years and 25% had left without any degree. Comparing these drop--out rates with those of students enrolled later (1962/63-1969/70) it was found that the proportion of the later students who had left without any degree already after three years surpassed that of the earlier ones after 11 years (30% as compared with 25%). This tendency towards higher drop-out rates mainly characterised the unrestricted university education. The level of the drop-out rate is also much higher for this type of education than for that with restricted entry. At faculties of liberal arts, natural sciences and social sciences 60% had taken a degree after 11 years as compared with 80-90% (sometimes 100%) in restricted university education. These frequencies of the 60s at unrestricted university education does not represent any improvement compared with those of the 50s when a certain numbers of measures were taken to counteract what was then considered to be very high drop-out rates.43) As was just mentioned these drop-out rates are, however, no longer regarded as unfavourably as before. To a certain extent the recent increases in the drop-out rates may only represent a temporary phenomenon. The factors that caused a general decline in university enrolments may also have provoked a higher propensity to leave the universities. Both among younger students and students with a declared intention to take a degree, very high, and increased, drop-out rates have been noted. This may be temporary. The proportion of students with the declared intention not to take a degree has, however, increased in unrestricted university education. In 1973 it was as high as 50% among those enrolling at the faculties of liberal arts, natural and social sciences. This fact points to a structural change. It can to a certain extent be explained by the increased share of older students. A greater proportion of them normally leave without taking any degree44) and in this case the drop-out could be interpreted as recurrent education.

The amount of educational reorientation is more difficult to evaluate. Therefore a few examples will only be given. Above was indicated that a certain number of students (10% during the period studied) gained their degrees at faculties other than that of enrolment. This flow mainly consists of students enrolling at the free faculties and ending up in restricted university education. For those who left upper secondary school before 1969 it is possible to improve the secondary school merits by studies at the free faculties. Because of the changed rules this educational route has and probably will continue to loose in popularity. However, even in the future it will certainly be used by some students (see below) and remain as an expression for true educational reorientation.[45]

Let us now cite the results of a follow up study of a number of new entrants in 1968 and 1971 at the free faculties in the University of Umeå.[46] When enrolling, 70-80% declared that they had the intention to take a degree. After 3 years more than 50% of the students had altered study and/or vocational plans. Changes in "external" factors (labour market and educational possibilities) dominated as explanations for these amended plans. The changed plans materialised in three ways:

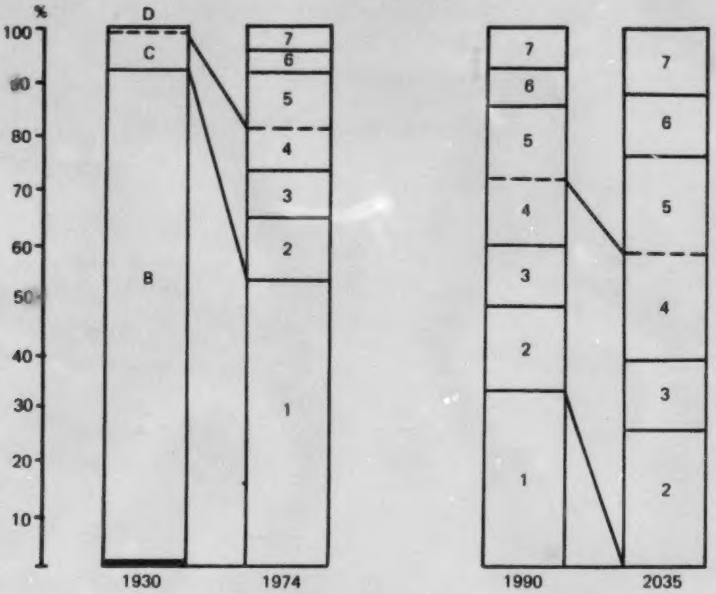
1. Transfer to other types of education. After 9 terms 30% had moved to other educational institutions. This flow of students was not only directed towards restricted university education and university institutions in other regions but nearly as often towards educational institutions outside the university sector. As for restricted university education 15% of the students stated that they had earlier asked for admission to restricted university education but only 8% motivated their studies at the free faculties by not having been admitted at restricted university education. 60% planned to ask for admission to such education later. Half of them could, however, not state exactly what type of education that would be.
2. Transfer to the labour market without having taken a degree. For some of the students this had been planned in advance but for many of them this was a reaction upon labour market signals.
3. Changed educational programme. At the end of the first year, on the average, 50% of the students had left the educational programme they originally enrolled for.

This information indicates that the educational system has an allocative function to fill during the educational process. To answer the question whether this function as well as that of providing possibilities for recurrent education are well filled would require more detailed investigations.

2.6 Effects of the expansion on the educational composition of the population

Graph 7 shows to begin with the educational composition of the population in 1930 and in 1974. It is noted that in 1930 only about 1% had passed the Higher Leaving Certificate or taken a higher degree (educational category D). The majority of the population (90%) had only primary education, and 1% had no registered education at all. The differences between the generations making up the 1930 population are very small at higher educational levels. The percentage with no registered education, however, decreases from about 10% in the oldest age-group to 0 in the youngest groups. For the older primary education also more often implies fewer years of education.x) The development, during the period these generations (born 1860-1910) represent, is above all characterised by the spread and consolidation of the primary school.

Graph 7. The educational composition of the population over the age of 15 in 1930 and the population aged 20–74 in 1974. "Projections" for 1990 and 2035



Educational categories:

- ■ no registered education
- B primary education and shorter vocational training
- C lower secondary education and longer vocational training
- D Higher Leaving Certificate or higher degree

1. Pre-upper secondary education less than 9 years
2. Pre-upper secondary education 9 (10) years
3. Upper secondary education 1 year or less
4. Upper secondary education 2 years or less but more than 1 year
5. Upper secondary education more than 2 years
6. Post-secondary education 2 years or less
7. Post-secondary education more than 2 years

Sources: Levnadsförhållanden. Årsbok 1975 op cit pp 123, 124 IFP 1974:7, 1974:8 op cit p 269

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x) Educational category B comprised in reality persons with 1-8 years of education.

In 1974 19% of the population belong to educational categories 5-7 which approximatively correspond to category D in 1930. This represents thus a rather important educational expansion. However, still only 9% have post-secondary education (level 6 and 7) and about 50% have only an education shorter than 9 years. 28% are found at the middle educational levels 2-4. Comparing the generations making up the 1974 population it can be noted that the intergenerational differences are very pronounced. As for educational categories 3-7 (upper secondary schooling or more) an important jump occurs between those born 1900-1907 and 1908-1924. Among the former, 13% belong to these educational groups and among the latter 24%. The proportion belonging to educational category 7 (post-secondary education longer than 2 years) is about the same for these generations (3%) but increases to 7% among those born 1925-1939. The younger generations are much more evenly distributed over educational levels than the older generations.

It can be added that on the average the education of women is somewhat shorter than that of men, 33% of the women belong to educational categories 3-7 which may be compared with 39% of the men.

Finally the "projections" made for 1990 and 2035 (graph 7) should be commented upon. They have been produced as an aritmethical example to show how slowly the educational composition of the population changes even though the education of youth may have expanded heavily. We have assumed that the high enrolment rates of 1972 become permanent and that people older than 20 years do not receive any further education. Our calculations thus only illustrates the effect of expanded education of youth permanented at the 1972 level. In 1990 only the 20-35 age group is new compared with 1974. Not until 2035 are all age-groups new and then the whole population would have received the same amount of education as the young in 1972. Thus in 2035 75% of the population would have more education than comprehensive school and 40% belong to educational categories 5-7.

2.7 International comparisons

Many problems arise when international comparisons of education are to be made. The OECD has, however, prepared statistical data describing the present situation in different countries, taking pains to apply uniform classification norms.[47) We will, to a large extent, base our observations on that material. The aspects we will deal with are: educational expenditure, enrolment rates within different age groups and at different educational levels and finally the population by educational levels.

Educational expenditure

The annual growth of total public expenditure on education was, in Sweden, rather high during the period 1965-70 (15%) but compared with other countries it was not exceptionally high. It was, however, sufficiently high compared with the growth in GNP to permit Sweden to advance from a shared third position (together with Canada) in 1965 to the second position (after Canada) in 1970 ranking the countries after the share of public educational expenditure in GNP. Sweden thus devoted 7.7% of GNP to education in 1970. The range of this share in the OECD countries was 2.0 - 8.3%.50)

In certain countries there exists, contrary to Sweden, a fairly important private educational sector. In the U.S.A. and Japan, for example, private educational expenditure amounted to 20% of the total (public and private) educational expenditure in 1970. If the private expenditure is included the share of GNP devoted to education in the U.S.A. (7.4%) becomes more comparable to the Swedish expenditure.50)

Enrolments

To get an overview of the amount of education received in various age-groups around 1970 in different countries table 1 can be studied. The average number of years of full-time education received in the 3-6 age group is very low in Sweden. To a certain extent this is due to the fact that compulsory schooling only begins at 7 in Sweden. Among countries similar to Sweden in this respect only the U.S.A. gives more education to the 3-6 year old children.51) As for the 7-14 age-group most countries (including Sweden) have an average of 7.8 - 8.0 years. If we look at the next age-group (15-18) only Canada, Norway and the U.S.A. have a higher average than Sweden. Japan is on the same level as Sweden (2.7 years). The average in the oldest age-group is somewhat higher in Finland, Norway, the U.S.A. and Yugoslavia than in Sweden. Despite the high Swedish ranking for older age-groups the overall average for the 3-25 age group is higher in several countries (e.g. Belgium, France, Japan, the Netherlands and the U.S.A.). Canada and Ireland have about the same average as Sweden.

In graph 8 the enrolment rates in Sweden and the U.S.A. can be compared both around 1950 and 1970. The expansion in enrolment rates has continued in the U.S.A. after 1950 but it has been much quicker in Sweden with the result that Sweden is much closer to the U.S.A. around 1970 than 1950.

In the figures above different types and levels of education were not distinguished. It can be useful to report some figures of that kind too. If we start with higher education (both university and non-university type) the enrolment ratio in Sweden was 23% in 1970. This figure is surpassed only by Canada (39%) and the U.S.A. (43%).52) A corresponding figure for the Soviet Union can be added. In 1968 the enrolment ratio for higher education (day, evening and correspondence education) was 30%.53)

Table 1. Average years of full-time education received per head of population in age-groups cited and average total. Expectation of education around 1970

		Average years of education received between ages:				Total
		3-6	7-14	15-18	19-25	3-25
Australia	1971	2.2	8.0	2.0	n.a.	n.a.
Austria	1969	1.8[a]	8.0	1.3	0.5	11.6
Belgium	1966	3.8	7.8	2.2	0.7[b]	14.5
Canada	1968	1.3	7.8	2.9	0.8	12.8
Denmark	1970	0.6	7.8	2.1	0.9	11.4
Finland	1967	0.3	7.9	1.9	1.3[c]	11.4
France	1970	3.5	7.9	2.2	0.7	14.3
Germany	1969	1.8[a]	7.8	1.2[c]	0.6	11.4
Greece	1968	1.1	7.0	1.6	n.a.	n.a.
Ireland	1971	2.6	7.8	1.9	0.5	12.8
Italy	1966	1.5	7.0	1.2	0.4[b]	10.1
Japan	1970	(2.0)	(8.0)	(2.7)	(0.5)	(13.2)
Luxembourg	1970	2.6	7.5	1.5	0.6	12.2
Netherlands	1970	2.8	7.9	2.1	0.7	13.5
Norway	1970	(0.2)	7.9	2.8[d]	1.2[b]	12.1
Portugal	1970	(0.3)	6.5	0.9	0.6	8.3
Spain	1970	2.2	7.2	1.1	0.7	11.2
Sweden	1972	1.2[a]	7.9	2.7	1.1	12.9
Switzerland	1970	(1.6)	7.7	2.5	0.3	12.1
Turkey	1968	n.a.	6.1	(0.8)	n.a	n.a.
U.K.	1970	2.3	8.0	1.6	0.4[b]	12.3
U.S.A.	1970	2.3	7.9	3.3[e]	1.3[e]	14.8
Yugoslavia	1968	(0.5)	6.9	1.5	1.3	10.2

N.B. In many cases education in special schools for handicapped children is excluded.

a	1970
b	19-24
c	Excludes part-time compulsory vocational schools
d	Includes a very small number of part-time pupils
e	Full and part-time
( )	Estimates

Source: OECD Educational Statistics Yearbook Volume 1, Paris 1974 p 14

For the university part of the higher educational system a few additional observations can be made:

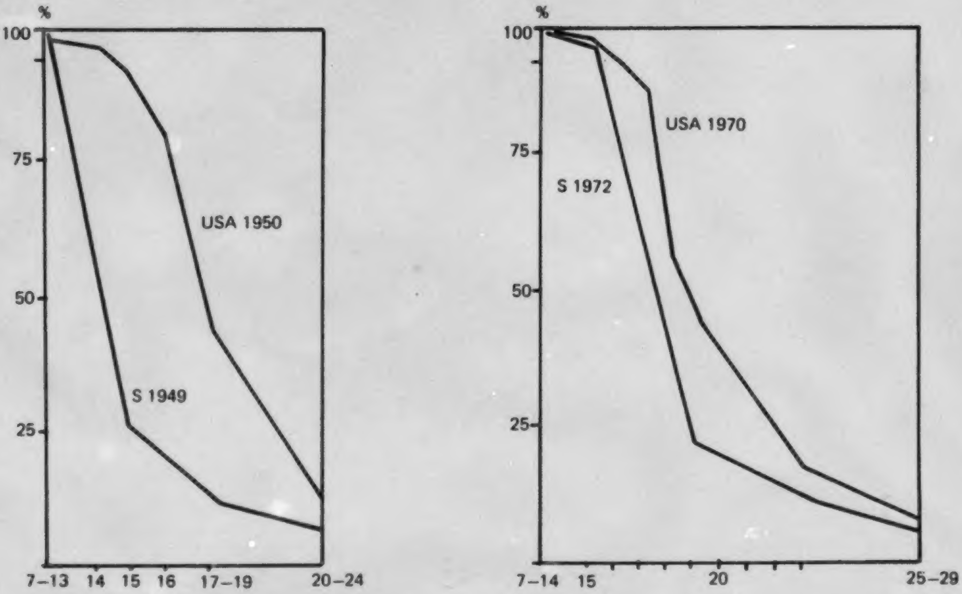
In 1970, 37% of the university students in Sweden were women. The same year the proportion of women was higher in Finland (48%), France (45%), Italy (38%), Portugal (45%), Yugoslavia (38%) and the U.S.A. (41%).54)

The number of graduates obtaining a first degree in university type of education as a percentage of a single-year age-group was 11% in Sweden, a figure that not surprisingly is surpassed in Canada (19%) and the U.S.A. (27%). Japan is at about the same level as Sweden. These figures are from 1970.[54)]

Enrolments and future number of graduates are influenced by the development of new entrants. The number of new entrants has declined in Sweden since 1969 and enrolment gone done as we earlier noted. Only for certain countries do we have information about the recent development of new entrants to university type of higher education. The number of new entrants has increased in Australia, Belgium, Canada, Denmark and Finland up to 1972. For Ireland and the U.S.A. there has been a decline in the number of new entrants from 1971 and 1970 respectively.[55)]

Even though the number of new entrants started to decrease in 1969 in Sweden it was not low compared to other countries in 1970 (the last year for which complete information about the OECD countries has been published). The annual entry to university type of higher education as a percentage of the relevant age-group was 31% in Sweden in that year. This figure is actually the highest one noted for all OECD countries that year.[56)]

Graph 8. Enrolment rates



Sources: SOU 1955:34 p 95
Educational Statistics Yearbook. Volume I op cit pp 27–30 (with corrections introduced according to Volume II pp 361, 412)

New entrants as a percentage of the relevant age-group to non-university type of higher education is on the contrary not especially high in Sweden. Belgium, Canada, Norway and the U.S.A. have higher percentages. The relative number of new entrants to the whole sector of higher education is, however, only higher in the U.S.A.56) x)

The expansion of secondary education means that a higher proportion of a cohort completes such education. Among European OECD countries Sweden has the highest rate of transfer to university type of education among holders of general academic secondary school leaving certificates.57) This might imply that in Sweden alternatives in terms of work and other types of education are relatively less abundant (see above) and attractive and the entry to university less restricted. For most of the European OECD countries (including Sweden) these transfer rates have declined 1965-70.57) Competition on the labour market and at non-university types of higher education between students who finished secondary school on lines normally preparing for university and students finishing more vocationally oriented lines may thus have increased.

Population and education

Graph 9 depicts the education in the population in different countries around 1970. The first part of the graph deals with the population aged 25-64. Sweden is found to the left and the other countries follow according to the average education in their populations. Among the OECD countries having a lower average than Sweden only Portugal has been included. Despite the fact that Sweden ranked high in most educational aspects we have treated so far (with the exception of pre-primary education which is not included here) the average education in the Swedish population is rather low. The present output of the educational system is higher in Sweden than in the U.K., Belgium, Denmark and Germany but apparently the educational expansion started earlier in these countries with the result that their stocks of educated are still higher.

The graph also shows the composition of the average consisting of years in primary, secondary and higher education. It is especially at the secondary level that Sweden, so to speak, lacks education. Higher education plays, in this perspective, a minor role in most countries. The normal pattern is that education at the primary level stands for a larger part of the average than education at the secondary level. This is, however, not the case in Germany and Denmark where both these levels have about the same share of the average. This means

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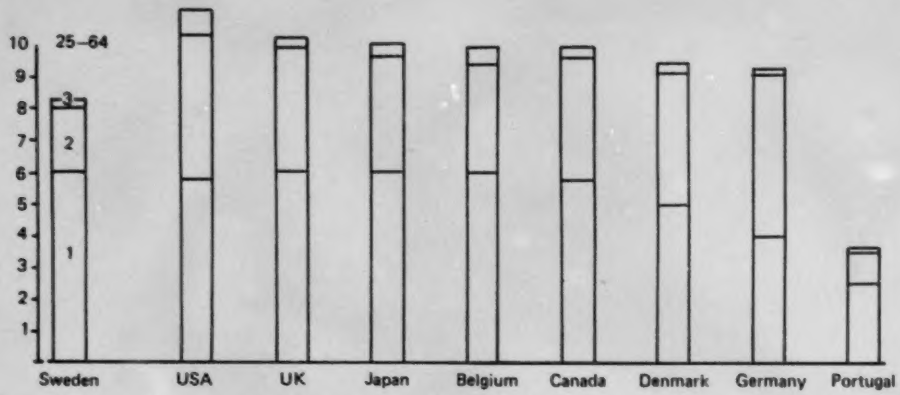
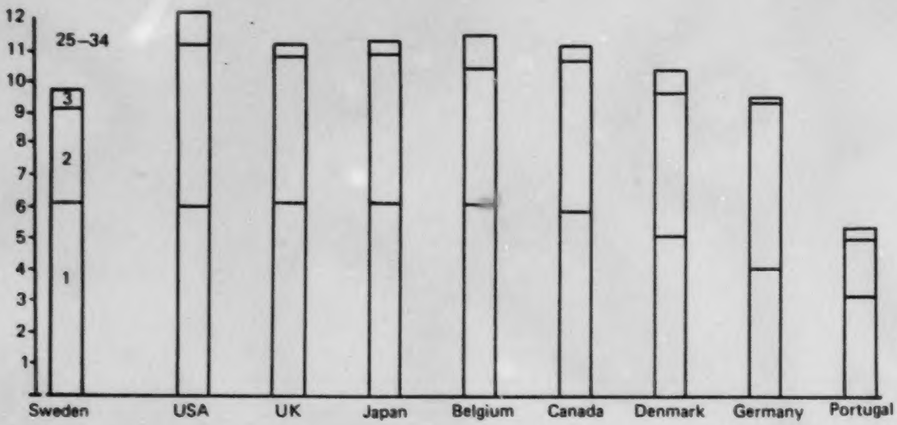
x) The figure for Canada is lower than that for Sweden but probably heavily underestimated.

Graph 9. Average years of formal educational experience of total population aged 25–64, 25–34 and 65–74

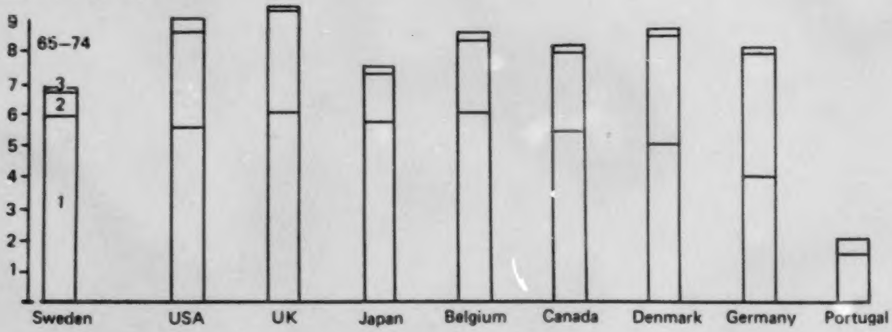
1 = primary education
2 = secondary education
3 = higher education



Graph 9. (page 2)



Graph 9. (page 3)



Source: Educational Statistics Yearbook. Volume I op cit pp 57–59 that in these countries both educational levels must have had a more parallel expansion.

If we look at the 65-74 age-group, not the U.S.A. but the U.K., followed by the U.S.A., Denmark, Belgium, Canada and Germany, has the highest educational average. Here too it is especially secondary education that is low in Sweden while primary education gives a higher average than in Denmark and Germany. For the 25-34 age-group the rank order is again reversed. The U.S.A. has the highest average followed by Belgium, Japan, the U.K., Canada and Denmark. It is this expansion of education in younger age-groups that permits the U.S.A. to surpass the U.K. in the average for the 25-64 age-group. Sweden has a higher average than Germany for the 25-34 age-group but not high enough to have a total average above that of Germany.

The difference in these countries between the 25-34 and 65-74 age-groups mainly consists of secondary education that has expanded.

In most OECD countries the education of women is shorter than that of men. Two exceptions can, however, be mentioned: Ireland and the Netherlands. In the former country the average education of women is higher and in the latter the same as that of men.58)

Considering the present volume of educational activities the OECD made projections for the development of the average education in the populations. Sweden is expected to have a faster growth in the average than countries with a higher average at present so the differences can be expected to diminish.58)

3 Education and allocation

Given the organisational and quantitative development of the educational system just described what have been the effects of this upon allocation, growth and distribution of income and welfare? This section and the next one are intended to bring forth empirical material that can tell us something about what has happened in these areas. The interpretation of the data will be discussed in the last section. While section 4 contains data mainly relevant from a distributional point of view this section is devoted to allocative aspects, that is how different educational groups have been utilised on the labour market. The changing position of different educational groups on the labour market will be described. Some data are also provided that may be useful when a link between education and economic growth at an aggregate level is sought. First, however, a brief summary of the general development of the Swedish labour market is given.

3.1 Some generalities about the Swedish labour market

Labour market organisations[59)]

The Swedish labour market is highly organised. Three labour unions - the Swedish Confederation of Trade Unions (LO), the Swedish Central Organisation of Salaried Employees (TCO) and the Swedish Confederation of Professional Associations (SACO) - dominate the picture. Broadly speaking the LO members have only compulsory education while the SACO members are typically university graduates and TCO members have upper secondary education. Centralised wage negotiations between these organisations and the employers organisations (private and public) take place at certain intervals.

For the employees of the public sector salaries are in principal fixed for every position and negotiations center around these salary schedules. Besides the general wage increases obtained in negotiations individual salary increases occurs as a result of automatic individual salary increases and individual advancement from one position to another. New positions are created by the public authorities not the individual agencies themselves.

The private sector does not have these rigid salary schedules. Besides the centrally and locally negotiated wage increases there is the so called "wage drift", wage increases accorded by the employer on top of those agreed upon in negociations with the trade unions. Especially the LO has pursued a wage policy aiming at decreased wage differentials.

General features

As in many other countries the labour market has been subject to considerable structural changes in the 20th century. The agricultural sector has continuously lost ground employing a smaller and smaller share of the total labour force while the industrial sector has expanded. After 1965, however, the expansion of the industrial sector stopped. Instead the service sector (especially public administration and services) increased. In 1973 56% of the employed labour force worked in the service sector. The public sector stood at half of this employment.60)

Parallel to, and to a certain extent due to, this change in the sector composition of employment there has been a structural change in occupations away from blue collar and towards white collar jobs. While in 1960 30% of the labour force was engaged in white collar jobs (professional, technical and related occupations, administrative occupations, clerical occupations and sales occupations) the share in 1973 had risen to 45%. Among these groups of white collar occupations the one covering professional, technical and related workers has had the most rapid growth rate during the period 1960-73. The administrative occupations expanded rapidly during the 60s but their share of the labour force declined in the early 70s. Instead, the clerical occupations expanded then.61)

The size of unemployment has been much lower after the Second World War than before. However, during the 60s and early 70s a tendency towards increased unemployment rates has been observed.62) During the 60s the relation between unemployment (u) and vacancy (v) rates also seems to have shifted outwards as illustrated in graph 10. This shift seems to have had less to do with changes in the composition of the labour force (by age-groups or sex) than with accelerating business concentration, productivity increases and factor substitution leading to an increased flow of unemployed and a decreased flow of vacancies. The length of the unemployment period thus increased.62)

This pattern seems to have continued during 1975 (a recession year) as far as decreasing vacancies and longer periods of unemployment are concerned. However, the inflow to the stock of unemployed did not increase with the result that the rate of unemployment actually decreased.

Labour seems gradually to have become much more of a fixed factor of production in Sweden. The tendency towards a decreased flow of vacancies, lower turnover and a growing importance of internal labour markets at the expense of external has been explained by: 63)

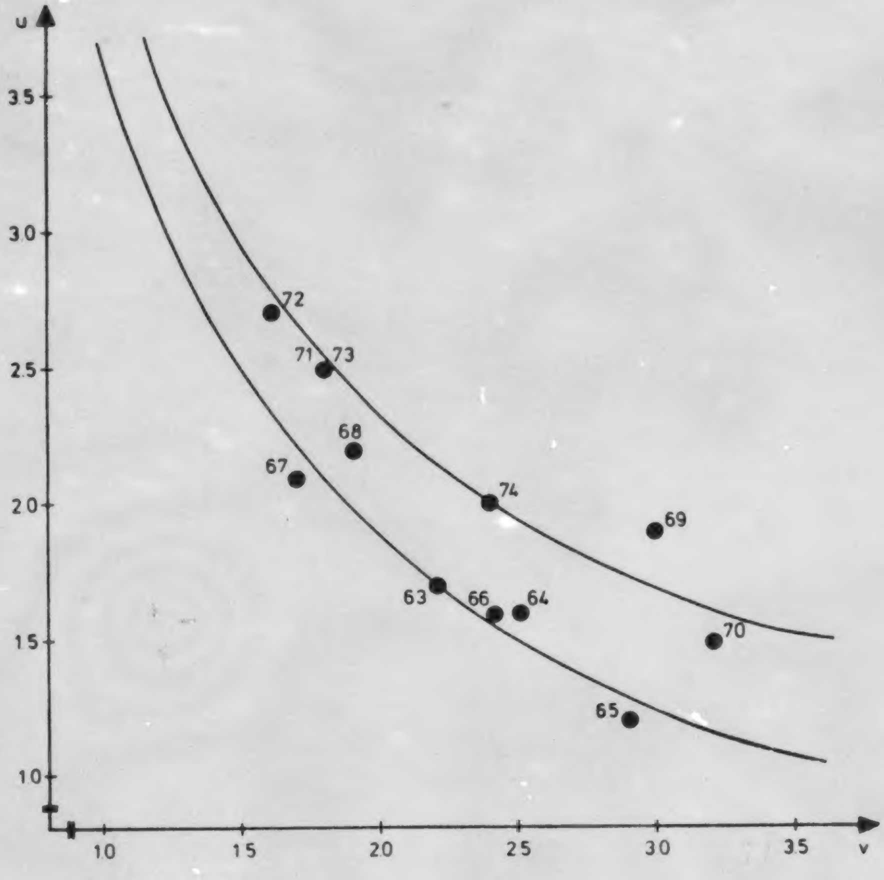
- the growing size of firms

- increased on-the-job training
- decreased incentives for mobility (progressive taxes in combination with smaller wage differentials)
- legislation to increase job security.

These trends, creating an increased security for large groups may lead to problems for certain marginal groups and give rise to a dual labour market. Those who are unemployed experience greater difficulties in finding jobs and have less to choose from.

Internal labour markets with career ladders can create problems especially for unemployed older people and those young who do

Graph 10. Unemployment (u) vacancy (v) rates in Sweden 1963–1974



Source: B Holmlund: Arbetslöshet och lönebildning Umeå 1976 p 82 not manage to get in. This may, however, be more of a problem for white collar employees than for workers.64)

As for regional differences, persistant, large, and relatively constant, differences in the rate of unemployment have been observed. Northern Sweden shows the highest rate of unemployment and an u-v curve furthest from the origin.65)

Differences also exist between demographic groups. In many respects, however, the situation on the labour market of women and the young are similar. Compared with middle aged men both groups:66)

- have higher rates of unemployment
- do more part-time work
- dominate relatively speaking the groups that look for jobs on the external labour market
- are more mobile in and out of the labour force.

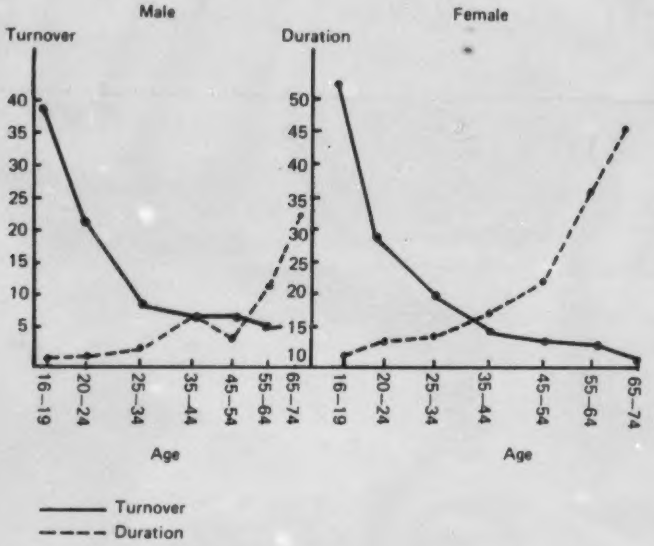
However, the development by age of turnover or risks of becoming unemployed and unemployment duration once unemployed is the same for men and women as can be seen from graph 11. Unemployment for the young is of short duration but occurs more frequently. The opposite is true for the older, both men and women. Studying different groups of unemployed it has been found that those who had lost their jobs had a longer unemployment duration than those who entered the labour market or were unemployed for other reasons. (These three groups accounted for approximately 1/3 each of the unemployed).66)

It can further be noted that the unemployment rates of the younger age groups are much more sensitive towards changes in the level of economic activity. Their rate of unemployment rises quickly and only slowly goes down as the economy recovers.67)

Another demographic group that in many countries experience a special labour market situation is immigrants. In Sweden they make up approximately 5% of those gainfully employed. They seem:68)

- to be overrepresented among workers and underrepresented among employees and business owners
- to a higher, and increasing, degree work in industry
- to be more mobile
- to be underrepresented in both low and high income classes
- as an average to have the same rate of unemployment as Swedes. However, their rate of unemployment fluctuates much more strongly. When unemployment goes up it rises much more quickly for immigrants. To a certain extent they and the young can be said to be marginal groups with less secure jobs
- to a lower degree have a job that they think themselves best suited for (69% versus 97% for Swedes).

Graph 11. Turnover and unemployment duration 1972



Source: N S Barrett, B Södersten: Uemployment Flows, Welfare and Labor Market Efficiency. The Swedish Journal of Economics 1975:3

However, they are not overrepresented among occupational groups that are characterised by especially low wages. Their first occupation may often have been less attractive but later they have been able to change.67)

Judging from these labour market data women, the young and immigrants seem to a certain extent to experience a different kind of labour market situation than middle-aged men. We will later present some data to indicate what the role of education may be in these respects.

However, to get a further perspective on the Swedish labour market let us first make a few comparisons with the U.S.A.69) 1965-70 the Swedish labour market was characterised by lower rates of unemployment than the American one. However, there seems to be less mobility on the Swedish labour market. Turnover is lower. Swedish workers change jobs less frequently. Fewer are unemployed but those who are remain unemployed for a longer period of time. Moreover, in the U.S.A. the relationship between turnover and unemployment duration is not the same as in Sweden. Both turnover and duration decrease by age. The low mobility on the Swedish labour market seems to have further decreased in the early 70s.

3.2 Labour market utilisation of different educational groups

Labour force participation[70]

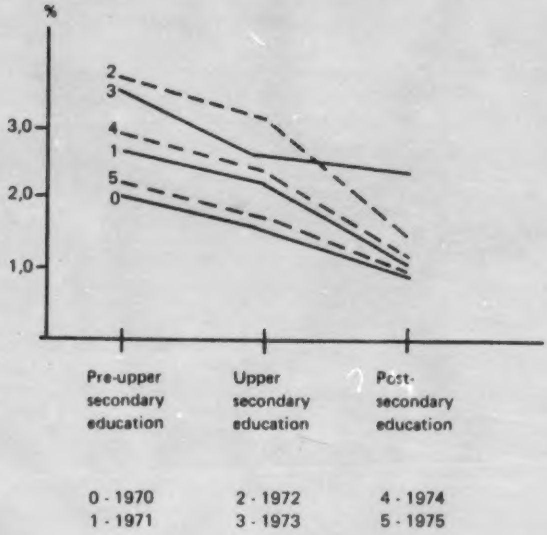
The labour force surveys during the 70s show that the rate of labour force participation varies with education. This is especially true for women. For women with a university education even the characteristic dip in the rate of labour force participation in the ages 25-35 disappears which is not true in for example the U.S.A. The proportion working at least 20 hours a week is also higher for higher educational groups.

Unemployment

During the 70s the rate of unemployment has also been found to vary with education being lower at higher educational levels (graph 12). When business activities go down the rate of unemployment increases in all educational groups but apparently more so for the lower educational groups. These differences in the rate of unemployment among educational groups remained when men and women, different age-groups and regions were studied separately. Among the regions distinguished (woodland, urban and other districts) the woodland districts had the highest unemployment rate but it is interesting to note that the greatest difference among educational groups was found in the urban districts.[71] As for occupations, the available data (1970, 71 and 72) show that in white collar jobs the rate of unemployment declines somewhat at higher educational levels while this is not true for manual jobs. Above all, the differences in unemployment rate between these two types of jobs are important.[71] Finally, the duration of unemployment was found to be somewhat longer for lower educational groups in 1970, 71 and 72 but not in 1973 and 74.[71]

We cannot say whether these differences in rates of unemployment among educational groups were greater or smaller earlier. They might have been more pronounced. The labour market situation of persons just graduating from university can be followed back to 1968 but there are no other educational groups to compare with. Their rate of unemployment increased 1968-72 from 5 to 10%. By 1974 it had again fallen back to 5%. At the same time the percentage share of the graduates who continued studying diminished from 32 to 17% and the proportion working increased from 59 to 73%. Unfortunately these figures do not tell us much about the long run changes of unemployment among highly educated persons neither in absolute terms nor in relation to other educational groups. This information, however, points to another interesting tendency. When the labour market situation deteriorated the propensity to go out looking for job increased. Similar tendencies have been noted for the upper secondary school too.[72]

Graph 12. Rate of unemployment



Sources: See foot-note 71

For the early 70s we can make some comparisons of unemployment rates among persons leaving the educational system at different educational levels. As the surveys are few in number and not completely comparable we will, however, only note a few tentative conclusions. 1971-73 those entering the labour market having completed comprehensive school seem to have had more difficulties (as judged by the rate of unemployment) than those leaving the upper secondary school. At times during this period, however, the situation for those graduating from universities and specialised university colleges can have been worse than for the two former groups. However, in 1973 and 1974 those leaving the comprehensive school again seem to have been in the worst position.72)

A special investigation67) into the unemployment situation in 1975 of young persons with only compulsory, some upper secondary and completed upper secondary education showed that there were some differences between these educational groups. For those with only compulsory education the risk of becoming unemployed was greater and the unemployment periods longer. The problem for them may not so much be to find a job as to find a stable job. The number of only temporary jobs increased over time among those at lower educational levels. Also, the risk of becoming unemployed increased with longer labour market experience while it decreased for those with upper secondary education. It looks as if it was more difficult for those at lower educational levels to get a qualifying first job leading to a stable labour market situation. Some may get caught in a vicious circle of temporary jobs and unemployment. For those with upper secondary education the problems encountered on the labour market seem more to have been related to how to find a job in accordance with educational specialisation. It can be added that among groups with different school marks at a given educational level no differences were observed.

The unemployment problem of persons with different educational specialisation can further be illustrated. For the graduates from the different parts of the upper secondary school the recent situation can be summarised in the following way:73)

1. 3- and 4-year lines. Among boys those with a liberal arts and social sciences orientation have had the highest rate of unemployment while among girls those with a technical, liberal arts and natural sciences orientation have had the highest rate of unemployment. The differences are, however, small.
2. 2-year "theoretical" lines (former specialised professional school). The annual variations are considerable. Among boys those with a technical orientation had the highest rate of unemployment in 1970 and the lowest in 1972. For girls it was the opposite.
3. Lines corresponding to the former vocational school. In 1970/71 the unemployment rate was above the average for those leaving the vocational school with a technical, clerical and service orientation.

Looking at higher education74) those graduating from the liberal arts and natural sciences faculties have normally had the highest unemployment rates during the early 70s. The relative position of graduates with a social sciences orientation deteriorated during this period. Among the groups surveyed low rates of unemployment were generally found for graduate engineers, Bachelors of law and of economics and business administration, journalists and social workers. Graduates, however, not only work or are unemployed. Some of them continue studying. If we look at the percentage distribution of graduates over these different types of activities75) the position of graduates from the faculties of liberal arts and natural sciences does not look worse than for many other groups during the period 1967/68-1972/73. The percentage among them being unemployed was neither at the beginning nor at the end of this period higher than the average. It is true that among them a higher percentage continue studying but this percentage has declined at the same time as the percentage working has increased. Graduates with a social sciences orientation are instead a group with a percentage unemployed among themselves above the average both at the beginning and at the end of the period. However, even for them the percentage that continued studying has decreased and the percentage working increased.

Recalling the development of the educational system the expansion especially at the post-secondary level was heavily inclined towards courses with a liberal arts, social and economics orientation. In spite of this the difficulties encountered while entering the labour market do not seem to have been greater for these groups than for natural scientists. The expansion of technical and natural sciences courses was slow and probably slower than politicians had desired. However, the labour market entry of these groups has not been without problems. At higher educational levels this is especially true for the natural scientists. At the university level the expansion of social sciences graduates, that at least once was regarded as desirable, has brought about a deterioration in labour market prospects for these groups and in certain respects they have had a more difficult time than other groups. Taking these facts into consideration it might be asked whether another composition of the educational output (more technicians and natural scientists) would really have resulted in lower average unemployment rates and if the differences in unemployment rates are such as to warrant a higher degree of steering.

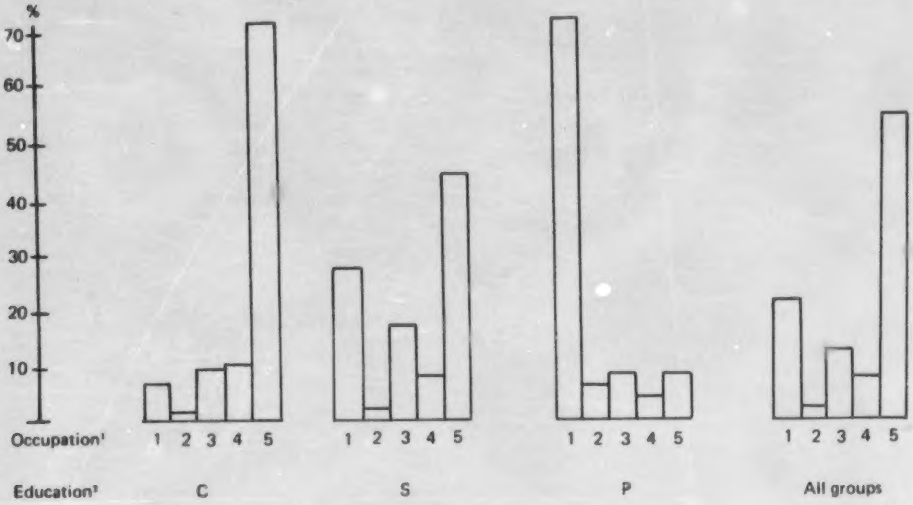
Occupations

The distribution of educational groups over occupations in 1975 is given in graph 13. The graph contains two types of distributions. One, which will be called the occupational profile of an educational group, indicates the percentage distribution of an educational group over occupations. The other is the distribution of employees with a certain occupation over educational levels. In the latter case we will talk about the density of different educational groups within an occupation. These two distributions do not necessarily develop in the same way. In a period of increasing output of for example university graduates their density within certain occupations may increase at the same time as their occupational profile shifts towards other occupations.

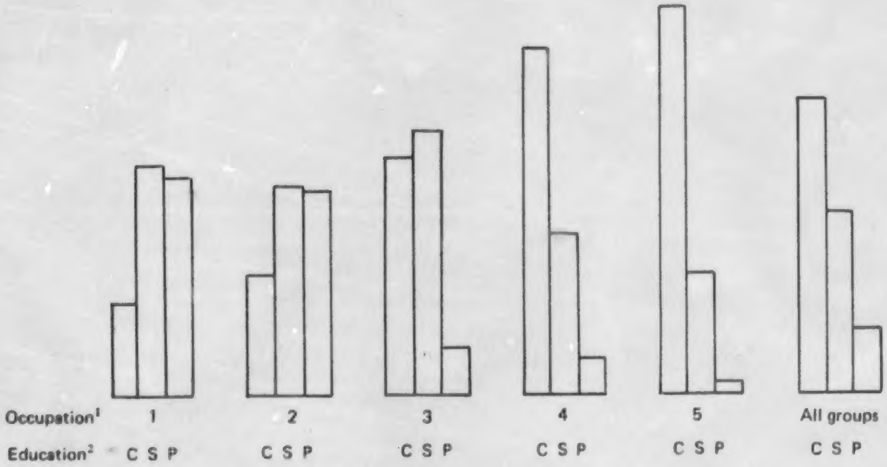
Four groups of white collar occupations are distinguished, professional, technical and related occupations, administrative occupations, clerical occupations and sales occupations, while all the manual occupations form one group. The occupational profiles of the different educational groups are very distinct.

Graph 13. Distribution of educational groups in the labour force over occupations, 1975

Occupational profiles by educational groups



Educational densities by occupations



[1] 1 = professional, technical and related, 2 = administrative, 3 = clerical, 4 = sales, 5 = manual

[2] C = comprehensive (or similar) i.e. pre-upper secondary, S = upper secondary, P = post-secondary

Source: AMS Meddelande från utredningsenheten 1975:24

To approximately the same extent as persons with post-secondary education are found in the first white collar occupational category (74%) the individuals with only comprehensive or similar schooling end in manual occupations (73%). Individuals at the upper secondary educational level are more evenly distributed over occupations.

Looking at the densities the pattern of distribution is not quite the same. Persons with only comprehensive (or similar) schooling predominate in the manual occupations but in none of the white collar occupational groups not even in the first one do persons with post-secondary education form a majority. In occupational groups 1 and 2 which have the highest density of post-secondary educated people (about 40%) there is nevertheless about 40% and 20% of individuals with upper secondary and pre-upper secondary education respectively. The fact that the selectivity to these occupations is not so high can be interpreted both positively and negatively. Positively, to the extent that capable persons lacking formal school merits are not discriminated against and negatively, to the extent that persons with higher education are more productive in these occupations, so much more productive that it would make up for the educational costs. Of course, also the fact that the profiles are so one-sided can be interpreted both positively and negatively from an allocational point of view. Positively, if the educational system adequately fullfills a sorting function and supplies education quantitatively and qualitatively adapted to occupational demand. Negatively, if persons with a certain education costumarily go to certain occupations whether they or their education are suited for them or not.

By comparing the situation in 1975 with that in 1970 we can get a certain idea about the way in which the labour market reacted upon the educational expansion during that period. The changes 1970-75 are rather small. For the occupational profiles we can, however, note that the occupational distribution of the total labour force over occupations shifted from the manual towards the white collar occupations. Within the latter the first, professional, technical and related, and the third, clerical, occupational groups expanded while the administrative and sales occupations decreased in importance. In spite of this the occupational profile of the highest educational group became a little less concentrated to the white collar occupations (92% against 94%) and the percentage share in manual occupations accordingly increased from 6% to 8%. Within the white collar occupations there was a shift of this educational group out of not only administrative and sales occupations but also out of professional, technical and related occupations. The result is that the occupational profile of the highly educated looked less one-sided in 1975 than in 1970. While 85% went to the first two occupational groups in 1970 this figure was 80% in 1975. The group at the upper secondary educational level has also seen its profile shift away from white collar occupations towards manual occupations (45% worked in the latter occupations in 1975 against 41% in 1970). For those at the lowest educational level the changes of the profile are negligable. It might have been expected that when the highly educated start spreading out over more occupations and perhaps must be content with simpler jobs they would at the same time push out the groups with lower educations from the occupations where the highly educated used to be concentrated. This might have happened to those with upper secondary education as their profile shifted away from the white collar jobs. The chances of ending up in white collar jobs have, however, not declined for the lowest educational group at least not up to 1975.

Considering now changes in density 1970-75 the proportion of the labour force at secondary and post-secondary levels have of course increased. The density of highly educated increased in all occupational groups. In absolute numbers the increase was greatest in occupational group 1, 2 and 3. In manual occupations it only increased from 1 to 2%. In spite of the fact that the share of white collar occupations in the economy as a whole increased and the density of highly educated rose in these occupations their profile shifted away from these occupations. For those at the upper secondary educational level the densities declined in the first two occupational groups and increased in the three others. Especially in manual occupations there was an important increase (from 19 to 27%). The density of people with only comprehensive (or similar) education declined in all occupational groups. While they constituted 25 and 80% of occupational group 1 and 5 in 1970 these figures were 17 and 72% in 1975.

It is difficult to say whether these changes in profiles and density should be given a positive or negative interpretation. The fact that the highly educated seem to end up a little less automatically in the highest groups of white collar occupations could be greeted as a sign of a more efficient and less status-oriented utilisation of this educational category. In such a case the educational system would not be regarded as a sufficient sorting devise. If it were, the development would indicate that the educational output was quantitatively too large which implies unnecessary costs. Moreover, if the educational content at different educational levels is adapted to certain occupations it could mean that an increasing proportion of the labour force gets an education not suited for the occupational tasks. The increased density of highly educated in white collar occupations could in the same way either signify increased discrimination and lower productivity or profitable educational expansion and increased productivity. We will in the last section come back to these problems of interpretation of data that have been hinted at here.

The increased density of highly educated in all occupations, especially in the white collar ones, shows that the inflated number of highly educated could not be absorbed by the economy only through the expansion of the occupational categories to which the highly educated were concentrated. At this level of aggregation our calculations show that, after having taken into consideration the general expansion of the labour market, changes in the occupational structure (1970-74) was responsible for about 40% of the absorption of the increase in the highly educated. This means that 60% of the absorption was due to increased density of the highly educated within the five occupational groups.

To make a further comparison over a longer period of time we can use the information in the 1960 census about the occupational distribution of graduates from universities and specialised university colleges. They can be compared with those at educational level 7 (post-secondary education more than 2 years) in 1975. These groups are, however, not exactly comparable. The educational definition used in 1975 is somewhat wider and moreover it refers to those in the labour force while in 1960 only those actually employed were included. As for the occupational profile of these "university" graduates the changes 1960-75 are rather small. Approximately the same proportion goes to the first occupational group in 1975 as in 1960 (82 against 81%). The high percentage in 1975 is to a certain extent explained by the fact that more people with teacher training are included in 1975. In 1975 a smaller percentage have administrative work (7 against 13%). The share in clerical occupations (5 against 1%) but even in manual occupations (2 against 1%) has increased. Even though the concentration to the traditional occupations thus still is very high there has been certain changes for those who cannot go to these occupations.[76)]

The development 1960-75 for the "university" graduates resembles that of the whole group with post-secondary education 1970-75 in that those who do not go to the first occupational group more often than earlier end in clerical and manual occupations. The "university" graduates but not the whole group with post-secondary education have managed to keep the same percentage in the first occupational group. Information concerning those who have recently graduated suggests, however, that this type of changes can be expected even for the former.[77)]

The fact that in spite of the expansion of "university" graduates the same high percentage has gone to the first occupational group is combined with a considerably increased density of "university" graduates in this occupational category between 1960-75 (14% against 25%). Even in the administrative occupations the density of university graduates grew rapidly, from 13% to 23%. In the other white collar occupations, clerical and sales occupations, the density of university graduates increased though they are still low in 1975 (3 and 2% respectively). In the manual occupations the density remained below 1% during this period.[76)] These changes in density 1960-75 agree with those earlier noted for the whole group of persons with post-secondary education 1970-75.

This means that during the 60s as well as the early 70s the educational output was much greater than changes in the occupational structure at constant densities would have demanded. Bearing this in mind the absorption process may appear surprisingly smooth as judged by the unemployment rates reported above.

To conclude this section on occupations let us add some information from a few special investigations.

The first concerns the absorption of individuals with higher education in manufacturing, construction and wholesale trade and the possibilities to relate it to the simultaneous increases in productivity.[78) 1959-70 the number of employees with certain types of higher education increased by 100%. Among the four main occupational groups distinguished (technical, commercial, supervisory and administrative-clerical) the expansion of the technical and commercial occupations absorbed 74% of the increase in highly skilled people. Looking closer into the technical occupations it was found that among the subgroups of the technical occupations those associated with increasing the efficiency in the utilisation of existing means of production expanded most. The number working in development work, while remaining at a high level, did not increase very much.

The association between education and productivity would thus in this case go via the control function. Whether this was so and whether there would exist any alternative such as implying more on-the-job-training and more of a workers' control of the working process can not be evaluated from this study.

Recently a number of reports,79) causing alarm and surprise, have shown that rather few people end up in occupations that could be more or less closely associated with the students' field of study: about 20% for certain types of labour market training courses and 43% for certain vocational courses at the universities. However, among the first degree university graduates about 70% reported that three years after graduating they held relatively qualifying jobs within a field approximately corresponding to their educational specialisation. The figures varied between 50-90% for different types of education.

Finally some findings can be reported from a retrospective longitudinal study of people in leading positions in private firms and the consumers' co-operative association.80) The career development in the co-operative sector was quite different from that in the private sector. Those in the co-operative sector followed more of a deliberately planned promotional sequence starting at rather low levels, the co-operative association itself supplying necessary educational courses. Among those working for private firms the starting level seems to have been fairly closely related to educational level and the starting type of job to educational orientation.

Those with a lower educational level also more often started in smaller firms. With the promotion from one occupational level to another changes of firm seem to have diminished but changes of type of job seem to have been augmented.

Branches

The distribution of educational groups over branches of economic activity in 1975 is given in graph 14. The same two types of distributions as for occupations are shown. Four groups of branches are distinguished. All goods producing branches form one group (1). As for the rest of the economy (i.e. the service sector) the commerce and transport branches (2), the banking, insurance, real estate and commissonary branches (3) and the public administration and other branches (4) are reported separately.

The branch profiles of the three educational groups are very different. Among those with comprehensive, or similar, education 51% go to the goods producing branches and 21% to public administration and other services branches. The corresponding figures for those at the post-secondary educational level are 14% and 68%.

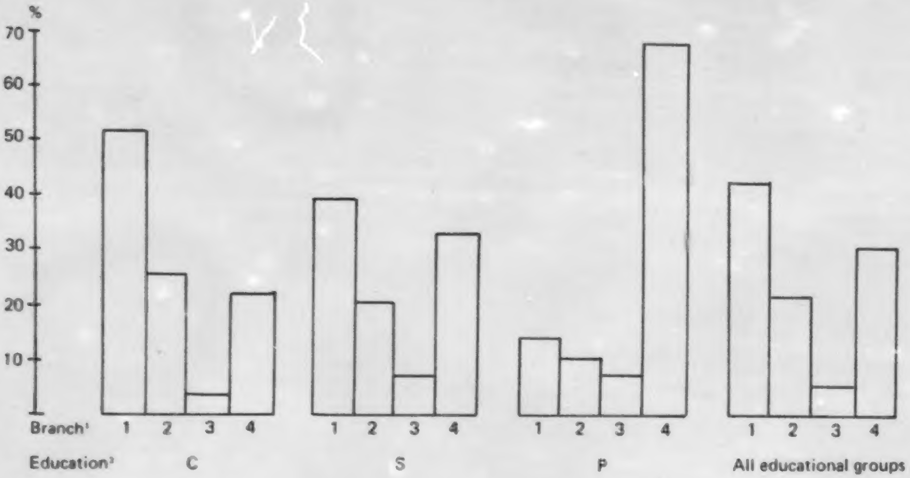
As for densities the group with comprehensive schooling dominates not only the goods producing branches (65%) but also the commerce and transport branches (62%). The density of highly educated is 19 and 26% in branch group 3 and 4 of the service sector.

Compared with 1974 the branch profile of the total labour force has remained about the same. This is also true for the branch profile of the lowest educational group. For those at the secondary and post-secondary educational levels there have been compensating reshufflings. Those with secondary education went a little more often to the goods producing branches in 1975 than in 1974 and those with post-secondary education a little less often in 1975 than in 1974.76)

Between 1974 and 1975 the density of the highly educated increased in the total economy. As the structure of employment by branches did not change increased densities of highly educated within branches occurred. Increased densities of highly educated was brought about in all 3 branch groups of the service sector especially in group 3 and 4. The density of highly educated did, however, not increase in the goods producing branches. The proportion of individuals with upper secondary education also increased in the total labour force but this only led to increased densities in the goods producing branches and in the commerce and transport branches. The density of the lowest educational group declined in all branch groups.

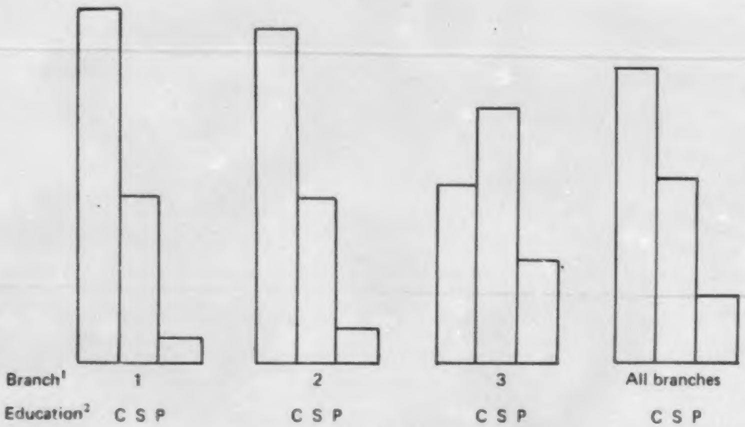
Graph 14. Distribution of educational groups in the labour force over branches of economic activity, 1975

Branch profiles by educational groups



Graph 14. (page 2)

Educational densities by branches



[1] 1 = goods producing branches, 2 = commerce, restaurants, hotels, transport and communication, 3 = banking, insurance, real estate and commissonary activities, 4 = public administration and other services

[2] C = comprehensive (or similar) i.e. pre-upper secondary, S = upper secondary, P = post-secondary

Source: AMS Meddelande från utredningsenheten 1975:25

To get more of a historical perspective on changes in the distribution of educational groups over branches we can report some results from an investigation of higher education and changes in branch structure between 1960-70.[78] For the graduates from universities and specialised university colleges the branch profile has shifted in the direction of the service sector away from the goods producing branches during this period. Whereas the former employed 79% of the university graduates in 1960 the percentage share had increased to 84% by 1970. The service sector received as much as 94% of the increase in graduates. The public sector which mainly belongs to the service sector traditionally employs a great number of graduates. The proportion of them working for the public sector has also increased (from 55% to 59% during the period studied). Among the goods producing branches the graduates are concentrated in the manufacturing industries (13% in 1970). The manufacturing industry was the only goods producing branch that took care of more than a negligable amount of the increased number of graduates.

The density of university graduates increased in most branches but rather slowly. Making the same type of calculations as for occupations but based on 15 branches it turns out that increased density of university graduates within branches can only "explain" 30% of the absorption of the increased supply of graduates. The rest, 70%, is related to the change in the structure of branches of economic activity. This means broadly speaking that the expansion of the branches making up the service sector could at given densities absorb most of the expansion of the university educatec Thus the educational expansion seems to have resulted in little disruption of the traditional density pattern within branches and more disruption within occupations.

Concerning the density of graduates from universities and specialised university colleges in manufacturing industry we can add a few figures from an earlier period. It has been estimated[81] that in 1930 it was 0.5% and in 1960 1% to be compared with 1.3% in 1970.[78] This density has thus not increased very rapidly. In connection with the survey of changes in the density of university graduates within manufacturing industries 1930-60[81] the development in different types of industries was related to various explanatory factors. A positive correlation was found between this density and productivity, capital intensity and different measures of the composition of the personnel (percentage of employees with higher education other than university education, the relative size of the administrative staff and the relative number of managers and technicians). The relative size of the administrative staff was positively correlated with the size of firms which indirectly links the density of university graduates to the size of firms.

Regions

The regional distribution of educational groups in the labour force in 1975 is indicated in table 2. The regions distinguished are: big town counties, woodland counties and other counties. This classification is of course very crude. The regional profile of the highly educated is, however, more concentrated to the big town counties than that of lower educational groups. Among those with post-secondary education 54% work in big town counties while the corresponding share of the lowest educational group is only 33%.

As for densities, the lowest educational group makes up 47% of the labour force in the big town counties and 57 and 60% of the labour force in woodland and other counties respectively.

During the period 1970-75 the distribution of the labour force over these three types of regions has hardly changed at all.71) This means that changes in the regional profile of one educational group must be compensated by changes in the profiles of other educational groups. In this case as well as for changes in occupational and branch profiles we find that the profile of the lowest educational group has not changed. In the other two profiles, however, there have been compensating shifts. The regional profile of the highly educated has shifted towards big town counties and that of people with upper secondary education out of big town counties.

Table 2. Regional percentage distribution of different educational groups in the labour force 1975

Educational group Region	Pre-upper secondary	Upper secondary	Post secondary	Unknown	Total
Big town counties	47 33	35 41	17 54	1 59	100 38
Woodland counties	57 21	34 20	8 14	1 14	100 20
Other counties	60 46	31 39	9 32	0 27	100 42
Total	54 100	33 100	12 100	1 100	100 100

Source: AMS Meddelande från utredningsenheten 1975:24

The fact that the regional distribution of the labour force has not changed at the same time as the percentage of highly educated in the labour force has increased further means that the density of highly educated must have increased within regions. As a matter of fact it increased in all types of regions but least so, both in absolute and relative terms, in woodland counties (from 7% to 8%).

Mobility

The mobility on the labour market of different educational groups has not been subject to very close comparative analyses. A few results from two surveys can, however, be reported. The first is a longitudinal study of a sample of men completing compulsory education in 1942-43 and followed up to 1956.[82)] The differences among educational groups related to geographical mobility and rural-urban migration was investigated. It turned out that in general the migrating groups had more education than the non-migrating groups. This was especially true for geographical movements over longer distances and migration out of rural districts.

The second survey,[83)] concerned with the gross inflow to the labour market in 1966-67, showed that in connection with the entry into the labour market 20% of those becoming economically active changed residency. Among the university graduates 43% moved geographically as they entered the labour market. This can be compared with 11% for the lowest educational group. Disregarding short distance movements (less than 50 km) the differences are even greater, 43% against 5%, for the educational groups mentioned above.

To this can be added some information about the propensity to change employment and type of work among certain types of university graduates even though there are no comparable data for other educational groups.[84)] A follow-up study of certain groups graduating from universities and specialised university colleges in 1967 showed that after 2 years 10-50% had changed employment. The figure varies somewhat according to type of degree being highest for Masters and Bachelors of Arts from the faculty of liberal arts and Bachelors of Arts from the faculties of natural sciences and law and lowest for graduate engineers. At the same time 5-20% changed type of work. This happened most often for social scientists and Bachelors of economics and business administration and least often for Masters of Arts from the faculty of liberal arts.

Among the groups of graduates from universities and specialised university colleges surveyed in 1971/72 and 1972/73 and who worked at the time of graduation only 60% of the men and 70% of the women had the same employment six months later.

We cannot get a very precise picture of the mobility of different educational groups from these data but it is possible that higher education leads to greater possibilities to change residency, employment and type of work as well as a higher propensity to take advantage of these possibilities.

International comparisons

Let us now finally make a few international comparisons of unemployment rates, occupational and branch distribution of different educational groups:

1. Unemployment rates. A negative correlation between educational level and unemployment rates is no unique feature for Sweden. In the U.S.A, for example, the situation has been the same. There too, the unemployment rates of the higher educational groups have usually been less sensitive to fluctuations in the level of business activity.[85] During the early 70s, however, the labour market situation of the highly educated has deteriorated in the U.S.A. The rate of unemployment among college educated and recent graduates from college has increased both in absolute terms and in relation to other educational groups. At times it has been higher for recent graduates from college than for their counterparts at lower educational levels.[86]

2. Occupational distribution. The OECD compiled data about the educational structure of the labour force and its distribution over occupations and branches in a great number of countries around 1960. Some of these data will be found in table 3, which shows the occupational distribution of university graduates in the labour force. In 1960 the share of university graduates in the labour force was very low in Sweden, only 2%. In many countries including Canada, France, Greece, Japan, Norway, the U.K. and the U.S.A. it was higher. Low rates are noted for the Netherlands and Portugal..

The occupational profile of university graduates is much more concentrated to the first occupational group (professional, technical and related occupations) in Sweden than in other countries. Countries with higher as well as lower density of university graduates in the economy as a whole have a lower percentage of the university graduates in this occupational group. The 81% for Sweden should be compared with the 60% for the U.S.A. To this corresponds the fact that in Sweden the share of the university graduates that go to clerical, sales and other occupations is very low (6% in Sweden, 27% in the U.S.A.). Even if the proportion of graduates in the first occupational group has recently declined it is without doubt still much higher than the figure of the U.S.A. for 1960. The link between education and occupation thus seems to be stronger in Sweden than in other countries, a fact that may cause trouble adapting to a higher average level of education in the labour force and achieving an efficient utilisation of such labour.

If we now look at the density of university graduates within occupations it can be noted that in countries with a high general density of university graduates the density in the first occupational group is often also high. The density of university graduates is for example very low in Sweden both in the first and second occupational group (14% and 13% respectively) compared with the U.S.A. (53% and 18%) but these densities are still

Table 3. Density of university graduates in certain occupations and occupational profile of university graduates around 1960

	Occupational profile					Density		
	Professional, technical and related occupations	Administrative, executive and managerial occupations	Clerical and sales occupations	Other occupations	Total	Total economy	in professional, technical and related occupations	in administrative, executive and managerial occupations
Canada	70	12	10	8	100	4.3	31	10
France	67	14	16	9	100	2.7	18	20
Greece	69	7	10	14	100	2.9	57	27
Japan	36	12	41	11	100	3.5	26	18
Netherlands	77	15	6	2	100	1.4	12	6
Norway	73	17	3	7	100	2.6	24	14
Portugal	76	6	6	12	100	1.2	33	6
Sweden	81	13	4	2	100	2.1	14	13
United Kingdom	73	6	9	12	100	3.2	27	8
United States	60	13	15	12	100	9.1	53	18
Yugoslavia	78	13	5	4	100	3.3	20	18

Source: OECD: Deployment and Utilisation of Highly Qualified Personnel: Statistical Annex DAS/EID/66.2 as quoted by M J Bowman and C A Anderson in Mass Higher Education: some perspectives from experience in the United States. OECD 1973

lower in the Netherlands with even fewer university graduates in the labour force. Thus even though in Sweden the university graduates are very concentrated to the first occupational group they have to a lesser extent prevented other educational groups from entering these occupations than in other countries.

Earlier we noted that as the average density of the highly educated increased the density in especially the white collar occupations grew. However, still in 1975 it seems to be lower than in the U.S.A. in 1960.

Of course these differences among countries are not only coupled to differences in the supply of graduates but also to differences in the occupational structure. An analysis made by the OECD secretariate showed that the factors that could best "explain" differences in occupational structure was output per worker and product per capita.[87)] Thus, Sweden had an occupational structure more ressembling that of the U.S.A. than that of Portugal. She also had an output/worker ratio closer to that of the U.S.A. than that of Portugal.[88)] However, the supply of highly educated labour was closer to that of Portugal. To put it bluntly: in Sweden there are many white collar jobs and few highly educated. This makes for a low density of highly educated in these jobs as well as in the economy as a whole at the same time as almost all university graduates work in white collar occupations.

The analyses of the OECD secretariate further showed that a certain economic level could be attained in many different ways. Apparently there exist great substitution possibilities among occupations and educational groups at least at the levels of aggregation used here. It is difficult in any absolute sense to talk about the need of the economy and of these occupations for labour with certain qualifications. Within an occupation the actual educational composition of those employed seems to be as much determined by the supply of labour with different qualifications as the "need" for certain qualifications to exercise a given occupation.89)

The data material used above illustrated the situation around 1960. We can add a few words about the development in the U.S.A. since that time. The occupational group "professional and managerial", that earlier was very expansive has stagnated and does not employ a greater share of the labour force in 1974 than it did in 1969. At the same time the supply of graduates from college has continued to increase. The percentage of the graduates from college that have been able to enter this kind of occupations has therefore been strongly reduced. It has been estimated to have decreased by 20% during the period 1958-70. Among young college graduates a much greater proportion will be found in sales and clerical occupations than before.86) During the 70s this change in occupational profile for the college graduates has been accompanied by increased unemployment and, as we shall see later, lower earnings. Already in 1960 did the highly educated have quite another occupational profile in the U.S.A. than in Sweden. However, only the further development in that direction during the 70s brought about a deterioration in their labour market position in relation to other educational groups in terms of earnings and unemployment.

3. Branch distribution. The density of university graduates within certain branches of economic activity is indicated in table 4. Once again the situation described is that of 1960 as recorded by the OECD.

In the manufacturing industries as well as in the commerce and transport sectors the percentage of university graduates is rather low in Sweden. In the manufacturing industries they only make up 0.9% of the labour force to be compared with 5.9% in the U.S.A. and 3.8% in Japan.

The Swedish service sector is a little bit better provided with university graduates. The density is 7%. Even this figure is, however, below that of the U.S.A. (19%) and Japan (10%).

The econometric analysis of these data on education and branches of economic activity made by the OECD secretariate showed that in this case as well as for occupations large possibilities of substitution exist. A certain economic level or a certain growth of the economy does not presuppose any

Table 4. Percentage of those employd within designated sectors with a university degree

Country	Branch			
	M	C	T	S
Belgium	0.99	2.35	0.05	6.29
United States	5.94	6.84	0.19	18.57
Japan	3.75	4.46	0.17	9.68
Norway	1.23	1.81	0.07	9.42
Netherlands	0.57	0.70	0.03	4.27
Sweden	0.89	1.24	0.03	7.17
Yugoslavia	1.02	1.29	0.02	9.44
Argentina	0.90	1.50	0.03	6.41
Egypt	0.84	1.51	0.07	7.12
Hungary	2.03	3.22	0.08	16.16
Israel	2.15	3.40	0.11	9.05
Pakistan	0.28	0.65	0.01	3.14
Panama	1.09	2.95	0.04	4.76
Philippines	2.10	5.80	0.10	18.64
Poland	1.97		0.05	15.83
Syria	0.20	0.85	0.02	4.68
Uruguay	0.76	1.36	0.02	4.41
Zambia	0.48	0.96		3.00

M - manufacturing	T - transport
C - commerce	S - service

Source: OECD Occupational and educational structures of the labour force and levels of economic development. Further analyses and data. Paris 1971 pp 124–127

precise branch structure or educational composition of the labour force within economic branches. The educational composition of the labour force within sectors is not only related to demand for but also to the supply of labour with different types of education.[89]

3.3 Some macroeconomic data

As education has often been conceived of as a major factor promoting economic growth some data relevant for such a discussion at an aggregate level have been put together in this section.

The growth rate of the Gross National Product (GNP) has fluctuated between 1861 and 1972 without revealing any trend. It is true that during the 50s and 60s it accelerated but during the period 1965-72 the growth rate fell back into line with earlier experiences.[90)]

During this period the labour force has also grown but the growth rate of the labour force has been lower than that of the GNP especially during the 50s and 60s and measuring labour by working hours.[90)] It is quite possible that without the educational expansion the number of persons economically active would have been still lower. As we noted in the preceding section labour force participation and educational level have been found to be positively correlated. However, as we have no figures for the educational composition of the labour force and rates of labour force participation within educational groups over longer periods of time we cannot even try to estimate to what extent the educational expansion can be said to have contributed to the growth of the labour force and thus to economic growth.

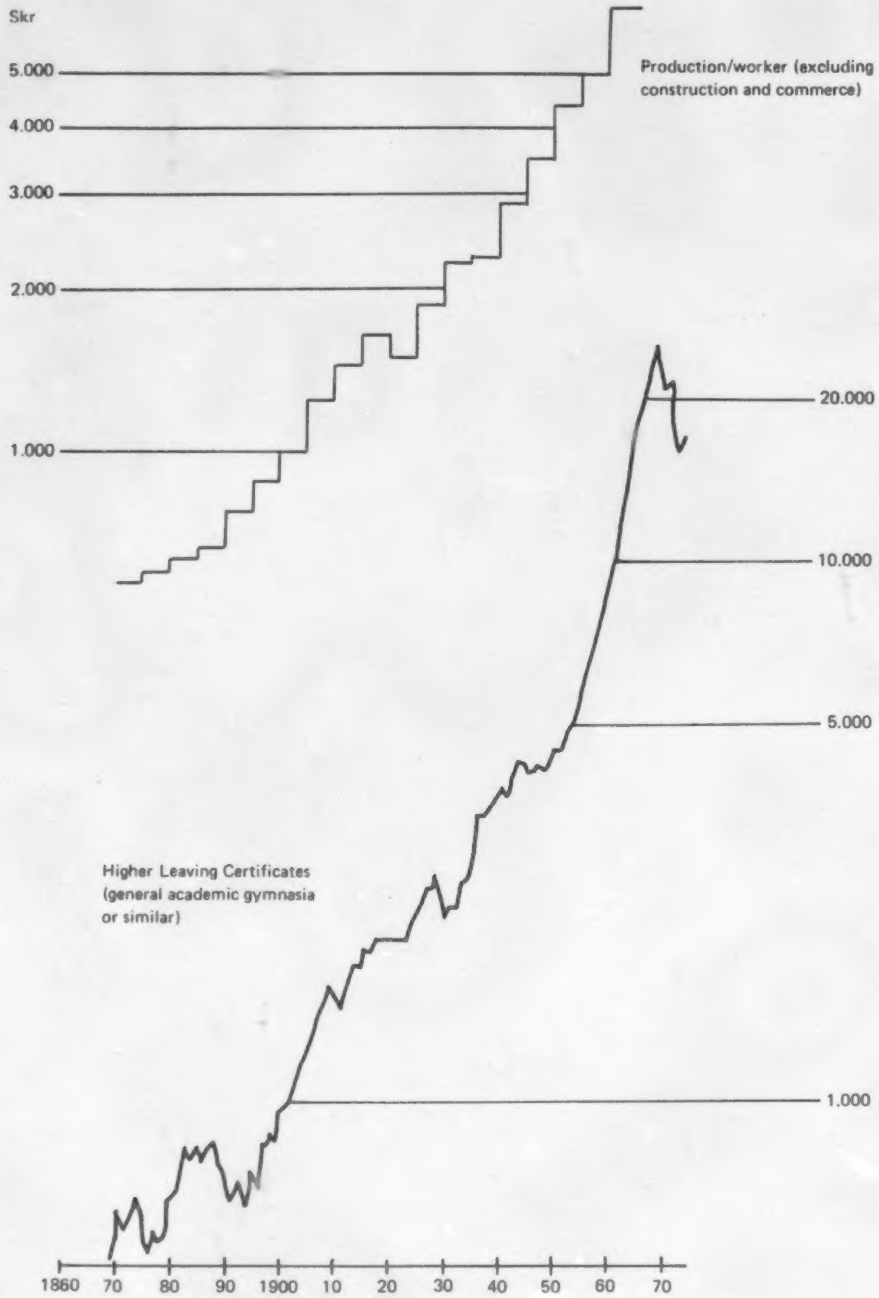
Another question that might be raised is whether the increased labour productivity could be related to the educational expansion. In graph 15 we let the number of Higher Leaving Certificates awarded by general academic gymnasia, or similar, represent the educational expansion and recent stagnation. Between 1870 and 1900 the number of certificates increased by jerks. The level in 1900 as compared with that in 1870 implies an annual compound rate of growth of 2.3%. Up to 1920 the expansion was smoother and faster, corresponding to an annual growth rate of 3%. The 20s and the beginning of the 30s were characterised by stagnation. During the period 1930-48 the annual growth rate was once more around 3%. During the late 40s there was again a certain stagnation while the period 1948-68 showed an unprecedented growth rate of 9.5% per year. The decrease in 1968-73 was 7% annually.

The top of graph 15 shows the corresponding development of production per worker. The same kind of pattern is visible: at first a rather low growth rate (1870-90), then a somewhat more rapid growth (1890-1920), stagnation, growth at about the same rate as before the stagnation (1920-40), stagnation and finally a much more rapid rate of growth. For the recent decades we can add an estimation of the development of production per working-hour:[91)]

	Annual rate of growth
1950-60	3.3
1960-65	5.6
1965-70	4.7
1970-72	4.0

As can be seen the rate of growth has not continued to in-

Graph 15. Production per worker (price level of 1913) and number of Higher Leaving Certificates awarded by general academic gymnasia (or similar)



Source: Å Sohlman: Utbildning och arbetsmarknad. Working paper No 1976:1 Department of Economics, University of Stockholm p 134 crease. However, it never was negative like that of educational expansion.

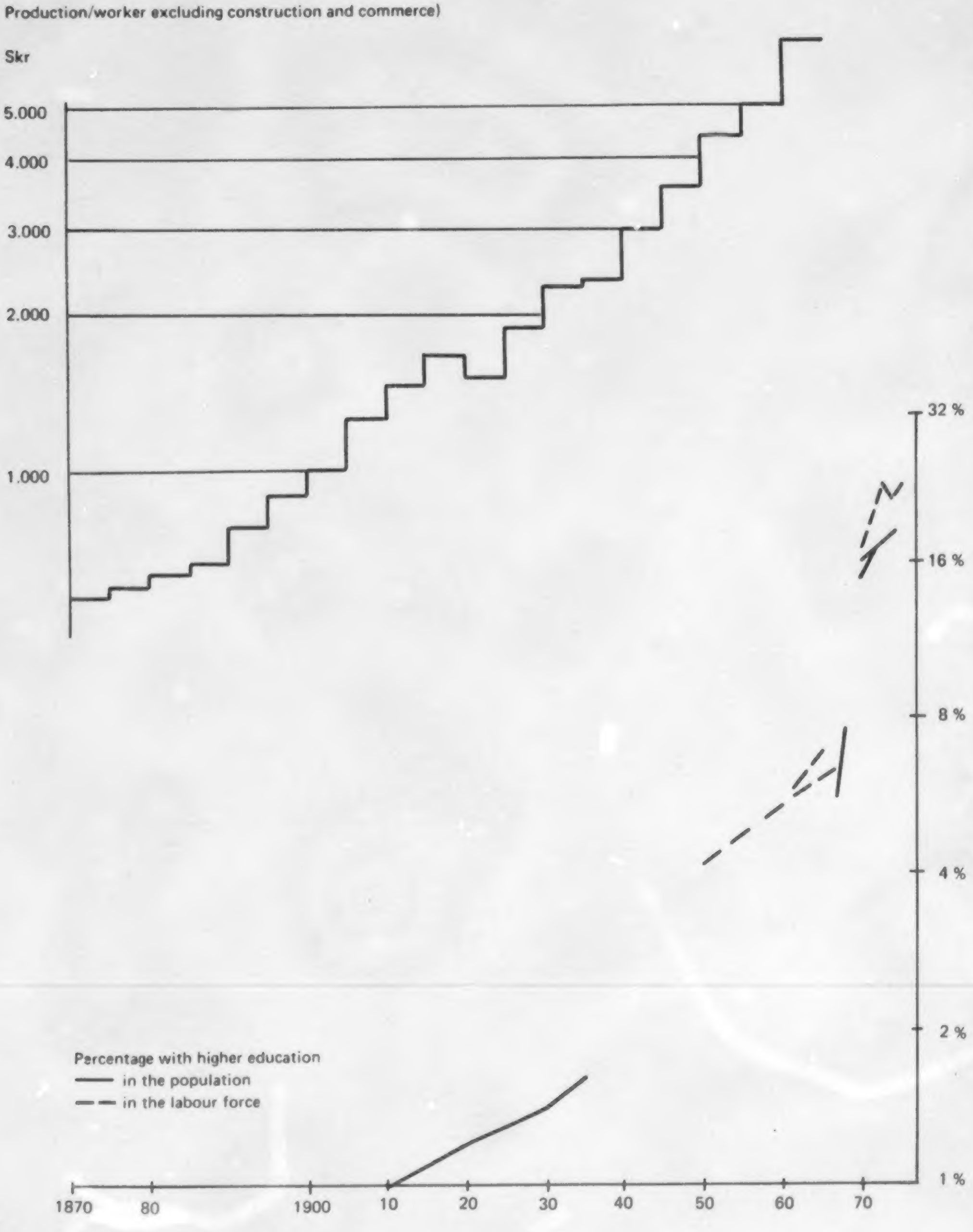
Hardly anyone has argued that increased education affects the productivity but with a certain lag. According to graph 15, however, educational expansion seems to parallel that of productivity or rather lag somewhat behind. A possible conclusion of this would be that rather than becoming "rich" from more education the Swedes have had more education because they have become "richer". Be that as it may, graph 15 at least makes clear that from this type of data we cannot directly conclude that more education results in higher productivity.

If education makes people more productive as they join the labour market comparing the development of the education of the labour force and that of productivity may appear a more appropriate strategy if you are interested in the consequences of educational expansion for economic growth. Actually we have too little information of a comparable kind about the education of the labour force in Sweden to really make such a comparison. In graph 16 we have, however, put together data about the development of productivity and the available information from various sources about higher education in the labour force and in the population. The graph seems at least to support the conclusion that productivity and the amount of higher education in the labour force are positively correlated.

Probably these attempts at finding direct traces of educational expansion in the GNP-development appear rather naive when at the same time so many other important events have taken place: the industrialisation of Sweden, the enormous capital accumulation accompaning the industrialisation, the First World War, the Great Depression, the Second World War etc. In many countries more detailed analyses of the GNP-growth have related the economic growth to different growth promoting factors including education. We cannot report on any such analysis explicitely considering education for Sweden. A less ambitious attempt has, however, been made to relate the growth of GNP to the growth of the labour force and of physical capital by means of a production function analysis. The part of the economic growth that cannot be "explained" by increases in labour force and physical capital the so called residual is of some interest to us here as it can be said to cover the effects of all other growth promoting factors including education. The result for Sweden looks as follows:[92]

	Annual percentage growth rate of production	Contribution from various factors to the annual growth rate of production:		
		physical capital	labour force	residual
1870-1913	2.81	1.35	0.28	1.18
1920-1939	2.48	0.98	0.54	0.96
1946-1964	3.44	0.85	0.43	2.04

Graph 16. Production per worker (price level of 1913) and percentage share of the population or labour force with higher education



Source: Å Sohlman op cit p 140

This means that the residual is responsible for 42, 39 and 59% of the economic growth during the respective subperiods. The effects of what factors the residual actually covers and how much of it could be attributed to education we do not have any information about from these figures. The data do not contradict the hypothesis that the increase in the residual during the last sub-period is due to the educational expansion and that without more education the growth rate would have been lower.

A few international comparisons can also be made with respect to education and economic development. Many studies of the development over time in one country or comparing different countries at a given point in time have shown as we earlier noted for Sweden that there exists a correlation between the volume of educational activities (either measured in financial or quantitative terms) and GNP or GNP per capita.93) Rich countries devote more resources to education than poor countries. Normally a certain variation between countries is, however, found. A survey of the situation in different countries around 1958 conducted by the OECD indicated that, for example, the Soviet Union and even Sweden at least for higher educational levels devoted more resources to education than other countries at a similar economic level.93) Estimating the connection between education and GNP in terms of the income elasticity (percentage change in educational activities/percentage change in GNP) one arrives at somewhat different results in longitudinal and cross-section studies. In the latter the elasticity has been around 1 while it has been found to be higher within countries over time.93)

With a few exceptions 94) this kind of results have been interpreted as indicating the existence of a causal relationship the direction of which goes from GNP to education not the other way round.93) There may exist an influence in the other direction (from education to GNP) but this is better shown by other means.95) As an example of studies which have been thought to indicate the existence of a causal relationship in the other direction we can mention:95)

- studies indicating that a certain level of literacy and primary education is a necessary precondition for economic development
- studies indicating that among countries at the same level of economic development those with more educational activities will have a faster economic growth than those with less[x]
- studies indicating a positive correlation between GNP-level and higher education in the labour force.

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[x] It should, however, be mentioned that a positive correlation between different measures of educational volume (or changes in educational volume) and later GNP-level (or changes in level of GNP) has not always been found.

The most influential support for the assumption that education causes economic growth has probably been that given by production function analyses which explicitely treated the educational factor. They began to be made in the U.S.A. in the 60s. This approach has, however, later been heavily criticised both for practical and theoretical reasons96) and authors have also come to rather divergent results. In one study it was found that for the U.S.A. during the period 1929-57 education could "explain" 23% of the economic growth (this being a lower limit as there was also a factor "advancement of knowledge" that "explained" 20% of the growth). In a somewhat later investigation the contribution of education to economic growth in the U.S.A. 1945-65 was estimated at only 14%.96)

For certain European countries low figures were also arrived at (Belgium 14%, Denmark 4%, France 6%, Italy 7%, the Netherlands 5%, Norway 7%, West-Germany 2% and the United Kingdom 12%) during the period 1950-62.97) If these figures are accepted not much of the economic expansion after the Second World War of at least these European countries would be related to the educational expansion. In the late 60s these countries (with the exception of Italy) have experienced a fall in the growth rate while the educational expansion has continued.98)

Altogether this gives rather frail support for the assumption that education is a direct and uncomplicated cause of economic growth.

4 Education and distribution

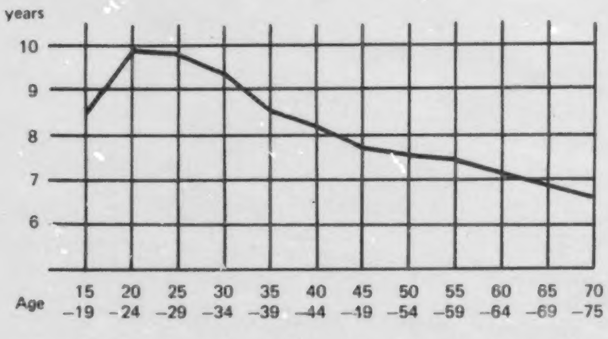
This section is concerned with the educational system as a distributional devise. First we will describe how the educational resources have been distributed both between and within generations and between individuals of different social origins. In the second part, distributional effects due to the connection between education and earnings will be considered.

4.1 The distribution of educational resources

Comparing generations

The education received by different generations is indicated in graph 17. The data come from a cross-sectional study in 1967 and shows the average number of years of education different age-groups have received up to 1967. That there are considerable differences among generations is hardly surprising. The fact that the educational expansion entered an accelerated phase rather late, however, means that there are great differences not only between the young and the old but also between the young and the middle-aged. As for the youngest generations it must be added that an important number are still studying (e.g. 13% of the 25-29 age group).[1]

Graph 17. Average years of education by age-groups in 1967



Source: Låginkomsturedningen. L Johansson: Utbildning – empirisk del Sthlm 1971 p 32

An educational policy aiming at eliminating these differences would devour enormous amounts of resources. "If all those approximately 3.2 millions aged 20-64 with only primary education should be given 4-5 years more of education and the approximately 1.1 millions with only lower secondary education 2 more years of education 'society would have to provide for 17 million years of full-time education'. This would, for example, imply that 10% of those aged 20-64 would have to be engaged in full-time education for over 30 years".2) Earlier was mentioned a rough estimate of 150.000 adults in full-time education. This figure included on-the-job training but let us assume that adult education at relevant levels was expanded to 150.000 full-time adult students. To obtain 17 million years at a pace of 150.000 per year takes about 110 years. This would be too slow to achieve the above mentioned reduction in differences between generations while they were still alive. Of course, if the average level of education of younger generations is not further increased the intergenerational differences will eventually disappear. However, the 20-year old persons of to-day will be 70 only in 50 years.

Differences within generations

Discussing the educational composition of the population it was mentioned that the younger generations were more evenly distributed over educational levels than the older ones. Thus in this respect the differences within generations have been reduced. Measuring the education received by numbers of years we can calculate the differences within generations both in absolute and relative terms. The standard deviation measures the spread around the average in absolute terms.

Table 5 shows that the standard deviation increases somewhat from generation to generation. Moreover, the figure for those aged 25-29 and even 30-34 could not be regarded as definite as 13% and 5% among them are still studying.1) The spread in years of education in relation to the increasing average as measured by the coefficient of variation also tends to increase from generation to generation at first but later this coefficient decreases.

Table 5. Variation in years of education by age-groups in 1967

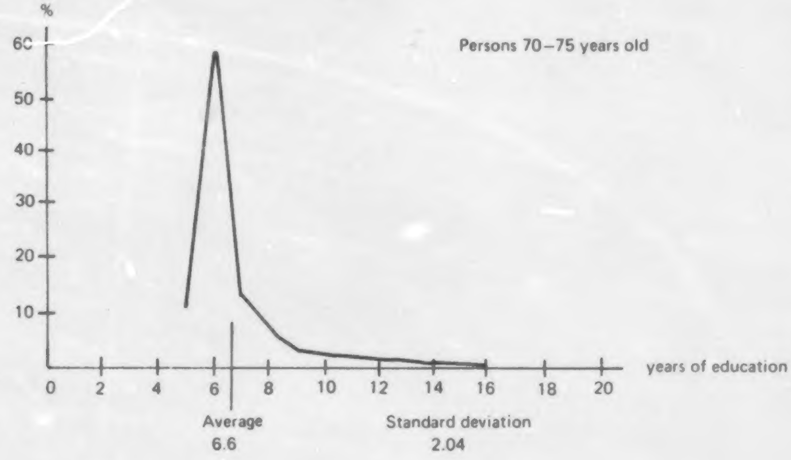
Age	25-29	30-34	35-39	40-44	45-49
Standard deviation	3.09	3.06	2.98	3.05	2.90
Coefficient of variation	3.15	3.29	3.46	3.71	3.76

Age	50-54	55-59	60-64	65-69	70-75
Standard deviation	2.50	2.38	2.34	2.27	2.04
Coefficient of variation	3.28	3.17	3.25	3.28	3.09

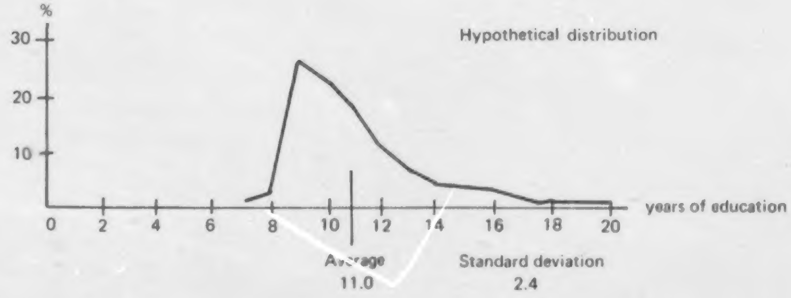
Source: L Johansson op cit pp 35, 162

Metaphorically the distribution for a generation over years of education can be seen as a caterpillar on a string (graph 18). It advances by streching forward and as a result the average and the spread increase. Thereafter, the behind is pulled up which also increases the average but reduces the spread. The development so far seems to have been mostly characterised

Graph 18. Distribution over years of education for different generations in 1967 and a hypothetical distribution



Source: L. Johansson op cit p 162



Source: see the text

by the first part of this movement. It is possible that for future generations the distribution will be more compactly situated around the mean. It must be remembered that among those aged 25-29 in 1967 as many as 47% had less than 9 years of education.[3] If, in future generations, practically no one has less than 9 years and the expansion of higher education is damped a reduction in the spread around the mean may be expected. The hypothetical distribution at the bottom of graph 18 has been constructed assuming that practically every one gets 9 years of education, 70% continue after 9 years and 20% after 12 years. The latter figure implies an increase by only one percentage point compared with those aged 25-29 in 1967. For this distribution the average is 11.0 years and the standard deviation has been reduced to 2.4 years.

Social differences

To describe the distribution of education between individuals of different social origins let us first use data from the 1967 cross-sectional study. The individuals were grouped into three categories according to the occupation of their father. In brief, social group III consists of those whose fathers were manual workers, social group II of those whose fathers ran small enterprises or had subordinate non-manual jobs and social group I of those whose fathers held leading business positions or non-manual jobs at higher hierarchical levels.[4] This means that comparing the different generations the distribution of individuals over classes of social origin does not remain the same. Among those aged 15-29 e.g. 9% belong to social group I and 54% to social group III as compared with 4% and 62% among those aged 55-75.[5]

In 1967 the average length of education was 12, 9 and 8 years among all individuals irrespective of age belonging to social group I, II and III respectively.[6]

Let us now compare the different generations. According to table 6 the absolute difference in average years of education between social group I and III was about the same for those aged 55-75 and 25-29 in 1967 (5.6 and 5.4 years respectively). As the share among those aged 25-29, belonging to social group I and still studying, is three times as high as among those belonging to social group III it may be concluded that at least up to those aged 25 in 1967 the absolute differences have not decreased. Considering the great social difference in percentages still studying among those aged 20-24 it can be doubted whether any reduction in social differences will result even for them.

As for relative differences table 6 shows that the average education was 80% higher for those aged 55-75 in social group I when compared with similar individuals in social group III. For those aged 25-29 it was only 60% higher. The final result when they get older will certainly be a greater relative difference but possibly lower than for the oldest generation. It takes much more extra education among those still studying to increase the average so that the relative differences remain the same as earlier than to increase it so that the absolute differences remain as before.

Table 6. Average years of education, percentage still studying, percentage with 9 and 12 years of education and with Higher Leaving Certificate in different age-groups by social origin

Social group	Average years of education					9 years of education (%)		12 years of education (%)		Higher Leaving Certificate (%)	
	Age					Age		Age		Age	
	20-24	% stud	25-29	% stud	55-75	15-29	55-75	20-29	55-75	20-29	55-75
I	11.9	75	14.4	35	12.2	97	84	67	54	45	31
II	10.3	30	10.0	13	7.4	75	19	29	9	12	3
III	9.3	19	9.0	10	6.6	62	7	15	2	7	1
I-III	2.6		5.4		5.6	35	77	52	52	38	30
I/III	1.3		1.6		1.8	1.6	12.0	4.5	27.0	9.0	31.0

Source: L. Johansson op cit pp 88–92

Thus, while the relative social differences in average education may be decreasing the absolute differences seem to have remained about the same. Similar comparisons between generations can be made for different educational levels. Table 6 indicates that comparing the percentage share:

- obtaining at least 9 years of education in social group I and III both the absolute and relative differences have been reduced. (The figures are based on the assumption that all those still studying will receive at least 9 years of education)
- obtaining at least 12 years of education in social group I and III there has been a reduction in relative differences while absolute differences have remained the same
- passing the Higher Leaving Certificate in social group I and III, the relative differences have decreased while the absolute differences have increased.

Many investigations have been designed to especially study the social composition of the student body at the universitie . The problem is that often they are not comparable and moreover build on approximations. Let us, however, look at table 7 where we have certain figures approximately describing the situation in the early 50s and late 60s and in 1968 and 1971.

Table 7. Percentage share going to university and specialised university colleges by social origin

	Early 50s		Late 60s		1968	1971
Socialgrupp						
I	28	9	79	33	27	25
II	5		20			
III	1		9		7	6
Total	4		21		17	16
I-III	27		70			
(I+II)-III	8		24		20	19
I/III	28		9			
(I+II)/III	9		4		4	4

Sources: SOU 1971: 61 pp 143, 144

B Gesser, E Fasth: Gymnasial utbildning och social skiktning. Sthlm 1973, p 69

During the 50s and 60s the percentage share going to university increased. The development seems to have implied decreasing relative differences and increasing absolute differences comparing either social group I and III or I+II and III. 1968-71, when university attendence decreased, the social differences seem to have remained about the same both in relative and absolute terms. This stability was achieved through compensating shifts among age-groups for persons with social background III, attendence declining among the young and increasing among the old.7)

Let us finally cite a few results from certain follow up studies.

1. In 1970 it was found that after having finished comprehensive school 82, 46 and 25% continued to the gymnasium in social group I, II and III respectively. Considering the gymnasium plus the specialised professional school and the vocational school the rate of transfer was 98, 87 and 80% in social group I, II and III.8) The social differences are thus much smaller for the upper secondary school as a whole than for the gymnasium. The social differences in transfer rates remained albeit somewhat reduced taking differences in school marks into consideration.8)

   In this connection it is interesting to note the results from an investigation in 1961. Pupils in grade six were administered an intelligence test. From these test results expected average school marks were calculated and compared with the school marks actually obtained. Similar comparisons were also made between intelligence test results and results on standardised curricula tests and between results on curricula tests and school marks. It was found that pupils from lower social groups obtained lower school marks and lower results on the curricula tests than could be expected from the intelligence tests and lower school marks than expected from the results on the curricula tests.9)

2. Social differences also appear among the university transfer rates of those finishing the gymnasium. In 1972 58, 43 and 36% in social group I, II and III respectively were actively studying at the university among those who finished the gymnasium in 1970. Here too, the differences remained taking school marks into consideration. The differences were small at low levels of school marks, somewhat higher at high levels and highest at intermediate levels.10)

   The development of these transfer rates to post-secondary education during the late 60s and early 70s can be summarised as follows:11)

   - the proportion continuing to post-secondary education has decreased. The proportion going to university education was heavily reduced while the proportion going to other types of post-secondary education increased somewhat

- the social differences (social group I compared with social group III) in transfer rates from the gymnasium tc the post-secondary education institutions increased both in absolute and relative terms
- the social differences (social group I compared with social group III) in transfer rates from the gymnasium to the university increased in relative terms but not in absolute terms.

The latter results with respect to social differences in transfer rates gymnasium-university do not necessarily contradict the earlier result: constant social differences both in absolute and relative terms in the percentage share of a cohort going to university. A change in the social composition of the student body at the gymnasium can compensate for increased differences in transfer rates.

3. Once at the university 7) students with different social background seem to obtain degrees to the same extent. As for educational reorientation (mostly from free to restricted faculties) students belonging to social group III seem to utilise these possibilities to adaptation to a lesser extent. There is also already at the university entry a difference as these students less often go to the restricted and more prestigeous faculties.12)

International comparisons

Let us first note the results of a comparison in financial terms. An investigation was made to find out how expenditure varied by schools having different social compositions of their student body. The information is from the middle of the 60s and concerns 13 year old pupils. For Sweden the method of classification used resulted in the distinction of only two types of schools: those where the majority of pupils had fathers with high status occupations and those where the majority of pupils had fathers with low status occupations Expenditures per pupil as well as achievements were higher in the former type of schools. The differences were, however, very small. The distribution of resources between various types of schools seems to be more egalitarian in Sweden than in England, West Germany or the U.S.A. However, analysing the effects of different modes of resource allocation (egalitarian versus elitist) and school system structure (comprehensive versus selective) it was concluded that differences in these two respects do not seem "to have much impact on overall level of achievement or the social distribution of achievement".13)

Statistics from OECD, for 1970, permit international comparisons of intergenerational differences. Comparing the 30-34 and

55-64 year olds it turns out that at the primary level differences are small in most countries. At secondary and higher levels differences are more important. Sweden has the highest differences after Japan and Finland at the secondary level and after the U.S.A. and France, at higher levels. The same OECD-statistics also show that within generations the dispersion (as measured by the Gini-coefficient)[x] has increased in Sweden (again comparing the 30-34 and 55-64 year olds). The same development was noted in eight other OECD-countries (e.g. Denmark, France, West Germany, Norway and the U.K.) while in ten other OECD countries (e.g. Canada, Japan, the U.S.A., Italy and Portugal) it decreased.

The OECD has also made an effort to describe on a comparable basis the social background of students in higher education and the share in different social groups going to this type of education around 1960. Two measures were constructed that can be used to compare the social differences between countries:

- the index of dissimilarity indicating differences in the social composition of the student body and active male population
- relative chances of upper-stratum and lower-stratum youth studying in a university.

Graph 19 summarises the results. It can be seen that Sweden does not occupy a leading position as to low social disparities. The development during the 60s as followed up by the OECD shows a tendency towards equalisation. Due to differences in social classifications we cannot, however, say whether this development was faster in Sweden than in other countries or if Sweden still would rank as low as in 1960.[15)]

4.2 Education and the distribution of income

The data at our disposal to illustrate the relation between education and income are of two types: individual and grouped. In the first part of this section a few studies based on individual data will be presented. An important question in these studies has often been how social background and education interact and affect occupation and earnings. With respect to the relation between social background and education they can thus complement the grouped data presented in the preceding section. The second and third parts of this section use grouped data to describe the average income of different educational groups for example by age and sex and at different points in time. In part four the development of the income

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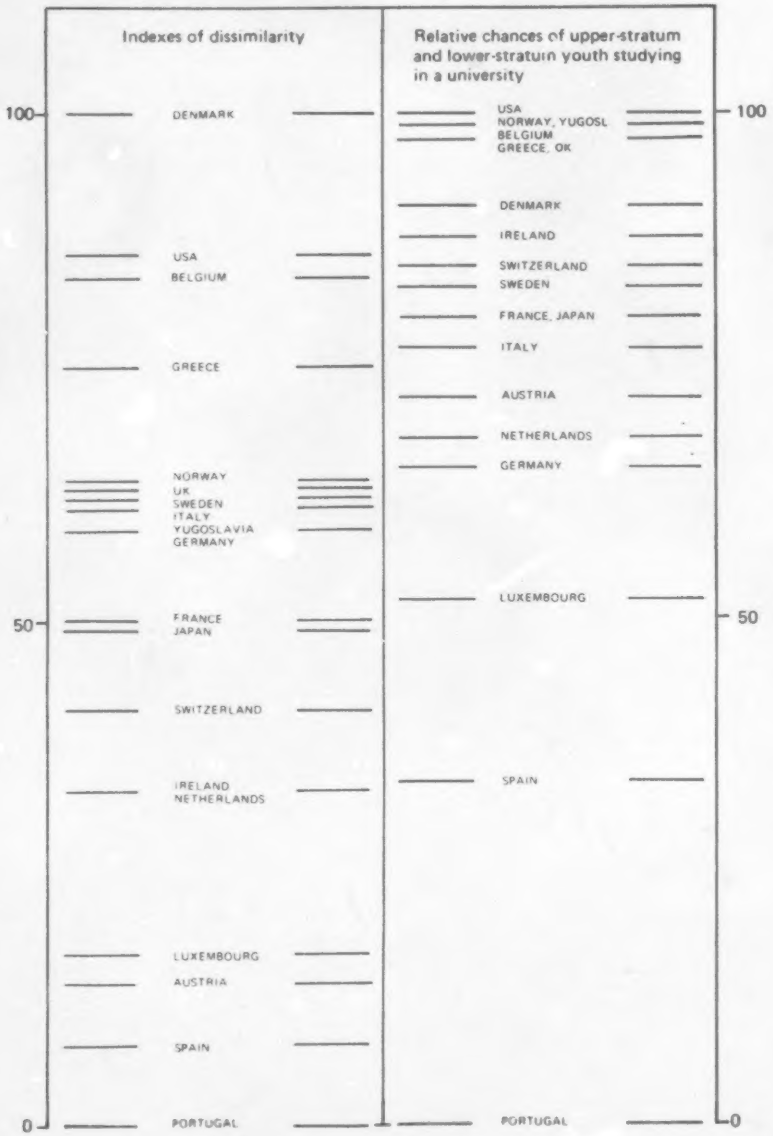
x) The coefficient of concentration or Gini coefficient varies from 0 - 1, a lower numerical value indicating less inequality.

Graph 19. Ranking of countries according to their index of dissimilarity and the relative chances of youth studying in a university



The country in which, according to the index adapted, social disparities seem lowest has been classified at 100, the country in which these disparities seem widest has been classified at 0. The other countries have been ranked relative to these two extrems.

Source: OECD. Group disparities in educational participation and achievement IV. Paris 1971 p 69 distribution in terms of educational changes is discussed. Finally in part five some international comparisons are made.

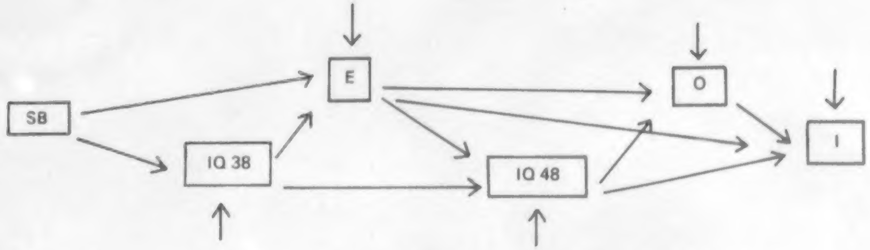
4.2.1 Individual earnings and correlated variables

There are several studies of this type. The first two presented here study the whole chain: social background, education and earnings and the last two only the relation between education and labour market variables.

Social background, education and earnings

A regression analysis based on dummy variables was used to investigate the relation between incomes in 1963 of a sample of men 35 years old and social background (4 classes), IQ test results in 1938 (5 classes) and education (4 classes).16) Only the coefficients for the highest social class, the two highest IQ-classes and the two highest educational classes were found to be significant. A man from the lowest social group, at the lowest IQ and educational level could be expected to have an income of 13,690 Skr whereas a man from the highest social group, at the highest IQ and educational level could be expected to have 46,880 Skr. 46% of the income difference could be "attributed" to differences in education, 32% to differences in social background and 22% to differences in IQ test scores. Studying interactions it was found that education in combination with high IQ and high social class contributed much more to income than what would correspond to additivity of the effects. Given these relations a general educational expansion would tend to increase income differences while increased education for those with the worst income perspectives would reduce income differences.

The same sample of men has later been further analysed when income data up to 1971 existed.17) A recursive type of regression analysis (path analysis) was used. The model finally adopted roughly looks like this:



SB = different variables indicating the family situation and the social background

IQ_{38}, IQ_{48} = IQ test results in 1938 and 1948

E = education (either number of years E_1 or educational level E_2)

O = occupational status (either in 1953, 58, 63, 68 or 71)

I = log of income (either in 1953, 58, 63, 69 or 71) excluding income of capital and those with income below 12,000 Skr in 1971

Certain interaction effects were also included ($SB + IQ_{38} \rightarrow E$, $IQ_{38} + E \rightarrow IQ_{48}$, $IQ_{48} + O \rightarrow I$ and $E + O \rightarrow I$).

It can be noted that the model does not include any direct relation between social background and occupation or income. The inclusion of such relations did not increase the "explanatory value" of the model for the given set of data.

The following amount of variation of different variables are "explained" by the model:

IQ_{38}	approximately	11%
E	"	40%
IQ_{48}	"	65%
O	"	40% (when the indivuduals are 30 years old or more)
I	"	5%, 10%, 35%, 38% and 52% in 1953, 58, 63, 69 and 71.

The rest of the results can be summarised as follows:

The SB and IQ_{38} variables "explain" an important part of variations in education. They are, however, not highly intercorrelated which would indicate that IQ variations rather "explain" differences among individuals with the same background than differences between groups of individuals with different social background.

There are certain interaction effects with respect to the educational system that can be of interest even though they are not very strong. Social background and IQ_{38} interact so that those who rank high in both these respects get more education than the simple additive effect of these two variables would suggest. IQ_{38} and E have a negative interaction effect on IQ_{48}. Thus the selection process in the educational system seems to have created certain losses of talent. The direct effect of E upon IQ_{48} is, however, positive as well as that of IQ_{38} (both directly and via E).

The E-variable has important direct effects upon occupation. Furthermore, it strongly affects income directly (especially E_2) as well as via occupation. Education cannot, however, be said to be the sole decisive factor in accounting for differences. Social background and the IQ-variables are also involved. Sometimes IQ_{48} has a direct effect upon income even though it is not as great as that of E. Moreover, important effects via E and O can be attributed to social background and the IQ-variables because of their correlation with E further back in the chain.

In certain respects the model gives different results when E_1 (years of education) and E_2 (level of education) are used:

- SB and IQ_{38} have stronger effects upon E_2 than upon E_1 (which agrees with our previous results that social differences are greater for Higher Leaving Certificate than for 12 years of education)
- using E_2, IQ_{48} has generally no direct effect upon O and I
- E_2 has a stronger direct effect upon I than E_1
- E_1 and O have positive interaction effects upon I but E_2 and O do not.

These results are somewhat intriguing. E_1 and E_2 are rather highly interrelated (the simple correlation coefficient is 0.81). E_1, however, also measures a certain amount of grade repeating and is less highly influenced by IQ_{38}. Therefore it might be less surprising that it has a lower effect upon I than E_2 and that IQ_{48} could be needed as a complementing explanatory variable. But why does not IQ_{48} influence I in case of E_2? Over 30% of IQ_{48} is not explained by the factors in the model. Because E_2 has more of a sorting function? Apparently not, as E_1 and E_2 have about the same (E_1 somewhat stronger) indirect effect on I via O. Does E_2 at the same time measure both quantity and quality of education? Does E_2 capture more of the effects of general education and E_1 more of the effects of vocational education? E_2 has stronger direct effects on I. E_1 also has somewhat more indirect effects via O. But E_1 not E_2 interacts with O in influencing I. To some extent general ability that in case of E_1 may be captured by IQ_{48} in case of E_2 could be confounded with E_2.

As a complement let us report a few results from studies analysing grouped data on social background, education and occupation. There is no information about income in these studies.

A sample of men living in Stockholm in 1949 was investigated as to their occupational position, education and the occupational position of their father. Similar data exist for a national sample in 1955.[18] In both samples, irrespective of the social origin of the individual, the percentage obtaining the highest occupational levels was greater at high educational levels than at low educational levels. The absolute level of these percentages were, however, higher for individuals with fathers in higher occupational positions.

An attempt was made at estimating whether the fact that children with different social background had different chances of reaching high occupational positions was mainly due to differences in education or to the impact of the social background at similar educational levels.[19] The result for the national sample was that the social background did not so much affect the occupational position by influencing the amount of education received as by influencing the occupational chances of individuals with similar education. For the Stockholm sample the result was the opposite. The social differences as to the occupational position reached could mainly be attributed to differences in education. Individuals with the same education had to a greater extent the same chances of ending up in different occupational positions. These calculations have been reproduced on a national sample from 1967.[20] The chances of reaching the highest occupational positions was mainly found to be related to educational differences irrespective of social background.

A lot of social mobility has, of course, taken place by other than educational routes. However, comparing the generations in the national sample from 1967 it appears likely that education has become more of a prerequisite for social mobility than before.[21]

Both the study based on individual data and two of the studies based on grouped data indicate that the social background has its most influential effect upon the education of an individual. Education then influences occupation and income with little further influence from social background. For the study based on individual data this was true after taking into account different factors describing the social background and personal characteristics of the individual. However, the important question from a policy point of view is what would happen if an educational policy aiming at income equalisation was built upon this relation. If the social differences in educational chances disappeared would the rest of the relations still hold?

Education and labour market variables

Income differences in 1966 were in one study[22] related to age, education, branch of economic activity and region, for men and women separately. In one regression model these variables and their interactions "explained" 72% of variations in income for men and 66% for women. Judging the relative importance of the different explanatory variables, either by their relative net contribution to the total explanatory power of the model or the range of variation for each factor, the conclusion was that education and age were more important than branch of economic activity or region for the variation in income. Certain interactions were also of some importance:

for men - education and age
- education and branch of economic activity

for women - age and branch of economic activity
- education and branch of economic activity
- education and region

The same type of results were arrived at by studying salary differences in 1971 among government employees. The range of variation was greatest for the age and education variables followed by area of activity, sex and region.23) These studies seem to indicate that persons of different ages and with different types of education operate in more or less separate sectors of the labour market. Regions and economic branches do not seem to give rise to different labour market conditions to the same extent. Mobility can, however, be a precondition for gaining a proper return to education.

4.2.2 Present income differences among educational groups

In an attempt to describe the income differences among educational groups at present we will first use a few cross-sectional materials (from 1972, 1971 and 1966), then we will compare the income of recent graduates at different educational levels. Finally, we report the results obtained from calculating the life-time earnings for different educational groups. This is, by the way, the only instance where we can compare what the differences look like before and after taxes.

Income differences in 1972, 1971 and 1966

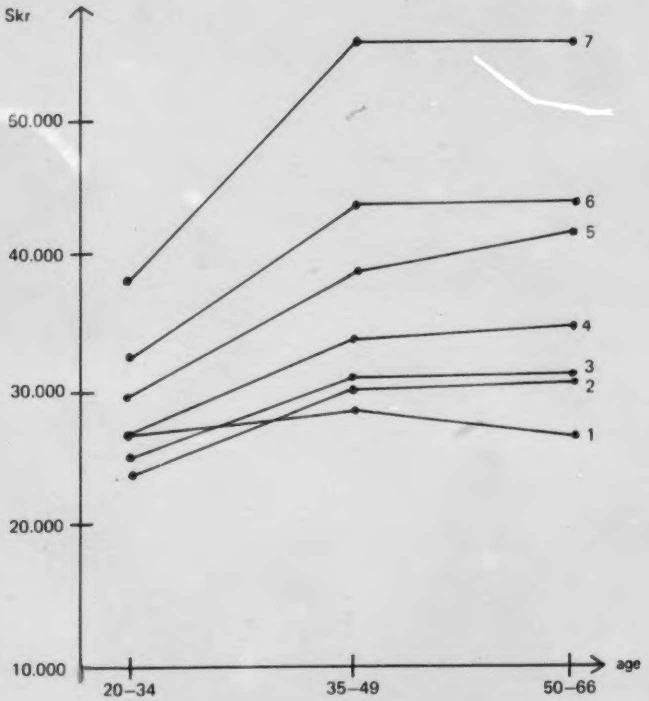
Graph 20 depicts the income situation, by age, for different educational groups in 1971. Only full year and full time employees have been included. Income refers broadly to labour income before taxes and transfers but after certain cost deductions.

The differences in median earnings among educational group 1-4 are fairly small especially for the youngest age-group. The distance from these educational groups to the highest one is considerable and greater among the older. The relative income differences in the three successive age-groups looks as follows:

5/2	1.3	1.2	1.3
7/5	1.3	1.5	1.3
7/2	1.6	1.8	1.8

The 1972 and 1966 materials give the same general picture for persons working at least half of the year and full year and full time employees respectively.[24] These data also permit men and women to be examined separately. At all educational levels and in all age-groups the income of women is lower. This is especially true at higher educational levels. This means that for women the income differences among educational groups do not increase with age to the same extent as for men.

Graph 20. Median income 1971 for full year and full time employees by age and education¹



¹ For the definition of educational levels 1–7 see graph 7

Source: SCB Statistiska Meddelanden Nr Be 1973:12 p 15

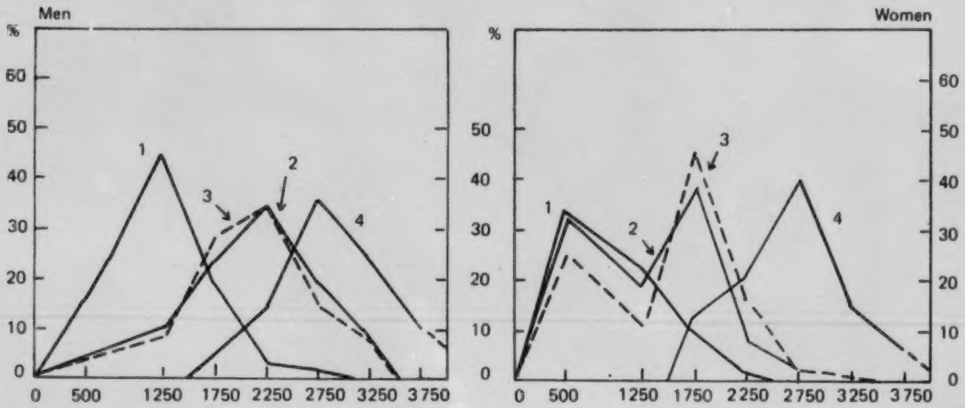
Let us also report some information comparing earnings in 1971 for employees at different educational levels working for the private and public sector respectively.[25] Three educational levels can be distinguished: university, upper secondary (excluding vocational lines) and other education (i.e. mostly pre-upper secondary and vocational upper secondary). For university male graduates earnings were, in all age-groups, higher in the private than in the government sector. The differences were most important in age-groups 35-55. For female university graduates earnings were about the same in both sectors. At the upper secondary level, for both men and women earnings were somewhat lower in the private sector up to 20-25 years and higher for older people, especially for men. The same pattern is found at the lowest educational level for men but not for women. For them the earnings in the government sector were higher in all age groups.

Let us also give a few indications about the income variations within educational groups. The 1966 material[26] shows for men both the absolute spread (as measured by the decile distance D_9-D_1) and the relative spread (as measured by $(D_9-D_1)/D_5$) to increase within a given age-group with educational level and within a given educational group with age. This means that the spread is quite substantial within the oldest age-group at the highest educational level. A more detailed study of the government employees (males) arrived at less regular results.[27] The standard-deviation was for each age-group higher at higher educational levels but only for those with secondary education did it rise monotonically with age. For those with high education it rose, leveled off and finally decreased. The same type of irregular pattern was also observed at the comprehensive educational level. The patterns as to relative spread (standard-deviation as a percentage of average salary) were even more irregular. For no educational group did it rise monotonically with age. For a given age-group those with secondary education most of the time had the highest relative spread. In this study it was also demonstrated that the overlaps in earnings for different age-groups was much greater at lower educational levels. Those with high education only seldom fell within the normal wage interval of those with lower education.

Recent graduates

The distribution over income classes of those leaving the educational system around 1972 at different educational levels, 6 - 12 months after graduation, is given in graph 21. As higher educational level roughly means that the whole distribution of monthly salaries is pushed to the right. The differences between those leaving the upper secondary school after 2 and 3-4 years on theoretical lines are, however, practically non-existant. It can be added that among those with lower education and among women there is a greater proportion gaining practical experience which presumably lowers their earnings.[28] However, judging by the previous figures this is not a fact which is compensated for later.

Graph 21. Monthly salaries in the spring of 1973 for persons working full time who have completed their comprehensive schooling (1), 2-year "theoretical" lines (2) and 3–4 year lines (3) in upper secondary schools and salaries 6 months after graduating for those having a first-degree from universities and specialised university colleges (4)



Source: SCB IFP 1974:8 Trender och prognoser 1974 p 180

For graduates from university and specialised university colleges we can make comparisons between certain groups with different educational orientation.29) No information, however, exists for graduates from the faculties of medicine, dentistry and pharmacy.

The highest median earnings among men with full time work in 1972/73 six months after graduation were found for Masters of Arts from the faculty of liberal arts, of technology, of natural sciences and Bachelors of Arts from the faculty of technology and the lowest for Bachelors of Arts from the faculty of natural sciences, of social sciences and of liberal arts. The corresponding highest median earnings for women were found for Bachelor of Arts from the faculty of technology and journalists and the lowest for Bachelor of Arts from the faculty of liberal arts, of Economics and Business Administration and of natural sciences. For every type of degree the earnings of women were lower than those of men. The difference varied from 1% to 27%.

The earnings of these graduates also vary with occupational field. The salaries of teachers, for both men and women, are far above other salaries. For educational lines preparing for this as well as for other occupational fields the labour market conditions for those who gain access to the educational field are very different from those in other fields, such as administration and clerical occupations. Investigating the income differences among graduates from the faculties of liberal arts, social and natural sciences excluding teachers the earnings do not vary much with educational orientation and earnings differences between men and women are also somewhat reduced.

Besides the educational field, salaries are also high in technical and mathematical fields for men and in the fields of journalism, psychology and law for women.

Life-time earnings

So far we have compared the earnings of different educational groups within age-groups. Higher education normally implied higher earnings and lower risks of unemployment. However, persons with higher education may be expected to spend a shorter time on the labour market and may, during the educational period, have certain extra costs besides the earnings foregone. It is interesting to take all these factors into consideration simultaneously. Summing up the earning differences over the whole working life of individuals with different education, including the direct educational costs as negative income during the period of prolonged education, would be one way of doing so. For this we can either use longitudinal or cross-sectional data. In the first case you describe the differences really experienced by a cohort of individuals but produced by an educational system different from the present one. In the second case you have in the younger age-groups individuals that recently left the educational system but the life-time earnings are calculated on income differences in older age-groups influenced by the educational system at an earlier period.

Let us therefore present both types of data beginning with life-time earnings calculated from a cross-sectional material for earnings in 1968. Table 8 indicates, for different types of higher education, how much higher the life-time earnings are compared with those of a secondary school engineer. Separate calculations have been made for men and women, before and after taxes. It should be noted that the results are:

- for undiscounted earnings
- including direct educational costs. Earnings before taxes are combined with total direct educational costs (both those of the individual and those of the state) and earnings after taxes with the direct educational costs of the individual only
- for women based on a comparison with the earnings of the male secondary school engineers reduced by 40% in the case of earnings before taxes and by 30% in the case of earnings after taxes
- obtained when the earnings of the highly educated are estimated for groups that include both economically active and not economically active individuals whereas the earnings compared with only refer to economically active persons. For the latter, however, earnings do not include overtime and other extra earnings nor are individuals for whom earnings

Table 8. Differences in life-time earnings between individuals with different types of higher education and secondary school engineers. Cross-section in 1968 (1.000 Skr)

	Women		Men	
	Before taxes	After taxes	Before taxes	After taxes
Type of higher education:				
Medicine	1,100	490	1,900	650
Business and Administration	- 80	- 50	860	340
Law	260	90	760	240
Technology	170	90	640	290
Dentistry	300	140	570	240
Natural sciences	240	80	320	100
Social sciences	30	- 20	230	40
Foresters	20	- 60	130	40
Liberal arts	70	20	60	- 40
Agriculturists	- 380	- 200	- 90	- 40
Social workers	- 160	- 110	- 130	- 100
Theology	- 160	- 120	- 240	- 160

Source: I Ståhl: U74. En samhällsekonomisk analys av den högre utbildningen. Malmö 1974 pp 39, 46
SOU 1972:23 pp 147–148, 186, 188, 194, 196

are negotiated individually included. However, a secondary school engineer, presumably has higher earnings than the average secondary school leaver

- the income concept used is income from all sources after deduction of certain costs and deficits.

Calculated in this way the life-time earnings before taxes are higher for all types of higher education except for the agriculturists, the social workers and the graduates from the faculty of theology plus Business and Administration for women. As a rule the differences in life-time earnings for women are lower than for men. Weighing the different types of education by the number of graduates we arrived at an average rate of return of approximately 4% for men and 3% for women.

After taxes the differences in life-time earnings are generally lower. This means that in case of a life-time earnings differential in favour of the highly educated it is reduced after taxes sometimes so much that it turns into a negative differential. When higher education yields a negative differential already in the calculation before taxes the negative differential becomes smaller after taxes. In many cases the differential, however, remains positive even after taxes. The only new negative figures besides those that were negative already in the calculation before taxes are for liberal arts graduates for men and for social scientists and foresters for women. However, altogether this means that for women 6 out of 12 and for men 4 out of 12 of the different types of higher education in table 8 give lower life-time earnings than those of secondary school engineers. A weighed average of the rate of return would very approximately be somewhat below 4% for men and 3% for women.

As a complement let us cite a few results obtained from cross-sectional material describing the salary differences of male government employees in 1971.30) Comparing university gratuates and graduates from upper secondary school with persons having only comprehensive education the rate of return before taxes was found to be 7.3% and 4.7% respectively. The larger investment thus gives a higher return. The ranking between different types of higher education was about the same as in table 8, that is high returns for medicine, business and administration, law, technology and dentistry and low returns for liberal arts, social workers and theology. At the upper secondary level the highest return was noted for those finishing longer theoretical lines. The returns after taxes were reduced for liberal arts graduates from 8 to 4% and at the upper secondary level for those finishing longer theoretical lines from 11 to 7%.

Finally let us report the results from a synthetic cohort investigation.31) Only certain groups of engineers and economists were studied and direct educational costs disregarded. The results thus arrived at in terms of rates of return to higher education for a cohort born in 1930 were:

	Before taxes	After taxes
graduate engineer/upper secondary school engineer	17%	12%
graduate engineer/lower secondary school engineer	19%	14%
graduate economist/upper secondary economist	18%	13%

Once more we note that the rate of return is lower after taxes. The absolute level of the rate of return is, however, much higher here.

4.2.3 Development of income differences between educational groups

Our possibilities to investigate the development of income differences between educational groups parallel to the educational expansion are rather limited. In table 9 we have, however, put together some approximately comparable data concerning the earning ratios for university graduates and persons with upper secondary and comprehensive education in 1934, 1960, 1966 and 1971. Before making the rather risky attempt at comparing these figures let us first comment upon the 1934 and 1960 data.

In 1934 salaries as well as other aspects of the labour market situation for different types of university graduates was investigated by officially appointed experts. Their appointment was to a certain extent due to apprehensions concerning the labour market situation for graduates and the increasing supply of graduates. This means that the income figures they collected may be low if they are to represent a more normal situation before or at the beginning of the educational expansion. This is, however, probably true to a lesser extent for the earning ratios. Moreover, the income figures are only for full time full year work at regular occupational posts. For an important group of university graduates (i.e. those employed in higher education) there are no earning figures. For the rest of the public sector and the private sector consisting of banks, insurance companies, railways and the manufacturing industry the data were indicated to have a good coverage or at least to be representative.

As can be seen in table 9 the median earnings of university graduates are normally higher than for persons with only a Higher Leaving Certificate. There are certain exceptions: the lawyer at the beginning of his career in the public sector and the liberal arts graduate in banking whose salary never exceeds that of a person with a Higher Leaving Certificate. Besides these cases the income differential varies from 10-90%. It is frequently higher in the lower age-groups which may be surprising. It also seems to be somewhat higher in the private than in the public sector.

Table 9. Ratios of median earnings in 1934 and 1971 and ratios of average earnings in 1960, 1966 and 1971

1934	Age	- 35	35 - 50	50 -
University graduates/persons with Higher Leaving Certificate by educational orientation of graduates and by economic sector				
medicine	P	1.4	1.3	1.2
dentistry	P	1.9	1.4	1.2
law	P	0.9	1.5	1.4
"	B	1.9	1.7	1.2
"	I	1.8	1.5	1.0
technology	P	1.3	1.3	1.4
"	I	1.9	1.5	1.1
"	R	1.9	1.6	1.6
"	M	1.8	1.5	1.2
liberal arts, natural and social sciences	P	1.1	1.2	1.2
"	B	0.7	1.0	1.0
"	I	1.6	1.3	1.1

P = public sector
B = bank
I = insurance company
R = railway
M = manufacturing industry

Table 9. (page 2)

1960 (Men only)	Age	- 30	30 - 40	40 - 50	50 - 60	60 -
University graduates/persons with Higher Leaving Certificate by educational orientation of graduates and total						
medicine		4.8	1.5	1.9	1.5	2.5
technology, law, social sciences		3.1	1.3	1.7	1.3	2.1
theology, liberal arts, natural sciences		2.0	1.0	1.1	1.0	1.3
all first-degrees		3.0	1.3	1.5	1.2	2.1

1966 (Only male government employees)	20	30	40	50	60
University graduates/upper secondary education	-	1.4	1.5	1.6	1.6
Upper secondary education/comprehensive education	1.2	1.2	1.3	1.4	1.4
1971 (Only male government employees)					
University graduates/upper secondary education	-	1.4	1.5	1.5	1.5
Upper secondary education/comprehensive education	1.0	1.1	1.2	1.3	1.3

Table 9. (page 3)

1971	Age	20 - 35	35 - 50	50 - 60	Total	
					Men	Women
Post-secondary education more than 2 years/upper secondary education more than 2 years		1.3	1.5	1.3	1.5	1.4

Sources: SOU 1935:52 pp 139–367, SOU 1963:74 pp 277–281, SCB:SM No Be 1973:17
S Gustafsson: Lönebildning och lönestruktur inom den statliga sektorn. Uppsala 1976 p 135

The 1960 figures are for average income but not only for those working full time and full year but all university graduates and persons with academic Higher Leaving Certificate irrespective of degree of economic activity. This of course makes comparison with figures for 1934 more difficult but they can be of interest in themselves, combining the effects of earnings per hour and hours worked. The average earnings are higher for university graduates in all instances except for the earnings in certain age-groups of graduates from the faculties of theology, liberal arts and natural sciences which are equal to those of persons with a Higher Leaving Certificate. The relative differences are highest for the youngest age-group and later decline though there is not much of a regular pattern in this decline.

Now, making comparisons the following complications should be kept in mind:

- differences in age-groups
- differences in definitions of educational groups. In 1934 the university graduates working in higher education are lacking. The difinition used in the second set of 1971 figures is wider and includes more types of higher education
- in all cases except for 1960 the earnings are for full time, full year employment. For 1960 only the results for men are shown as they can be assumed to be less influenced by differences in labour market participation. The data for 1966 and the first set of 1971 figures are only for male government employees
- the ratios are for average earnings except in 1934 and 1971 (second set) where median earnings ratios are reported. The 1960 income concept is wider including earnings from different sources while for other years only labour income (though not exactly defined in the same way) is reported.

Together all this means that these data do not constitute any ideal material for drawing conclusions about the long term development of income differences. However, let us consider the pattern of wage ratios by age-groups. In 1966 and 1971 it looks different from the one in 1934 and 1960. In 1966 and 1971 it is not highest among the youngest and then falls but first rises and then levels off or even falls back in the older age-groups.

As for the absolute level of the wage ratios university/upper secondary education it appears to be lower in 1971 than in 1960 and 1966. Compared with 1934 the level of the 1971 wage ratios seems to be much more about the same.

Wage ratios for upper secondary level/comprehensive level can only be compared for the period 1966-71. Here we can definitely notice a reduction in the ratios in all age-groups. This further means that during this period the wage ratio university/comprehensive level also decreased from 1.7, 2.0, 2.2 and 2.2 to 1.5, 1.8, 1.9, 2.0 in the respective age-groups.

As a complement to this information let us add a few results from certain special investigations as to the development of certain wage ratios in the private sector, life-time earnings and the situation of recent graduates on the labour market.

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In table 10 for certain educational groups we can follow the development of wage ratios in the private sector 1952-70 and 1966-71. For the period 1966-71 we can compare this with the corresponding development in the government sector. These figures are only for males and standardised for age and 1952-70 for the total even for changes in the distribution over occupational levels. The development in the private sector 1952-70 is characterised by rather stable wage ratios. Towards the end of the 60s a certain tendency towards lowering ratios for graduate and upper secondary school engineers as compared with the average can, however, be observed. The over all stability is all the more impressive as considerable fluctuations in the numbers recruited were registered during this period.32) However, the data 1966-71 show that the decrease in wage ratios may have accelerated towards the end of that period. The development is about the same in the government sector though the level of the ratios are lower especially for university graduates.

Table 10. Average wage ratios 1952–70 and 1966–71 in the private and government sector (corrected for changes in age distribution)

	Private sector							Government sector	
	1952	1960	1965	1970		1966	1971	1966	1971
GE/T	1.88	1.86	1.83	1.76	U/O	1.88	1.70	1.76	1.64
USSE/T	1.29	1.26	1.25	1.20	S/O	1.33	1.24	1.32	1.21
LSSE/T	1.12	1.13	1.14	1.11					
GE/USSE	1.46	1.47	1.46	1.47	U/S	1.42	1.36	1.34	1.36
USSE/LSSE	1.15	1.12	1.09	1.09					
GEc/T	1.69	1.66	1.65	1.64					

GE	graduate engineer
USSE	upper secondary school engineer
LSSE	lower secondary school engineer
GEc	graduate economist
T	average wage for all non manual workers corrected both for changes in distribution over age-groups and occupational levels

U	university graduates
S =	USSE + LSSE + upper secondary school economist
O	other education

Sources: SOU 1971:62 p 217
S Gustafsson op cit p 206

As for the development of life-time earnings we have certain information from 1934, 1960-68 and 1966-71.33) The calculations for 1960 and 1968 (see table 8) are comparable and show that for males and most of the different types of higher education considered the differences in life-time earnings increased. Exceptions were engineers and social workers. For 1934 the approximately corresponding gross difference for the lawyer and the upper secondary school leaver in public employment was estimated at 33,000 Skr. In 1968-prices this would correspond to about 120,000 Skr which is very much less than the figure for the lawyer both in 1960 and 1968 (760,000 Skr).

The calculations for 1966 and 1971 are also comparable. They show that for the male government employees the rate of return before taxes fell from 10.4 to 8.3% and from 10.3 to 6.6% for respectively university and upper secondary school graduates as compared with the comprehensive level.

Several investigations have been made concerning the development of the labour market situation for university graduates during the 60s and early 70s. However, we cannot draw any conclusions from them as to the development of the relative position of university graduates, i.e. relative to other educational groups. As for the development of the absolute position of graduates on the labour market they point to a certain deterioration:

a) The development 1966-73 within the public sector with respect to the educational composition of persons under 30 recruited for certain appointments. Here the percentage "university graduates" rose considerably (from 21 to 45%). Above all they have come to make up a much greater percentage of those recruited for lower salary grades. This is especially true for the men. The share of persons with other types of higher education has also increased which means that the percentage for lower educational groups has been reduced among those recruited for these appointments.[34)]

b) Similar figures exist for the field of the Swedish Confederation of Employers for male university engineers and economists. Between 1960-70 the average occupational position for newly appointed university engineers declined while it first rose and later declined for the economists being about the same at the beginning and end of this period. Further, keeping the level of occupational position recruited constant, it was found, for engineers that at the occupational level formerly dominated by upper and lower secondary school engineers, more than 50% were university engineers by 1970. Similarly among economists, lawyers and social scientists recruited for certain occupational levels. In 1970 university graduates made up a greater share than secondary school economists which they did not do in the beginning of the 60s.[35)]

c) The salary position of university graduates 6 months after graduation that we already described for 1972/73 can be followed back to 1967/68. For most

> groups the nominal salaries in 1972/73 are but a few percentage points above those in 1967/68. The exceptions are for men university engineers, lawyers and Masters of Arts from the liberal arts faculty and for women social workers, lawyers and Bachelor of Arts from the liberal arts faculty. Among the liberal arts graduates the more positive development mainly occurred for tachers.36)

This information indicates that the entry into the labour market has become more difficult for university graduates even though this might not be so in relative terms. Moreover, it may be compensated later. It has been noted that at least for certain types of higher education those who are recruited at lower occupational positions have a more rapid career development.37) Also, the differences after 3 years of labour market experience between certain classes of university graduates were much smaller than at the entry into the labour market. Those who graduate when business activities are low may get lower salaries and, to a higher degree, unqualified tasks but later the differences may disappear.38)

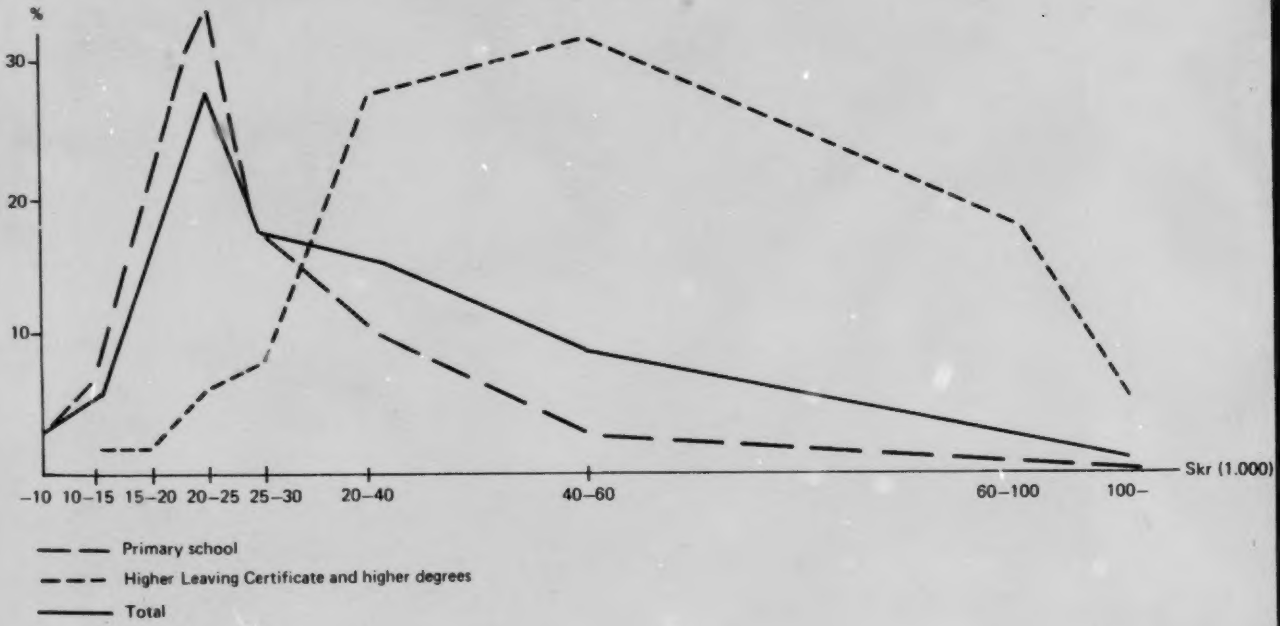
4.2.4 The income distribution

So far we have discussed income differences among educational groups measured in different ways. What about the income distribution? The distributional objectives mostly refer to some income distribution that ought to be equalised. Which one is less clear. When you describe the income distribution you have to decide upon what group to study (the whole population, the labour force, those employed, men, women, a certain age-group etc.) and how to define income (total income from different sources, income from employment, life-time earnings, annual earnings, income before or after taxes etc.).

Graph 22 illustrates the 1966 distribution of income from employment for males full year and full time employed aged 14-74. The distribution within the highest and the lowest educational groups are also shown. Given the shape of these distributions and the fact that most individuals belong to the lower educational groups and very few to the higher educational groups the over all income distribution is centered around the lower income classes but with a prolonged right-hand-tail.

Fundamentally, and for the same reasons the income distribution would have the same shape if we draw it up for the 1971 earnings data, for more restricted age intervals in 1966 or 1971, or for those who in 1972/73 had recently left the educational system at different levels. Our possibilities to relate the development of some such over all income distribution during the period of educational expansion to changes in the distributions within and between educational groups are, however, practically non-existant.

Graph 22. Income distribution 1966 for males full year, full time employed aged 14–74



Source: SOU 1970:34 p 115

The Swedish income distribution may be said to have become less uneven during the period 1951-73. If we study the annual income (income from all sources after certain cost and deficit deductions but before taxes and non taxable transfers) of the population aged 20 years or more the following can be noted:39)

- the share of total income accruing to the highest deciles has decreased while it has increased for the lowest deciles. Studying men and women separately this development has, above all, characterised the distribution for women. For men, the situation in 1973 was about the same as in 1958 as a result of a certain tendency towards inequalisation up to 1968 and towards equalisation after that
- the coefficient of concentrationx) continually declined from 1951-73 and more rapidly so after 1968. Again this tendency was strongest for women. For men, the coefficient rose somewhat up to 1968 and later fell being about the same in 1958 as in 1973.

Also the after tax income distribution can be said to have become more even. The differences in income inequality between the income distribution before and after taxes also seem to have widened especially after 1964 which may be interpreted as an increase in the equalising effect of taxes.40)

This type of income statistics, of course, do not include any educational variables. As was seen above we were not very successful in finding earnings data by education from other sources. If we want to describe the long term development of the income distribution as related to educational changes we must know both the quantitative development of different educational groups and the income differences between educational groups. Neither type of data exists in abundance. What is especially lacking are data concerning the wage differences among lower educational groups (primary, comprehensive and upper secondary school). It is at these levels that the greatest educational changes have taken place. For higher educational levels more information exists but uncertainty arises due to the incomparability of data. Instead of drawing any definite conclusions this tentative interpretation of data is proposed:

- the expansion of higher education could have been accompanied by increasing wages relative to the next lower educational level. These differences might later have stabilised or decreased
- in the late 60s the wage ratios university/comprehensive and upper secondary/comprehensive school seem to have decreased

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x) The coefficient of concentration or Gini coefficient varies from 0-1, a lower numerical value indicating less inequality.

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- the expansion of higher education might, to begin with, have tended to increase the income inequality in the population. Later, as more individuals received higher education at constant or decreasing wage ratios, there might have been a tendency in the opposite direction. As a whole these effects on the income distribution of the population should have been small as only a minority was involved in higher education
- the equalising effects of expanded upper secondary education in combination with decreasing wage ratios in the late 60s could have been of much greater importance
- education can also have affected the income distribution by influencing the rate of labour force participation. The equalisation of the income distribution for women can be assumed to be partially caused by higher rates of labour force participation. Higher education can have contributed to this development
- if the income ratios do not increase or continue to decrease the income distribution for younger generations will be more equal than for older generations as they are more evenly distributed by educational levels and the relative spread in educational years probably also will be lower.

4.2.5 International comparisons

After commenting upon studies in different countries relating different explanatory variables to individual earnings we will make a few international comparisons as to relative wages by educational level, rates of return to education and education and income distribution.

Individual earnings and explanatory variables

In numerous studies, especially in the U.S.A., variations in individual earnings have been related to different explanatory variables. As methods, definitions and types of dependent and independent variables, populations investigated and time for the investigation have varied it is not surprising that the authors arrive at diverging results. Among the more extreme points of view we can note the following:[41)]

- neither social background, ability nor education affect the earnings of individuals
- the social background is of prime importance for individual income
- education and training are decisive for the earnings of individuals
- ability (eventually as certified by education) is of highest importance.

We cannot go into all the details that would be required to try to explain these differences. Moreoever, in most cases there would not be any comparable studies in Sweden. Let us, however, note that the methodology by which it was found for the U.S.A. that neither social background, ability nor education "explains" earnings differences to any greater extent, very much resembles the path-analysis technique applied to the Swedish longitudinal data for individuals.42) In the Swedish investigation the explanatory variables did, as we already pointed out, "explain" a considerable part of the variation in individual earnings. Social background mainly affected education and education affected occupation and earnings.

Relative wages

Table 11 shows relative wages by education in different countries. The figures for all countries but Sweden are taken from an international study. There are of course innumerable difficulties involved in making comparisons between countries. Definitions of educational levels differ and the coverage and comprehensiveness of earnings data vary between countries. For Sweden we have added two sets of data, those for 1972 are based on average earning for people working at least half of the year. The fact that the age distribution may vary from one educational level to another has not been taken into account. This was done for 1971 but these figures are only for the government sector. The only difference of any importance between the Swedish figures are for males where wage ratios are lower in the government sector.

The wages are about 40% higher at the secondary level. Sweden and Japan are on the low side with about 20% only. The figure for Belgium is exceptionally high.

The premium on higher education over secondary education is, in most countries, higher than on secondary over primary education. As for the ratio higher/primary level the figure for Belgium is again exceptionally high. For other countries the figure lies around 210-220. Sweden is again on the low side this time in company with Japan and the Netherlands. No clear tendency towards a lower ratio in richer countries is discernible. Compared with less developed countries the OECD ratios are, however, low. In the former countries figures around 600 have been noted.43)

We can also try to make a few more comparisons between the

Table 11. Index of average annual earnings of labour by level of education in OECD countries

Country		Year	Educational level Primary	 Secondary	 Higher
Belgium		1960	100	251	502
Canada		1961	100	144	263
France		1968	100	183	289
Greece		1960	100	139	220
Italy		1969	100	141	244
Japan		1968	100	117	161
Netherlands		1965	100	131	152
Norway		1966	100	140	213
U.S.A.		1967	100	129	200
U.K.		1967	100	140	225
Sweden	total	1972	100	122	189
	men		100	123	202
	women		100	118	173
	men	1971	100	123	175
	women		100	121	170

Sources: G Psacharopoulos, Earnings and Education in OECD Countries, OECD, Paris, 1975 p 165
SCB PM 1975:11 Utbildningsstatistik 1970–1974 p 24 S Gustafsson op cit p 181

U.S.A. and Sweden. In table 12 relative wages for full year, full time, male workers are given. In the U.S.A. the wage ratios remained remarkably stable during the 50s and 60s but between 1969 and 1973 the relative mean income of college graduates fell compared with either of the lower educational groups as can be seen in the table. The same happened in Sweden and a similar development has been reported from countries such as Japan and Italy.[43)]

As for the level of the wage ratios, and their fall, the differences between Sweden and the U.S.A. are small. Perhaps the wage ratios could be said to be somewhat lower in Sweden but the fact that only the government sector is included may overstate the case.

Rates of return to education

In an international study information about rates of return to

Table 12. Relative wages in the USA and Sweden. Full year, full time male workers by age groups

U.S.A.	25-34	35-44	45-54	55-64
College/high school graduate				
1969	1.4	1.5	1.7	1.7
1973	1.2	1.5	1.5	1.6
College/grade school graduate				
1969	1.7	2.1	2.2	2.2
1973	1.6	1.8	2.0	2.0

Sweden (government sector)	30	40	50	60
University/upper secondary				
1966	1.4	1.5	1.6	1.6
1971	1.4	1.5	1.5	1.5
University/comprehensive				
1966	1.7	2.0	2.2	2.2
1971	1.5	1.8	1.9	2.0

Sources: R B Freeman: Overinvestment in College Training? in Journal of Human Resources, Vol X:3
S Gustafsson op cit p 135

education as calculated in different countries were gathered. In table 13 some of these results are illustrated. Certain information about Sweden has been added. The comparability of these data can, of course, also be doubted even though a great effort was made to achieve figures comparable across countries. The following limitations to comparability can, however, be noticed. The profitability measures relate to different points in time. The coverage of the studies also vary. In the U.S.A., for example, the calculations are only for white males and in West Germany only for males. In certain countries (e.g. the Netherlands and Belgium) the return to higher education has been calculated as the average for only certain university educations. The Norwegian, Danish, Japanese and British figures for higher education are only based on earnings in a more or less restricted number of sectors or occupations.

Table 13. Rates of return by educational level and country

Country	Year	Gross rates of return			Net rates of return		
		P/O	S/P	H/S	P/O	S/P	H/S
U.S.A.	1959	18	14	10	155	20	14
Canada	1961		12	14		16	20
Great Britain	1966		4	8		6	12
Norway	1966		7	8		7	8
Denmark	1964			8			10
Netherlands	1965		5	6		9	10
Belgium	1967			9			17
West Germany	1964						5
Japan	1961		5	6		6	9
Sweden	1968			4			3
born	1930			12			7
	1971		5 - 7				

O no education
P primary education
S secondary education
H higher education

Sources: G Psacharopoulos, Returns to Education. New York 1973, p 62 SOU 1972:23 pp 149–156
A Klevmarken m fl: Industritjänstemännens lönestruktur. Uppsala 1974 p 90 S Gustafsson op cit p 131, 134

As for the Swedish figures, the following should be said. The figures for 1968 are an average for males with certain types of higher education. They are based on cross-sectional data which, in this respect, makes them comparable to those of other countries. However, as was already pointed out the Swedish figures for 1968 are low because the group with higher education includes all individuals irrespective of degree of economic activity. The earnings compared with refer only to persons employed. For other countries the earnings data used seem to reflect both differences in wage rates and degree of individual economic activity but for both groups compared. This procedure can be expected to result in higher rates of return than if the Swedish 1968 method had been applied.

The Swedish figures, for those born in 1930, are based on comparisons between groups of employed persons only. They are, however, only for male engineers and economists. The figures have been adjusted downwards to be comparable with cross-sectional results. The figures for 1971 are only for male government employees.

In table 13 both gross and net rates of return are given. The gross return has generally been calculated on total educational costs and earnings before taxes and the net return on that part of the educational costs the individual pays and earnings after taxes. There are, however, exceptions to this. In the case of the U.S.A., Canada and Belgium the net return was indicated to have been calculated on earnings before taxes. For the U.S.A. it was, however, noted that the net rate of return would have been about the same even if calculated after taxed earnings.

All these differences in methods and definitions make comparisons extremely difficult. Comparing the gross and net rates of return it can, however, be noted that only in Sweden are the latter lower than the former. The fact that the individual in most cases only pays for part of the direct educational costs would tend to rise the net return above the gross. The effect of taxes can be ambiguous as they, to varying degrees, both lower the earnings foregone during education and the earnings received after education.

In most countries both the gross and the net rates of return are lower for secondary compared with primary education than for higher compared with secondary education. This means that an individual taking higher education has the opportunity to make a more profitable investment. An exception in this case is the U.S.A.

The variations in net rates of return to higher education are considerable. Sweden, whether judged by the figure 3% or 7%, has a low net rate of return. The variations in gross rates are also large. The Swedish figure of 12% belongs to the higher ones but the figure of 4% to the lowest ones.

Later calculations for the U.S.A. showed that while the rate of return to higher education (college graduates compared with high school graduates) remained stable from 1959 to 1969, it was reduced by about 1/3 from 1969 to 1974. In 1974, the gross rate of return was 7.5% and the net was 8.5%.[44] (In this case the net return was actually calculated on after tax earnings.) Similar tendencies have also been noted in Sweden.

Returning again to table 13 it can be seen that also for secondary education both for the gross and the net rates of return the differences among countries are quite large. The Swedish figure 5-7% is rather low.

Distribution of income and education

We will end this section on international comparisons by reporting the results from an attempt made by the OECD at investigating the existence of direct relations between the distribution of income and the distribution of education within and between countries.[45] Information about the distribution of education could only be gathered for five countries: Canada, Denmark, France, the Netherlands and the U.S.A. Measuring the inequality in either distribution by the Gini coefficient it was found that the inequality in education was lower than the inequality in income. However, countries with the lowest income inequality did not necessarily have the lowest educational inequality. In all of these countries the income inequality decreased during the period studied. No clear relation between the rate of decrease in income inequality and reduced educational inequality could, however, be established. These data did not support the hypothesis that equalisation of the distribution of education directly influences the income distribution.

5 Summary and discussion

In this section we first summarise the development, as far as we have been able to document it in this report. Thereafter, we turn to the more complicated question of how to interpret our data. By what mechanisms are educational and other economic variables related? Do we know enough about these relations to be able to speak about the effects of education and educational policy?

5.1 The development and present situation

In the very first section of this paper we described the objectives of educational policy with respect to the labour market under the three headings: social change, distribution, and allocation and growth. Now, the expectations as to what the educational policy would achieve within these areas were never exactly stated. Of course, factors other than educational policy measures also influence these goals sometimes supporting sometimes counteracting the possible beneficial effects of the educational policy. In this part we will, however, abstract from these complications and only briefly summarise what has happened within these areas parallel to the educational expansion.

The educational system in Sweden has certainly expanded considerably. Nowadays Sweden is one of the countries with the highest educational frequences in age-groups 7-25 years. Educational frequences are, however, higher, for example, in the U.S.A. Moreover, for children of pre-school age and adults, the educational frequences are still low.

The expansion of the Swedish educational system took place during a rather short period of time. At the lower secondary level, the upper secondary level, and the university level it can be said to have started in the 20s, 30s and 40s respectively. Once the expansion started it accelerated up to the late 60s. The fact that the Swedish educational expansion started relatively late also means that the average level of education of the whole population or labour force has not yet been raised so much. Compared with other countries it is not especially high either.

Vocationally oriented education now dominates over more general education both at the upper secondary and post-secondary levels. However, while its share increased at the former level it actually decreased somewhat at the latter during the 60s. Comparing the humanistic-social sector with the science and technical sectors we noted earlier that the former dominates both at the secondary and post-secondary levels and increased its share during the 60s especially at the post-secondary level.

The flexibility both within and in and out of the educational system has increased. Though recurrent education as, for example, measured by persons aged 30 or more at the universities has a large share of student numbers (about 20%) its volume in relation to that age-group is low (0.4%).

Summing up the development in certain distributional aspects it can for educational resources be noted that:

- The differences between generations are very big not only comparing the young and the old but also the young and the middle-aged. The volume of adult education is very small compared with this gap. The advantage of younger generations is also higher in Sweden than in many other OECD countries.

- The distribution over educational levels within generations has evened out. However, measuring by years of educational experience much less of equalising tendencies could be discerned. While the spread within an age-group (measured by the Gini coefficient) seems to have increased in Sweden it has decreased in a number of other OECD countries.

- The comprehensive school was not introduced in all districts until 1972/73. Its introduction should guarantee that for the future practically all children get a common basic education of 9 years. What this means in qualitative terms is, of course, more difficult to say. Certain investigations indicate that the results are not quite satisfactory in that respect. Moreover, there was a drop-out rate of 1-4% in grade 9. These children together with the 15-20% who do not continue to the upper secondary school do not get any formal vocational education which an education of that duration normally would have given earlier. The labour market situation for this group may become very difficult especially in the future as other forms of job training may disappear at the same rate as the vocational education at the upper secondary level consolidates.

- At the upper secondary level the distinction still exists, though in a somewhat milder form, as to who gets a practical and who gets a theoretical education. At the university level, this distinction never had the same actuality. Not until recently were the vocational technical programmes for people in trade and industry introduced on an experimental level. (276 students began such education in 1975.)55)

- The distribution of educational resources over individuals with different social background is very unequal. Approximately 11 out of 100 children from social group III as compared with 89 out of 100 children from social group I go to post-secondary education. Also, compared with the situation in other countries the inequalities in Sweden are rather large. Comparing generations it was noted that at the lower and less formal educational levels equalising tendencies between social groups could be observed both in absolute and relative terms. At the university level the development has brought decreasing differences in relative terms but increasing differences in absolute terms. Why this diffusion process has been so slow our data do not tell us. An intensified study of cohorts could give more information about the way in which the aspirations of parents are influenced by their educational and work experience and how these in turn influence the educational choice of their children. If the social distribution of educational resources is closely linked to such adaptation mechanisms it may indeed be less surprising that it has been so slow.

As for other indicators of social welfare we noted certain differences between educational groups:

- Earnings. The differences in earnings between educational groups are rather important in Sweden though they seem to be much more substantial in other countries. The figures for Sweden and the U.S.A. are, however, very similar in spite of the fact that the proportion of highly educated is much lower in Sweden than in the U.S.A. Life-time earnings, whether calculated on cross-sectional or longitudinal data, have normally been found to be higher for those with secondary and higher education than for those with shorter education. The differences are smaller after than before taxes. The opposite situation seems to prevail in many other countries. It also seems plausible that the Swedish educational expansion has taken place at a level of rates of return after taxes lower than that in other countries.

- Unemployment. The differences among educational groups on the labour market were in this respect rather large in 1975 and about the same as they had been in 1970. However, for recent graduates the differences between educational groups in the early 70s seem to have been small although this might have been a transitional phenomenon.

- Occupations. At higher educational levels people more often end up in white collar than in blue collar jobs. Those with higher education seem to be much more concentrated in certain types of white collar jobs (professional, technical and related occupations) in Sweden than in other countries. During 1970-75, however, certain changes occurred. The occupational profile of those with higher education shifted somewhat out of the white collar jobs and into the blue collar ones. At the same time the share in white collar jobs of the highly educated increased reducing that of other educational groups. For those with secondary education this led to a rather more substantial shift of the occupational profile out of white collar and into blue collar jobs. The occupational profile of those with only comprehensive education did not change.

- Labour force participation and mobility. The rate of labour force participation increases with educational level, especially for women. Those with higher education also seemed to be more mobile on the labour market. These factors are mentioned here as possible indicators of additional welfare effects to be associated with high education. Higher education presumably permits the individual to participate, to a greater extent, in search processes on the labour market whereby a job with not only higher earnings but also more in correspondence with personal preferences can be found. In the same way a longer educational period can permit the individual to better discover his own interests and preferences.

While these differences can be said to exist at present we found it more difficult to describe their long term development. It is possible that the differences in earnings before taxes for people with a university and a secondary education increased for a long period of time perhaps up to the middle of the 60s. In the late 60s and early 70s the differences in earnings before taxes for both university graduates and persons with secondary education, compared with those with less education, were probably reduced.

Lack of data prevented us from relating changes in wage ratios between educational groups and the relative size of different educational groups directly to long term changes in the over all income distribution. The educational expansion possibly increased the inequality of the income distribution before taxes to begin with but may have worked in the opposite direction lately. We could observe that the income distribution before taxes during the last 20 years has become more equal for women but remained rather stable for men. In the case of women the contribution of education might have been greater through influencing the rate of labour force participation than through changes in wage ratios and the relative size of different educational groups. For the after tax income distribution the equalising tendencies have probably been stronger especially lately. However, the long term development of differences between educational groups in after tax earnings, which are smaller than those before taxes, has not been possible to follow.

As for growth and allocational matters the following factors may be of interest. We note some of them once again as they were of relevance also from a distributional point of view:

- The rate of labour force participation that rises with educational level. The educational expansion may have increased the supply of labour.
- The high mobility of higher educational groups that may increase the efficiency in production.
- The low rate of unemployment of higher educational groups. The average rate of unemployment has been much lower after the Second World War than before although a certain tendency towards increasing unemployment rates have been observed during the 60s and early 70s. As we lack longer time series for unemployment rates in different educational groups we cannot say to what extent the reduction of the average unemployment rate can be associated with increasing shares of groups with low unemployment rates. The more complicated question whether lower unemployment rates for the highly educated really indicate a fuller capacity utilisation or is to be interpreted as a pure redistributional effect our data cannot answer.
- The occupational structure of the labour force has changed in the same way in Sweden as in other countries. White collar jobs have increased their share at the expense of blue collar jobs. At the level of aggregation we used (4 groups of white collar and 1 group of blue collar jobs) we found that the increased supply of highly educated 1970-74 was to a greater extent absorbed by increased densities within occupations than by the structural change in the composition of occupations at given densities. However, the density of higher educated in the economy as a whole as well as in the different occupations is still much lower than for example in the U.S.A.
- The development of the branch structure has led to an increasing share of the service sector at the expense of the goods producing sector. During the 60s the increased supply of higher educated not much more than fully corresponded to changes in the branch structure at given densities within branches. The density of highly educated within branches is also low in Sweden compared with the U.S.A. and Japan. However, it looks as if it was less behind and more rapidly approaches the densities in these countries in the service sector than in the goods producing sector.
- The above factors give a few indications about how the increased educational supply has been utilised. If we now at the macro level look at what has happened during the educational expansion we find that both GNP and labour productivity increased (except for the early 20s) during

the period we have studied. During the 50s and early 60s there were certain tendencies towards increased growth rates but later they fell back to more normal levels. This reduction thus occurred during a period when the additions of highly educated to the labour force were at its height and the educational level of the labour force grew with increasing speed. This fact, of course, only demonstrates that educational expansion by itself is not a powerful factor enough to guarantee increasing growth rates under whatever conditions. In this connection we would, however, like to add that the conventional GNP measure and measures of labour productivity based on GNP are ill suited for studying the present relation between education and production growth. In the public sector and other parts of the service sector as well, where most of the highly educated work, the value of production is estimated from the cost side which by definition excludes productivity increases.

- The educational expansion might have contributed to the growth of other aspects of welfare besides production such as job security and satisfaction. The fact that lower rates of unemployment, greater mobility and flexibility are associated with higher education point in that direction.

As for the more long term and fundamental social changes in a spirit of democracy and equality our material does not permit any firm conclusions to be drawn. While income differences between educational groups may have decreased, changes in unemployment rates and the distribution over occupations seem to have been small. A more detailed study of the development in these respects would be motivated. Especially investigations into changes of work content in different occupations and for different educational groups would give interesting information. To what extent has the increasing level of education in the labour force achieved or does it prepare the ground for new types of work organisation and changes in the structure and distribution of job tasks? What changes in attitudes are taking place and what do they indicate about what could be expected in this respect? Of course it must be remembered that only recently was the comprehensive school completely introduced and the educational expansion only slowly rises the average level of education of the whole population.

5.2 Alternative ways of interpreting the data. Possible theoretical explanatory models

Let us start by presenting four alternative (not necessarily irreconcilable) approaches to education:

1. Education as an investment in human capital for production purposes. As far as education has a durable effect

upon the individual it could be considered an investment. The effects can be of different types such as changes in stock of knowledge, in attitudes, in the ability to solve problems and to adapt. The value to the individual of this investment for production purposes, of course, depends upon a lot of things in an individual's labour market situation and his possibilities to make a career letting the services of his human capital equipment. There could both be a quantitative and a qualitative side to human capital.

2. Education as sorting. The individuals leave the educational system at different levels equipped with various types of certificates. In an educational system with a pure sorting function the classification would be based on innate ability or some other characteristica not influenced by education e.g. social background. It may be appropriate to make a distinction between capacity sorting and social sorting as those phenomena are likely to appear for quite different reasons and in different social contexts.

   Education could of course also fulfil a double function both improving e.g. cognitive abilities and certifying the existence of these cognitive abilities. The value to the individual of the sorting depends on his alternatives being sorted or not.

3. A search and information perspective could be applied to education as well as the whole life of an individual. The individual successively finds his way at the same time as his information about himself and the world around increases. Education is part of this process as well as it furnishes information.

   This search approach would of necessity lead to a multidimentional perspective on education and educational output that may or may not be incorporated into the human capital and sorting approaches. It would tend to stress dynamic elements in the educational process and education as a means to reduce uncertainty. It would include both investment and sorting elements.

4. Education as a value for other than production purposes. Individuals can participate in educational activities for the pleasure of so doing. It can also be a value of more durable character for activities in other than the production sphere such as the cultural, political and consumption sphere.

To be able to discuss the value of education outside the school both to the individual and the society at large you apparently also need to have an idea about the way the rest of the economy and the labour market works. One extreme view of the labour market is to regard it as a smoothly functioning market with perfect competition and mostly characterised by general equilibrium. Supply and demand for different factors of production would determine their respective rewards. Another extreme would be to completely deny the influence of market forces, and to see the conditions on the labour market as set by existing institutional conditions, the type and stage of development of the economic system. Education might influence the labour market but only indirectly via institutional change. In less extreme versions the existence of barriers to a smoothly functioning labour market and the influence of institutional factors may be given more or less emphasis.

The Swedish educational policy with respect to direct effects upon the labour market can be said to have been based mainly on a human capital approach, that is, education as an investment for production purposes. This has been coupled to a view of the labour market where institutional factors have a large influence especially on the long run development and where market forces do operate within the given institutional framework but not with perfect smoothness.

Let us first discuss what view of the labour market seems warranted by our data and then against this background more in detail analyse the role of education and possible explanations of the association between education and earnings. Of course these two questions are intimately related but a division of the discussion along these two lines may clarify certain issues.

5.2.1 The labour market

Whether the labour market functions in a smooth way according to the competitive market model reasonably close to general equilibrium or not has important implications for the interpretation of data.

Guidelines for educational policy in view of reaching efficiency and distributional goals might also be more easily obtained in case of a competitive market. Income differences could in that case be interpreted as differences in productivity. When a given production factor is paid the same price in all uses production could be regarded as efficiently organised at least from the point of view of firms. Of course policy makers might want to apply another valuation when planning to increase or decrease the supply of different types of educational groups or weighing educational expansion against other types of activities but the valuation made by the market may be a good starting point. Moreover, if private and social valuations were in line with each other the government may just supply the amount of education demanded by individuals and let the labour market take care of the adaption of supply to demand for different educational groups at least as far as only efficiency goals are considered. Distributional effects of changes in educational policy could also be predicted.

Our intention is not, however, to make a full-fledged evaluation from a welfare economics point of view of the results of the Swedish market system in educational respects. In this part we are going mainly to discuss the more restricted question whether the same or different prices are being paid for one factor of production. Such an analysis is of interest as it could indicate to what extent the Swedish labour market operates smoothly according to the competitive model.

In a competitive market one would, with certain qualifications, expect a single price to be established for every factor of production and that this price would correspond to the marginal productivity of the factor in question. For labour it would be possible to distinguish different factors of production according to varying qualifications or productive capacities. Individual differences in productive capacity could for example be due to differences in innate ability and education. For every such factor of production we would thus expect one price to reign. Before we get to the qualifications and the difficulties of identifying different factors of production in case of labour let us schematically illustrate the difference between a competitive and a less competitive market. In figure 4a two factors of production, for example individuals with longer and shorter education, are supplied (S_1, S_2). For each category there is only one price. In figure 4b there are two demand curves (D_1 and D_2) which results in two different prices being paid for each factor of production. Such a segmentation could

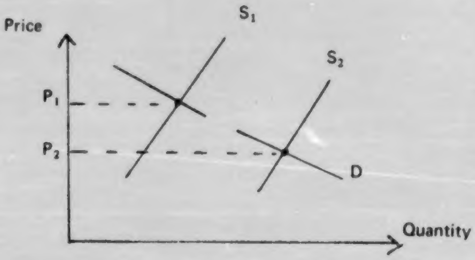


Figure 4a

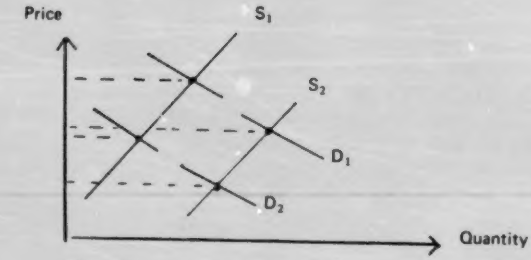


Figure 4b

indicate inefficiencies in production. Moreover, income differences now do not only stem from different demand and supply conditions for different factors of production, i.e. people with different productive capacity, but also from different demand and supply conditions for the same factor of production.[46)]

In certain instances, however, even with a highly competitive market different prices could be found for the same factor of production:

- when non-pecunary aspects are to a varying degree associated with different jobs
- when the reward, pecunary or non-pecunary, in different jobs are evaluated differently
- when adaption costs, both on the side of demand and supply, prevent factor price equalisation
- in the short run when adaption has not yet taken place.

Observing a situation like that in figure 4b we cannot always be sure that we are really dealing with different situations for the same factor of production rather than different situations for different factors of production. Within educational groups one should try to standardise as far as possible for personal capacity related characteristics. As for education, the situation of people with the same type of education could be compared. However, in a competitive market certain relations between the earnings at different educational levels and for different types of education at a given educational level would also be expected. The private rate of return to different types of education could be expected to be equalised.

Our data only permit a limited number of comparisons to be made. Out of the data presented earlier in this report we would like to point to the following evidence that seems to indicate the existence of segmentation:

1. As we saw earlier there may exist a dual kind of labour market where some people, the young, women, and immigrants, can get caught unwillingly for shorter or longer periods of time without having worse qualifications than people not working there.

   An investigation of individuals with low pay[47)] (less than 7 Skr/hour) made in 1967 showed that 15% (14% excluding students) of those employed got such a low pay. Low pay was more frequent among women than among men and much more frequent in younger age-groups (76% as compared with 8% in age-groups 25-44 and 25% for those older than 45 years). No one with a Higher Leaving Certificate or higher degree belonged to this group. In the other educational groups the percentage belonging to this group was reduced from one age-

-group to the next older one except for those with only primary education where the percentage stayed around 14% in all the older age-groups. Even for those with lower secondary education plus vocational training as many as 19% had low pay in the age-group 20-24. The risk of low pay may be influenced by education and other personal characteristics but it is probably also a problem of economic and industrial organisation.

2. If men and women just leaving the educational system with the same type of education are compared we can assume that differences in personal characteristics are small. Our evidence showed that for all those different types of university education investigated the salaries of women six months after graduation were lower than for men. Only those working full time were included in this comparison.

3. In the longitudinal study standardising for personal characteristics such as education and IQ test scores as much as 90-50% of the variations in logaritmic earnings could not be "explained" by the model. The explanatory power of the model was lower at younger ages than at older.

4. The other analyses relating earnings to supply and demand characteristics showed that such personal characteristics as age and education were more important in explaining earnings differences than region and type of activity. However, the effect of especially branch or type of economic activity was not negligable. Only in the public sector were regional variations practically zero. Labour market conditions apparently differ depending on where the individual worked.

5. The comparison between the private and government sector also pointed to the possible existence of a long term situation of disequilibrium. Differences in earnings were greater in the private than in the public sector among male educational groups and between males and females (standardising for age and education). The 1966-71 period, characterised by excess supply of highly educated but probably no more fierce competition between the private and public sector, brought a reduction in wage differences between men and women (standardising for age and education) and educational groups in both sectors. The rate of change was higher in the private sector but not high enough to erase the sector differences.

6. For a very long period of time (1934-68) rates of return to higher education probably increased. Also great differences have always existed between different types of education at a given educational level.

   While for male government employees in 1966 the rate of return to a longer (from compulsory level to higher education) and a shorter (from compulsory level to upper secondary level) investment seems to have been the same this was not so in 1971. By then the level of both returns had been lowered but in such a way that the return to a longer investment had become relatively higher than the return to a shorter investment. To this corresponds the fact that the wage ratios in different age-groups were much more reduced comparing the secondary and the compulsory level than comparing higher and secondary education.

7. Within a simple market model an increased supply of highly educated persons would be expected to drive down their wages. As the highly educated become relatively cheaper, employers would substitute them for other factors of production. Also, as the goods and services made with intensive use of highly educated become less expensive to produce a change in the composition of final demand in favour of such goods would be expected. Both these types of adaption would, to a certain extent, counteract the fall in wages for the highly educated. Such mechanisms have certainly been at work and contributed to the changes we observed in wages (reduced wage ratios in the early 70s), in occupations (changes downwards on the occupational profile for the two highest educational groups) and in branches. However, the adaption process often seems to have been rather slow. During the 60s when the output of highly educated grew rapidly, the wage ratios seem to have remained stable for a certain period of time, quantitative instead of price adjustments taking place. When wage-ratios fell, the higher and secondary level were brought closer to the compulsory level while the gap between the university and the secondary level remained about the same.

The occupational data also showed that the market adaption to excess supply may be reduced average wages for a certain educational group but not so much by reducing wages within occupations as by changing the occupational composition of the educational group. In doing so, the next lower educational group can also be pushed out of its normal occupations. Rationing of jobs would occur at different levels and wages be kept stable in various types of jobs48) instead of an equalisation of wages received by a certain educational group irrespective of occupation. This kind of behaviour creates a labour market disequilibrium and moreover stifles the adaption on the side of final demand.

8. The information about unemployment rates by education and occupation showed the same type of phenomenon, that is, treatment not according to qualifications but according to position. In manual occupations, the unemployment rates for highly educated were not lower than for those with lower education.

9. The persistent regional differences in unemployment rates can also be an indication of labour market segmentation.

Many factors may of course contribute to the creation and persistence of a situation where rates of return are not equalised and different prices are paid for the same factor of production: the non-existence of competitive forces, institutional and other factors restricting competitive forces, the slow working of competitive forces etc.

Let us discuss a few of these factors that seem to have influenced the Swedish situation.

The educational system

The institutional framework within which the educational system works may have hampered a rapid and efficient adaption to changes on the labour market. Obvious examples of institutional barriers to adaption are compulsory education and education with restricted entry. The high returns to certain types of higher education (e.g. in medicine and dentistry) must at least partly be explained by the restricted entry. The expansion of the "free" faculties was probably also initially held back by restrictions at the secondary level. The possibilities to get round formal entrance qualifications to higher education were introduced rather late. The supply of total numbers of people with different types of education seems, however, by and large to be interpretable as reactions upon market signals though such adaptions have generally been rather slow. Institutional restrictions have operated as barriers in periods of increased demand. A new phenomenon that may cause inefficiencies in the supply of individuals with different types of education in the future is short term overreactions on the part of students in periods of slack demand.

The educational expansion might also have been more rapid if it weren't for the lower participation in certain social groups. The fact that children from social group III, only to a limited extent, go to post-secondary education is, however, not necessarily only related to factors within the educational system. Factors on the labour market as well as the individual's and his family's perception of the value of higher education can also be of importance. If those who experience worse labour market conditions at higher educational levels were more often from social group III this might be of some relevance in this connection. The investigation of those with low pay47) does not permit this type of questions to be analysed. We can only note that among women 25 years or more 70% of those with low pay came from social group III compared with 50% among women not having low pay. For men there were no such differences.

Moreover, the differences among social groups as to participation in higher education may lead to inefficiencies as those best suited for higher education and most able to make a productive contribution once educated might not proceed to higher education. We know that IQ and higher education are far from perfectly correlated. We have earlier seen that the social background of an individual is of great importance for educational achievement. School marks may not do justice to those from lower social groups but even at equal school marks the percentage going to higher education is not the same. Especially at school marks of medium level there were great differences among social groups in this respect. Also, students from lower social groups seem to have less possibilities to utilise reorientation possibilities. Of course little is known about the importance of IQ and IQ plus education for work productivity. In at least some instances, however,49) education and IQ have been shown to have a positive interaction effect upon earnings. This would thus indicate that the selection process within the educational system is to a certain extent inefficient.

The educational system may also have had other internal inefficiencies as indicated by the negative interaction effect of education and IQ at an early age upon IQ at an older age.

The labour market

On the institutional side on the labour market we have already mentioned the labour unions and the commitment of especially the LO to a reduction of wage differences and increased employment security. The government has also increasingly taken stand in favour of these endeavours. Legislation to increase the security of employment has, for example, been passed. This legislation restricts the scope of action of market forces. Furthermore the whole public sector itself with its rigid salary schedules and seniority provisions is less adaptable to market forces.

As for the supply of labour with different types of education we have just noted that the flexibility has not always been very high. To this can be added the fact that the general mobility on the Swedish labour market has been (and perhaps increasingly so) low compared with the U.S.A. for example. Low mobility and flexibility on the side of supply would, of course, tend to create non-competitive results on the labour market.
The behaviour of private firms and other producing units can, besides the behaviour of supply and institutional factors on the labour market, be more or less conductive to the persistence of a segmented labour market with different conditions ruling for the same type of factor of production.50)

Long run efficiency considerations in cases of a stochastic supply of labour, continuous and substantial amounts of on-the--job training may lead the organisations to practice a certain rationing of jobs. Preferences on part of clients and employees in favour of discrimination may have the same effects. In situations with uncertainty, information and search costs may prevent factor price equalisation. Transaction and mobility costs have the same effects. Owners and managers of production might also have an interest in order to retain or increase control over the production process to organise work hierarchically. The wage structure, the number of hierarchical levels and of jobs at different levels could be arranged with that purpose in mind. In these situations labour market segmentation is not necessarily created for lack of competitive forces but for efficiency and control purposes as seen by managers of production and owners of firms.

In other situations, segmentation could be caused by the lack of competitive forces or the slow working of competitive forces on the demand side of the labour market. Discrimination based simply on prejudice and low mobility of capital would be examples of this.

Segmentation of the labour market can create not only different conditions within educational groups but also differences between educational groups on top of those that would have existed in a competitive market. Differences between educational groups will be further discussed in the next section.

The share of employment in the service sector and in white collar occupations has increased. The educational supply was more in line with the structural changes of branches than with the structural changes of occupations which could be expressed as the latter having changed more slowly than the former. The rationalisation and concentration process may have contributed to this. The public sector and bigger companies have come to play a more important role for employment. Certain tendencies towards the creation of internal labour markets have also been noted. These tendencies and the declining importance of small firms may have led to a more highly structured labour market. The changing composition of final demand and technical development possibly calling for more on-the-job training certainly influenced this development. How exactly all these factors are to be related to the organisation of work within various types of organisations and this in turn to the demand for labour with different types of qualifications and the way this demand expresses itself we know less about.

In many cases it would be possible to interpret our data in terms of a simple competitive model though adaptions to new conditions seem to be very slow. However, a closer investigation of the mechanisms or changing mechanisms behind such results would be instructive. For example, the recent reduction in wage ratios for the highly educated following a period of increased supply may not, or only still later, have come about if it were not for the commitment of the government and the larger labour unions to income equalisation. Certain tendencies on the labour market would seem to guarantee other forces than regular market forces a greater influence on the future development: the growing size of the public sector, the general interest in income equalisation, legislation to secure employment, the growing size of firms etc. The effects of these institutional changes upon the labour market should be further investigated. They could in the future be results less, rather than more, readibly interpretable as "competitive" results.

Some of our data also indicate the existence of segmentation and of non-competitive results on the labour market. The male-female differences would be one example. The occupational data also seem to warrant a closer investigation into the behaviour of organisations. The development of the occupational structure seems to have followed the same pattern in many countries being more related to economic development than to e.g. educational supply. Moreover, when people with higher education are in excess supply some seem to continue to get the same types of job and the same pay as before. Others get simpler jobs and are less paid. In this way average wages may be lowered but hierarchies kept. The treatment of the individual (wage, risk of unemployment, on-the-job training, advancement possibilities) may go more with the job he manages to get than with his capacities.

5.2.2 The role of education

Having discussed some general features characterising the Swedish labour market let us now pursue the analysis of education and earnings. This discussion will center around the more direct relationship between education and earnings as we have not material enough to tackle the indirect effects via changes in attitudes and institutions. - Whether or not institutional factors and other barriers to mobility give rise to different labour market conditions for a given educational category what lies behind these demand curves and why are on the average earnings higher for highly educated?

Already for a priori reasons it would appear highly remarkable if the same type of mechanism were at all times and for all types of education to explain the association between education and earnings. The material we have presented earlier in this report seems to warrant the search for an explanation from three starting points:

- education as a search and information process
- education as a human capital investment
- education as sorting

We will concentrate our discussion on the two latter types of approaches and one-dimensional versions of them. The relative merits of the first approach will not be further explored here. We can just note that we earlier saw that certain Swedish investigations pointed to the possibilities of this type of effects being important. We can also refer to studies made in the U.S.A.51) that show the importance of education for labour market information and of the latter for earnings. A regression analysis showed, for example, that labour market information among young men was correlated with length of education and social background, though not with the amount of vocational guidance. Labour market information in turn was correlated with earnings a few years later.

Now, before we can start discussing whether our data may be better interpreted as supporting a human capital or a sorting approach we must make clear what exactly is to be understood by either approach. Different interpretations of them are possible and give rise to different implications to be tested.52)

Let us start by presenting a rather simplistic version of the human capital model. Basic to the human capital approach to wage differentials is the compensation principle. Human capital is acquired at a cost. If firms want to hire individuals with higher education they have to pay a wage differential high enough to compensate for the costs involved in obtaining higher education. Other types of human capital investments, e.g. on-the-job training, would also enter into the determination of wage differentials. In a perfectly competitive market in equilibrium the marginal rate of return to every type of investment and for every individual would be the same.

Actually what is acquired, e.g. skills or attitudes, by the individual as human capital equipment is, of no great importance to the human capital approach. However, much of the interest has centered around cognitive abilities and additions to the stock of knowledge.

With this approach we would expect individuals leaving the educational system after a shorter period of time to have lower earnings (1 in figure 5a). The individual pursueing education for a longer period of time has no earnings and perhaps some extra costs to begin with. Later his earnings will be higher (2 in figure 5a) so much higher as to compensate for the earnings foregone and the extra costs during education. Both individuals may, however, undertake investments after leaving the educational system (e.g. on-the-job training). This investment will also cost something, thus lowering earnings, but later yield a return, thus increasing earnings, as illustrated if 1b and 2b are compared with 1a and 2a respectively in figure 5b. In equilibrium the lifetime earnings implied by all these curves (1a and b, 2a and b) would be the same. Investment incentives could be expected to be reduced as the individual gets older. Thus including returns to on-the-job training we would expect the earnings profile to rise but at a decreasing rate. An often used functional form to test the human capital hypothesis is:52)

$$\ell n Yt_i = a + bS_i + cF_i + dF^2_i + v$$

where:

i = individual i

$\ell n Y_t$ = logarithmic value of income in year t

S = years of education

F = years of labour market experience

v = error term

a, b, c, d = constants

If we make the specific, but somewhat arbitrary, assumption that individuals devote the same share of earnings to human capital investments once on the labour market and this share later decreases at the same rate. those with higher education would show faster growing earnings profiles than those with shorter education (figure 5c). At first the wage differences between educational groups may get smaller but later they would continually grow. If we compare individuals with the same type of education we would expect those having lower earnings early in life (= making large investments) to have higher earnings later.

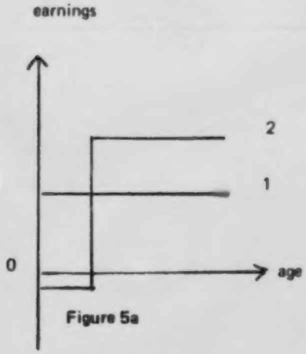


Figure 5a

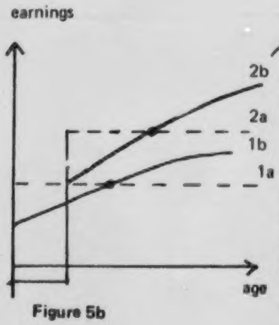


Figure 5b

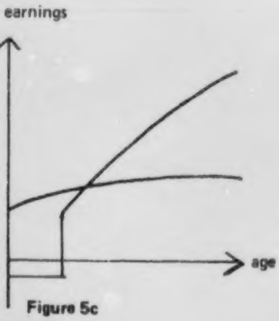


Figure 5c

With the existence of on-the-job training we could only obtain information about the differences in earnings caused by education alone by comparing individuals that have been out on the labour market a certain number of years. (In figure 5b by comparing the earnings at the intersection of curve 2b and 2a with those at the intersection of curve 1b and 1a. These intersections have been called the overtaking point.) Earnings of individuals after approximately 10 years on the labour market have sometimes been used to see whether they could reasonably be explained by differences in schooling period.

It might be of interest to see how far these implications of a certain simple human capital model can be reconciled with our data. However, let us just at once point out a number of factors that might be added to the model and should be kept in mind when interpreting data.53) The above mentioned implications have not taken into consideration e.g.:

- innate productive capacity, depreciation of human capital, non-pecunary aspects of jobs and education as a consumption good
- child rearing, household production and other non-market activities
- uncertainty, lack of information
- human capital investments paid for by others (e.g. parents, employers) or human capital accumulation paid by no one (e.g. incidental learning by doing, biological maturing)
- individual variations in preferences, access to information, possession of innate productive capacity, ability to profit from education and on-the-job training, opportunity to finance human capital investments etc.

Ideally one should test the model on as homogeneous groups as possible with reference to the above mentioned individual factors.

The implications are also supposed to hold only in equilibrium situations. In disequilibrium situations the human capital model only indicates the direction of change. An increased demand for educated people would e.g. at first be expected to raise their wages. The wage increase would be mitigated to the extent that there exist substitution possibilities (i.e. possibilities to replace this production factor by other now relatively cheaper factors of production). Later the wage increase would be expected to bring forth a greater supply lowering wages until a new equilibrium position is reached. In the same way an increased supply of educated people would first tend to lower wages and later to reduce the supply until equilibrium is restored.

Within this type of human capital model a rental price for human capital would be established that equalises total demand for and supply of human capital services. The resulting income structure would be determined by the distribution of human capital and the latter by individual differences in human capital production and tastes. No special requirements for certain types of income hierarchies would be expected to come from firms and other organisers of production.

As for the sorting hypothesis let us sketch a few versions:

Longer education or better school marks could be treated by employers as an indication of the possession by the individual of favourable characteristics. The individual would get at least a first job in correspondence with the length of education and/or school marks. What would happen later to the individual, whether education would work as a permanent or only a temporary screen would depend on the employer's interest in and possibilities to discover what the suitability and the real capacities of the individual are. If the follow-up is strict any unwarranted sorting effect of education would be of rather short duration suitable individuals advancing faster whatever level they start at. In this case the correlation between education and the individual's position could be expected to decrease over time and the correlation between position and suitability or capacity (possibly IQ) to increase. For a given educational level the correlation between "suitability" and position would increase over time and for a given level of "suitability" the correlation between position and education decrease.

If the employer's interest in and possibilities to follow the individual's performance were small the correlation between education and earnings could be expected to remain stable over time.

Finishing an education of 4 years and obtaining a certificate could also be expected to give a greater return than expected by the additional effect of extra educational years as measured by the three first years. Earnings may also be expected to be more highly correlated with traditional educational levels than with educational years.

With a permanent screen the development over time may look like the one in figure 6a and with only a temporary screen like the one in figure 6b.

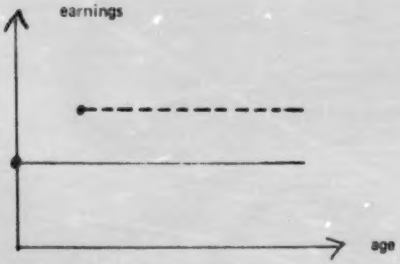


Figure 6a

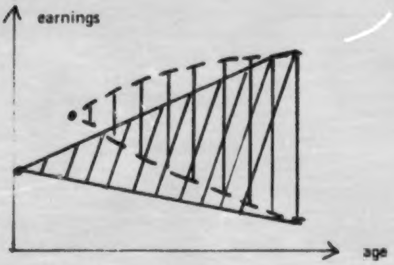


Figure 6b

Sorting hypotheses do not necessarily have to be combined with a complete denial of every investment element in education nor with the absence of on-the-job training and post-school investments. The sorting function of education may instead precisely have a bearing upon not only who gets a good job but also on who gets a good job with development and on-the-job training possibilities. Certain jobs may be richer in this respect than others. For different reasons by-product of the organisation of work, at the same time as the value to the firm of this training is high and much higher than to the individuals, competitive forces may be prevented from driving down wages in these occupations. Those who get into jobs with large development possibilities will see their capacities and earnings increase. Human capital construction occurs but not necessarily at a cost for the individual. Other individuals get into jobs with less development possibilities perhaps even jobs where human capital destruction results. In this situation productive capacity and earnings will also be related but for different reasons than in the human capital model. If a criterion for assigning individuals to these different types of jobs were education increasing differences between educational groups over time would be expected as in the human capital model. However, at a given educational level there would be no special reason to expect those who start low to surpass the others later.

A reason why employers might use education as a criterion for choice of individuals for the more promising jobs could be that they are of the opinion that on the average highly educated people are socially better suited for jobs in leading positions and can adapt more quickly and at less costs. This opinion may have been born out by previous experience and we get what is called statistical discrimination. It may also be based on tradition and conventions without any verification as to efficiency. Educational merits can furthermore have the advantage of being accepted by large groups as a fair criterion for choice.

With a sorting approach factors other than education would be expected to play an important role in determining the occupational and resulting income structure in firms and the economy in general. From an educational point of view the occupational structure could to a large extent be taken as given being designed to assure a controlable and smooth organisation of the productive process.

A common argument against different types of sorting hypotheses might as well be dealt with at once. It has been maintained that if education only sorted private test agencies would be set up as they could do the same thing much quicker and cheaper. In Sweden no such development has taken place. The lack of test institutes might indicate that education is something else than pure testing.

Let us now try to relate our data to these different types of hypotheses:

1. An analysis of earning profiles

   First a word about the possibilities to use cross-sectional data to test the human capital model that actually predicts what would happen to the earnings of an individual over his lifetime. An individual can be seen as advancing over time from one cross-sectional curve to another (figure 7). The increase in earnings that results (e.g. F to G) can be separated into a movement along the original cross-sectional curve (F to H) and a jump to a higher curve (H to G). If this second step always produces a parallel shift in the cross-sectional curve that is the same for all educational groups we could, for all practical purposes, disregard it as not influencing decisions about education. Certain evidence shows that at least for certain higher educational groups the percentage shifts have been about the same.54) However, in absolute terms and for all educational groups they have probably not been identical which is a fact that is not taken into consideration below.

The general shape of the age-earnings profiles coincides with what would be expected from the human capital approach presented above: a positive but declining rate of growth of earnings. In some instances, however, a depreciation factor would have to be introduced that actually lowers earnings at older ages, such as

- in 1971 (all employed) for the lowest educational groups
- in 1971 (government male employees) for higher education and certain types of upper secondary education.

The age-earnings profiles for different educational groups seldom cross which means that higher educational groups get higher starting salaries than the earnings of lower educational groups that have been out on the labour market a certain number of years. The curves for higher educational groups especially for those with university education also increase faster than for lower educational groups. The latter fact would, with a human capital approach, be interpreted as the effect of greater on-the-job investments. The costs to these groups for this investment is not to be seen in a comparison with other educational groups but with the salaries they would have obtained if they did not do any on-the-job investments. We will return to this question below.

Now, why should there be a positive correlation between length of schooling and amount of on-the-job investment? Introducing an ability factor an explanation within the human capital model could run as follows: some people are more able than others. This ability permits them to get more out of education than other individuals. They therefore find it profitable to take a longer education (more investments are possible until the marginal return is driven down to the minimum required). Those talented for education would also be more gifted for on-the-job investments thus being induced also to undertake more post-school investments.

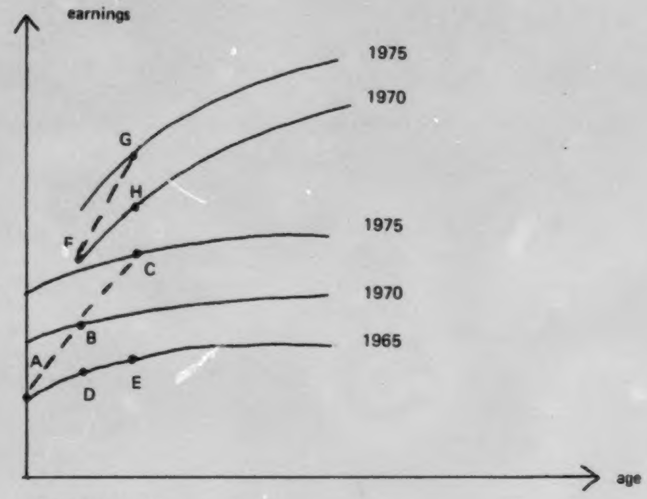


Figure 7

Ability differences of the kind just discussed may be a conceivable explanation of the different shape of earnings profiles at different educational levels. However, at equal educational levels the earning profiles of women rise less rapidly than those of men. You might argue that men and women, who have, for example, finished higher education, are as able to profit from education but women less so from on-the-job investments. Therefore they undertake less post-school investments. However, this does not show up in women with a university education having higher earnings the first years after graduation, on the contrary.

The only exception to women with a university education having lower initial salaries than men with the same education was for government employees with a medical education.56). In this case, the interpretation could thus be that women take jobs that pay well but have few training possibilities which results in a flat age-earnings profile. Men pay for their more sharply increasing earning profile by accepting lower starting salaries. However, training should, in this case, be understood in a very wide sense including gaining commercial skills as women seem to take jobs where the possibilities to develop medical skills are as important.

Other evidence points in the same direction, normally not those with low starting salaries at a given educational level have the highest earnings later on. Among male university graduates, working full time with low, medium and high earnings 6 months after graduation (1972) 13%, 44% and 93% respectively had high earnings three years later.[57)]

Certain investigations include some approximate measure of on-the-job training. At lower educational levels the percentage taking jobs especially indicated as giving practical experience was investigated in 1973. This investigation showed, as we have mentioned earlier, that the percentage with such jobs was higher among those with a comprehensive than with a upper secondary education and at each level among women than among men. In none of our materials has there been any sign of these groups, with eventually more "on-the-job training", having faster increases in earnings. To get more precise information it would, of course, have been preferable if these 1973 groups themselves could have been followed up.

Data from 1974 contain information about earnings and the percentage indicating that they have great possibilities to learn new things in their jobs by education, age and sex.[62)] This percentage was higher in each age group at higher educational levels but always lower for women than for men. Within each educational category the percentage declined with age. However, a comparison between men and women shows that initial earnings are not, as might be expected by a human capital approach, higher at equal educational level for groups with less training possibilities.

With a sorting approach these differences in age-earnings profiles could be interpreted in terms of some people getting into jobs with promotion possibilities while some do not. Some people have the ability to utilise these promotion possibilities others do not. Education can be one of the factors determining what type of job people get and their possibilities to profit from given opportunities but not the only one as the male-female differences at given educational levels would tend to show. In contrast with the human capital approach the faster increase in earnings would not be interpreted as returns to investments on the job paid by the individual.

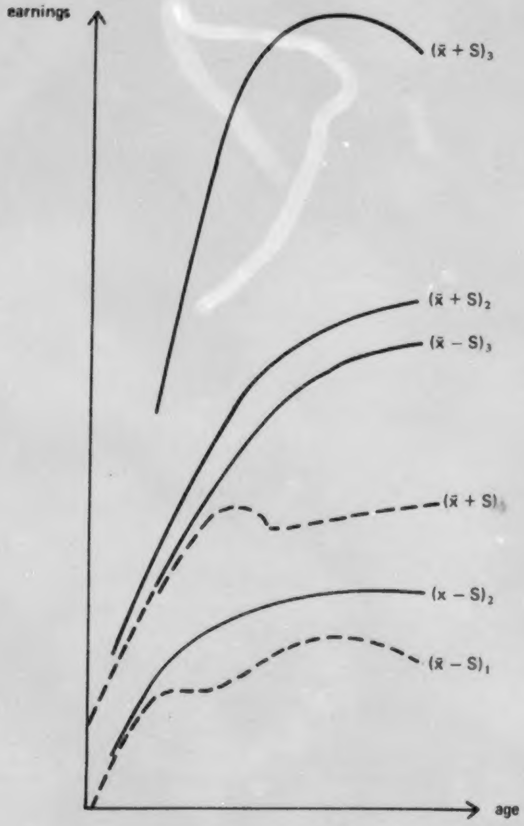
2. Variance around age-earnings profiles

With the human capital approach differences in earnings between educational groups could be expected to be U-shaped over age, first decrease slightly and then increase. This does not fit in very well with our data especially not those from 1934. There the differences were greatest in young age-groups and later decreased. In 1971 (all employed) they first increased and later decreased. Higher education might have had more of a sorting function in the 1930s than later working as a permanent screen protecting those with higher education but with the possibility for capable individuals, at lower educational levels, to advance.

As for the development within an educational group, only with a permanent screening hypothesis, would we expect the variance not to increase over age. This, however, happened in the 1966 data (males) especially for the highest educational group. The material from 1971 for male government employees showed, as we have already pointed out, more diverse tendencies. Both the standard deviation and the standard deviation as a percentage of the average salary first increased and later decreased in the highest educational group, continually increased in the intermediate educational group and first decreased, later increased, and finally decreased again in the lowest educational group.58)

Assuming that the earnings within educational groups, at different ages, was lognormally distributed the overlap in earnings for different educational groups was calculated on the 1971 male government material.58) Figure 8 shows the variation in earnings in different educational groups

Figure 8



(3 higher, 2 intermediate, 1 comprehensive education) as measured by average earnings ± the standard deviation in each educational group. While 85% of those in the highest educational group was calculated to have earnings higher than $(\bar{x}-S)_3$ the figure varied over age approximately from 15% to 30% to 20% in the intermediate and from 9% to 20% to 0% in the lowest educational group. Earnings less than $(\bar{x}+S)_1$ had 85% at the lowest educational level. The figure varied over age approximately from 80% to 70% to 40% in the intermediate educational group and from 8% to 15% to 1% in the highest educational group.

While there is a large overlap in earnings for the two lowest educational groups the chances of people with such education to get over $(\bar{x}-S)_3$ are rather small. These findings are reconcilable with the human capital approach assuming that people at lower educational levels could at a sacrifice in earnings take jobs with large training possibilities bringing them up to the higher income levels but that they generally lack the incentives to do so.

3. According to the human capital model the effect upon earnings of educational investments alone could be studied when the individuals had been out on the labour market a certain number of years at the so called overtaking point (see figure 5b). The earnings of male governments employees have been analysed along these lines.[59] The overtaking point was assumed to occur after alternatively 6, 8, 10 and 12 years of labour market experience.

The formula (using the same symbols as earlier):

$\ln Y_i = a + bS_i + v$ was tested .

The results were very good in the sense that the correlation coefficient (R^2) was high (0.94-0.93). They also indicate that the rate of return on investments in educational years above compulsory schooling was 8%. Analysing the situation for different types of education certain divergences were found to exist. The earnings at the overtaking point were higher than might be expected given the number of years of education for physicians, dentists and economists and lower than expected for upper secondary economists, social workers and theologians. In the former case the divergence could be explained by restricted entry and in the latter perhaps interpreted as a situation of disequilibrium or the effects of non-pecunary aspects.

It may also be interesting to note that the rates of return for the different types of education over the whole working life were positively correlated with higher or lower than expected earnings at the overtaking point.[59)]

As we saw above the results concerning the pure effect of educational investment as studied at the overtaking point were not much influenced whether the overtaking point was set at 6 or 12 years of labour market experience. This stability in results rather naturally leads to the question: what would have happened if the regression was run on wages obtained immediately after graduation i.e. at 0 years of labour market experience. If these were as highly correlated with education you might start wondering if the higher educational groups ever pay for their faster increases in earnings. Unfortunately this question was not raised in the study. Inspection of the data published gives, however, the impression that the same type of correlation might have been obtained even in this case.

4. As for the question whether the returns to education are more tied to obtaining a special certificate or the amount learnt the following facts could be recalled:

   - Education as measured by educational level was more highly correlated with earnings than education measured by number of years. It was also more highly correlated with social background and early IQ.
   - The more prestigious academic Higher Leaving Certificate gave a higher return to education than other upper-secondary types of education.

   Unfortunately, we have no Swedish data for earnings of drop-outs as compared with people finishing a given education.

5. An important factor for how to interpret earnings data is individual capacity. If we could measure individual capacity we would directly know whether education increased individual productive capacity or if it only sorted individuals and if the sorting was done according to productive capacity. However, it is difficult to know what exactly should be meant by individual capacity, if such a one-dimensional thing exists and how it should be measured. The only information we have concerns the relations between variables such as social background, IQ, education and earnings. Unfortunately, not much is known about these variables either as all information comes from different analysis of one and the same material (i.e. the follow up of the Malmö sample).

   According to the path-analysis social background and IQ-test at an early age (themselves not highly intercorrelated) influence education (explaining 40-50% of the variance) but not directly occupations and earnings.

Education influences earnings and increasingly so over age. About half of the total effects of education upon earnings goes via occupation obtained. With education measured by educational level the direct effects of education on earnings are somewhat stronger than with education as measured by educational years. The effects of IQ-test scores measured later in life are also different depending on what educational variable is used. When education is measured by educational level IQ has no significant effect at any age. When education is measured by years IQ influences earnings and increasingly so over time. Interaction effects were also somewhat different. While in both instances a positive interaction effect between occupation and IQ upon earnings was found education and occupation only interacted upon earnings in case of education measured by years. No significant interaction effect of education and IQ upon earnings was found. In other analyses not considering occupation and interactions at that level a positive interaction effect upon earnings of education and IQ[49] or alternatively of high IQ and social background in combination with education has been found.

This material is, as we have already discussed, suggestive of both educational sorting and investment effects. The former could be connected with the indirect effects of education on earnings via occupation obtained. However, as both general and vocational training effects of education are involved the indirect effects via occupation are difficult to interpret. Moreover, the occupational categories were not constructed especially to analyse possible sorting processes. Also, the direct effects of education could contain both sorting and investment elements. In our opinion this information does not permit any firm conclusions to be drawn as to the underlying mechanisms that for this group of men connected education to earnings.

6. From the retrospective study of persons in leading positions two observations could be retained:

   a) For those working in the private sector the occupational level first attained was associated with educational level. Those with a shorter education that later advanced to leading positions had more often begun their careers in small enterprises. This might indicate that with a greater dominance of big companies the labour market may become more highly structured and alternative career paths might tend to disappear.

   b) The comparison between the co-operative and the private sector showed that careers can be organised in different ways. In the co-operative sector people in leading positions had often started at low occupational levels and later advanced as they took advantage of in-service-training.

7. With more disaggregated data other types of interesting comparisons could be made. It might, for example, be instructive to study:

   - whether the simple human capital earnings function presented above better describes the situation for certain parts of the labour market than for others. In an American study60) this was found to be the case. Cross-sectional earnings data for occupations at higher hierarchical levels fitted the human capital earnings function better than cross-sectional data for jobs at lower hierarchical levels or on the dual labour market. What is the mobility between these segments?
   - more in detail what worker traits are rewarded with higher pay, training possibilities and advancement? Can they be related to education?
   - at each educational level what are the training alternatives? To what extent can the jobs individuals do or have been able to choose at a given educational level be said to contain more or less opportunities for acquiring skills or worker traits rewarded by employers? Inspection of figure 9 (based on German data)61) seems to suggest that for those with higher education there might be more of a choice. As self employed they seem to be able to undertake great human capital investments. For other educational groups there is not the same difference being self employed and not. In the Swedish data from 1974 the percentage indicating that they had great possibilities to learn new things in their jobs was also lower in lower educational groups.

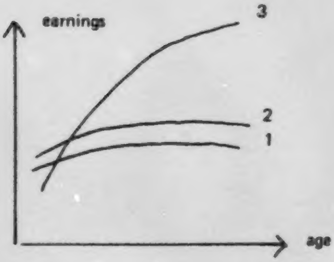


Figure 9a

Self employed

3 higher, 2 secondary, 1 primary education

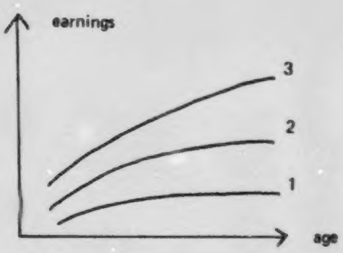


Figure 9b

Others

- what things learned and what type of labour market experience is of value? In one instance we saw, for example, that at the comprehensive level the risk of unemployment did not decrease with the length of labour market experience.
- whether education can be said to give rise to different effects for those types of jobs where individual performance is more difficult or costly to observe than for other types of job.
- the wages obtained by individuals with a certain type of education who go to jobs corresponding to their educational specialisation and those obtained by individuals who go to other types of jobs. This would be of interest to be able to analyse the cognitive version of the human capital model. If individuals gain the same returns to education in both situations this would indicate that, if education is to be regarded as an investment, it would have to be for other reasons than adding to the stock of direct vocational skills. Of course, there are many problems involved in making such investigations such as how to determine what occupations correspond to a certain education. For Sweden, we only know that even in vocationally oriented education an important number of students seems to get jobs not related to their education. We know less about wage differences in these cases. In certain cases they seem to have been lower in occupations less related to the educational orientation.

5.2.3 Provisional conclusions

As will be clear from the previous discussions we are not in a position to draw any firm conclusions as to the mechanisms by which education and other economic variables have been related and how they interact. A forteriori a rigorous evaluation of the educational system and the educational policy from efficiency and distributional points of view must wait.

Both sorting and investment elements do seem to have been and to be involved in education corresponding to a less than perfectly competitive labour market. In the labour market differences have remained even over time between regions, men and women, the private and public sectors, and possibly within educational groups between those get into good and stable jobs or careers and those who do not. However, given this type of segmentation market signals are at work and seem to have influenced the supply of different types of education and wage ratios. Market adjustments seem, however, to be rather slow and both on the supply and the demand side hampered by different institutional arrangements, traditions and other than efficiency considerations. The low level of unemployment, the active labour market policy and the fact that the Swedish government mainly used general instead of selective means in its attempts to achieve full employment has, however, probably to a great extent, counteracted segmentation tendencies. The increasing elements of regulations and selective policy measures may work in the opposite direction as well as the growth of the public sector and big companies with internal labour markets.

Our information about the relations between education, occupations and earnings before or at the early beginning of the educational expansion, is rather scanty. In those days vocational training for occupations at lower occupational levels took, to a great extent, place outside the formal educational system. A limited number of individuals were able to continue formal education to the university level. The higher educational system provided vocational training for teachers, lawyers, engineers, physicians, dentists etc. The selection operated by the educational system may have been more highly associated with social background than with educational or productive capacity or IQ.

It is quite possible that a rather pure cognitive human capital approach to the development of earnings and on-the-job training might be applicable at lower educational levels for this period. Most people left the educational system at these levels and many different types of jobs combining work and on-the-job training in various proportions stood open. A lower rate of technical and other changes also created less uncertainty about jobs and job perspectives for workers. At the same time the need felt by employers for any initial sorting might have been small as productive capacity probably was rather easily observable and recruitment mistakes could be corrected at low costs. Higher education, while providing opportunities for human capital formation, probably created monopoly rents and had selective effects permitting individuals to get into more advantageous careers than warranted by their productive capacities.

The development has changed this situation. By now the formal educational system carters for a great deal of the vocational training already at lower occupational levels. For those who only get comprehensive education and thus do not participate in the formal vocational education the labour market situation can be quite difficult. There are fewer alternatives and possibilities for skill formation left for them on the labour market. They risk never getting into the primary labour market.

With the expansion of the upper secondary level and its diversification in connection with the integration of vocational education a new element might be said to have been introduced at this educational level: occupational choice. Earlier this took, for most people place, on the labour market. Now,

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the educational system has to cope with this activity. The educational system might not, at least not initially, have been especially well fitted for fulfilling these tasks efficiently. There are two sides to this problem. One is how to adapt the types and numbers of courses within occupational fields. Here, the drawback of the educational system is its remoteness from the labour market. The other is how to help individuals to choose and how to select individuals among those applying, when there are more applicants than places available. Here, the educational system might be expected to give priority to its own criteria (e.g. educational suitability), which might be something quite different from productive capacity within a certain occupational field.

Many more individuals nowadays continue to higher educational levels. The selection of these individuals may have become somewhat more correlated with educational capacity and somewhat less with social background than earlier. At this level too the tendencies towards integration and diversification have been felt. After a period of loosening ties between higher education and occupations efforts have been made to strengthen them as both the volume of shorter and longer more directly vocationally oriented courses has been increased and education at the "free faculties" has been given a more vocational orientation.

This diversification and specialisation creates uncertainty and information problems, especially during periods of rapid structural change on the labour market. The students may be uncertain about their possibilities to fulfil the educational requirements of a certain type of education and about the labour market perspectives within a particular vocational field. This uncertainty would tend to reduce the possibilities of rapid and efficient adaption of supply.

At the same time employers may, for different reasons, have become more interested in using educational merits as at least an initial sorting devise. Sorting after general ability (not specific vocational training) may be valuable with a rapid rate of technical change, more on-the-job training and legislation creating rigidities and increasing the costs of lay offs. The expanding public sector and the big companies may also use a more formal recruitment policy and for organisational and other reasons rely on educational credentials.

The greater amount of diversification, specialisation, change and uncertainty may thus have reduced the scope for a simple human capital approach relying on perfect markets as both the number and length of disequilibrium situations may be expected to have increased and as education may more often be used as a general capacity signal. However, it is possible that at the same time the educational-occupational sorting of a more social character has lost some of its importance.

Within a segmented labour market the chances of education having sorting effects would seem to be greater. However, the investment type of effects of education increasing skills and capacities of individuals would not necessarily be smaller in a segmented than in a competitive labour market. The problem is rather how to identify these effects in a segmented market when earnings are more loosely related to the productive capacity of individuals. The simple human capital model could turn out to be less applicable in such a situation not for the reason that education does not improve the skills of individuals but for the reason that no common rental price for human capital services is established and the returns to educational investments are not equalised. To gain a better understanding of the effects of education given the structure of the Swedish labour market and current development tendencies we think it necessary to conduct more detailed studies of the effects of education at the level of individuals and firms (see below micro level studies). At the same time the mechanisms behind the operations of the labour market, whether the results be interpretable within a competitive model or not, would merit a closer follow-up (see below intermediate level studies).

To the extent education has sorting effects more information is also needed if we were to evaluate these effects. We would have to know what sorting criteria are used. Are they relevant from the point of view of efficiency in production? Furthermore we must ask whether or not the same sorting could be achieved at smaller costs. In the same way for an evaluation of education as a human capital investment we need more precise information about what the social returns to educational resources alone are for different types of educations and individuals to be able to compare it with the returns yielded if the resources had been used for other purposes.

To evaluate the distributional effects of the educational system we need a better understanding of the factors influencing the individuals' participation in education, the organisation of work, hierarchical structures and job content.

5.3 Educational research and future educational planning

There can be many reasons why policy objectives are not reached. Educational research, may in some instances and on certain conditions, be helpful. A first requirement would seem to be that the collection of statistics is geared towards the fields of interest. In this report we have on several occasions noted that unfortunately little is known about what is going on, if the objectives are being approached or not.

The existence of relevant data permits an investigation of relations between variables. However, gathering and analysing data will never be enough if you want to base educational policy on a firm ground. Explanatory models are needed if you are to understand the existence or non-existence of relations between variables and mechanisms working beneath the surface.

With a theoretical understandning of the workings of education, a discussion of effects of educational policy becomes meaningful. Means can be related to goals. A better theoretical understanding can also permit a more precise formulation of goals. This need will probably also be felt more strongly in the future. Earlier, when the level of educational activity was low, educational expansion almost irrespective of what type could be recommended both for efficiency and distributional reasons. Now that further educational expansion is less self evident a more difficult balance between different types of actions will have to be struck. Decisions more marginal in character will have to be taken and need a more precise view of education.

Theory will also be useful when educational policy is to be coordinated with that of other sectors for reasons of consistency and efficiency. One might ask whether education is a reasonable and possible means to affect earnings.

Certain conditions should, however, be fulfilled by theory:

- It should aim at being all-inclusive. Different approaches should rather be integrated than each tried to see how far it carries one, leaving the rest in the dark. This probably implies that simple relations have to be abandoned.
- It should cover all the levels and areas that educational policy has aimed at. In the case of Sweden this would include both long run institutional effects and medium and short run efficiency and distributional effects.
- It should be precisely stated and thus ameanable to discriminating tests.

Theory at the following interdependent levels would thus be needed:

1. Micro level. Given the educational system and society at large how do individuals and firms behave? What are the effects of education upon individuals and what difference do these effects make in production?

   How do firms determine their occupational structure? What are the important considerations when organisational changes are introduced and new people are hired?

2. Intermediate level. How do markets behave? What

factors govern the supply and demand for different educational categories? Are changes in the structure of relative prices for educational groups related to changes in demand and supply and if so by what mechanisms? Decentralised decisions by individuals and firms? Decisions at more centralised levels?

3. Macro level. How is institutional and technical change taking place and what does institutional change and technical development mean for the functioning of markets, individuals and work organisations?

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4) S Johansson: Om levnadsnivåundersökningen. Sthlm 1970 p 84
L Johansson 1971 op cit p 79

5) L Johansson 1971 op cit p 81

6) L Johansson 1971 op cit p 82

7) SCB PM 1976:5 Högskolestatistik II pp 19 - 34, 42 - 73

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13) R D Noonan: School Resources, Social Class and Student Achievement. Sthlm 1976 pp 52 - 83, 111 - 119

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19) G Carlsson: Social Mobility and Class Structure. Copenhaguen 1969 pp 132 - 137

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23) S Gustafsson: Lönebildning och lönestruktur inom den statliga sektorn. Uppsala 1976 p 113

24) SCB PM 1975:11 Utbildningsstatistik 1970-1974 p 24
SOU 1970:34 pp 118 - 119

25) S Gustafsson op cit pp 209 - 226

26) SOU 1970:34 pp 118 - 119

27) S Gustafsson op cit pp 128, 137, 138, 183

28) SCB IFP 1974:8 Trender och prognoser 1974 pp 375 - 376

29) SCB IFP 1974:8 op cit pp 179 - 184, 373 - 376

30) S Gustafsson op cit pp 130 - 132

31) A Klevmarken m fl: Industritjänstemännens lönestruktur. Uppsala 1974 pp 88 - 90

32) SOU 1971:62 p 223

33) SOU 1935:52 p 368
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S Bowles: Schooling and Inequality from Generation to Generation in Journal of Political Economy 1972:3 II
J Mincer: Schooling, Experience and Earnings. Mimeo, National Bureau of Economic Research, New York, 1974

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43) G Psacharopoulos: Earnings and Education in OECD Countries. OECD, Paris 1975 pp 166 - 168

44) R B Freeman: Overinvestment in College Training? in Journal of Human Resources vol X:3

45) OECD Education and Distribution of Income VII. Paris 1971 pp 15 - 16

46) A B Atkinson: The Economics of Inequality. Oxford 1975 p 113

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49) I Fägerlind op cit p 76
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M L Wachter: Primary and Secondary Labor Markets: A Critique of the Dual Approach. Brookings Papers on Economic Activity 1974:3

51) H S Parnes, A I Kohen: Occupational information and labor market status: the case of young men. Journal of Human Resources. Vol X no 1
Z Griliches: Wages of Very Young Men. Journal of Political Economy 1976:4 Part II

52) R P Dore op cit
OECD Education, Inequality and Life Chances. Paris 1975. Vol II pp 220 - 227, 439
R Layard, G Psacharopoulos: The Screening Hypothesis and the Returns to Education. Journal of Political Economy 1974:5

53) Å Sohlman: Utbildning och fördelning i ett humankapitalperspektiv, UKÄ rapport 1975:1 and background paper to that report: Humankapitalteori

54) A Klevmarken: Statistical Methods for the Analysis of Earnings Data. Uppsala 1972 pp 110 - 121

55) Meddelande från universitetsförvaltningen 1976:13

56) S Gustafsson op cit p 165

57) SCB SM:U 1975:51 p 17

58) S Gustafsson op cit p 128, 136 - 139

59) S Gustafsson op cit pp 121 - 124

60) P Osterman: An Empirical Study of Labor Market Segmentation. Industrial and Labor Relations Review, 1975:4

61) Quintessenzen aus der Arbeitsmarkt- und Berufsforschung, Heft 3, Nürnberg p 33

62) SOS Levnadsförhållanden. Rapport nr 4. Utbildning och studiedeltagande 1974. Sthlm 1976 pp 242 - 243

Abbreviations

AMS	National Labour Market Board
OECD	Organisation for Economic Co-operation and Development
Prop	Government bill
SCB	National Central Bureau of Statistics
SI	Federation of Swedish Industries
SM	Statistical Reports
SOFI	Social Research Institute
SOS	Official Statistics of Sweden
SOU	Official investigation
Sthlm	Stockholm
UKÄ	Office of the Chancellor of the Swedish Universities

II

HUMAN CAPITAL MODELS AND EDUCATION

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Introduction

This study is a theoretical sequel to the first study based on the research project "Education and the Labor Market", presented above. The initial study aimed at investigating the parallel development of the Swedish educational system and the labor market mainly during the past thirty years. The relations between education and the labor market were described and various interpretational schemes proposed for these relations. The following - not necessarily irreconcilable - alternative approaches to education seemed plausible: education as an investment in human capital for productive purposes, education as sorting, education as a search and information process, education as an asset for purposes other than production, and education as a guardian institution.

Logically, the next step in this research would have been to make a theoretical statement of various plausible approaches in order to make a comparative evaluation of them with respect to Swedish data. However, given the state of the art in the field of educational models, a theoretical study and presentation of one of these approaches represents, in itself, a major research effort.

In this report we have therefore restricted ourselves to studying one class of educational models - i.e. models using "the human capital approach." That we initially concentrated our theoretical studies on "the human capital approach" was motivated not only by the apparent appeal this type of analysis has for policy makers, and its potential empirical relevance in the case of Sweden, but also its dominant role in the economics of education. The approach has even been said to have "no genuine rival." 1)

The empirical application of human capital models has spread at an impressive rate. Surveys of empirical results are now beginning to appear. 2) However,

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1) M. Blaug, "The Empirical Status of Human Capital Theory. A Slightly Jaundiced Survey." Journal of Economic Literature, 1976, No.3, p.849.

2) M. Blaug, ibid.
G.C. Cain, "The Challenge of Segmented Labor Market Theories to Orthodox Theory: A Survey." Journal of Economic Literature, 1976, No.4.
S. Rosen, "Human Capital; Relations between Education and Earnings," in E.M.D. Intriligator (ed.), Frontiers of quantitative economics, Vol III B, Amsterdam, 1977 /a/.
S.Rosen, "Human Capital: A Survey of Empirical Research," in R.Ehrenberg (ed.), Research in Labor Economics, Vol. 1., Greenwich, Conn., 1977 /b/.

despite the fact that human capital theorists share certain ideas, there are many different types of human capital models - some more or less partial, others more or less complex.[1] Little work has been done to consolidate and integrate the various models. For this reason we found it necessary - before continuing with the empirical work - to try to analyze the models and give them a unified presentation. By doing so, it will be possible to identify suitable empirical testing strategies with regard to existing models and further theoretical development possibilities.

In Section 1, as an introduction to our investigation of human capital models we will clarify our concept of "the human capital approach" in greater detail and briefly survey the historical development of this type of analysis, naming the more important contributors, works, and dates of publication. This survey will indicate the common general framework for our models and show how the models presented are related to their various sources of origin. Moreover, in each section where different types of specific models are presented, we will mention the authors whose work inspired the respective models, and note the resemblances and dissimilarities between the models.

In our combined survey and synthesis of human capital models we are especially interested in clarifying:

- the assumptions used;
- the explanatory variables that emerge with respect to educational choices and such related variables as earnings;
- the relations, qualitative as well as more exact, that result between explanatory variables, education, and earnings.

Our reasons for concentrating on these aspects are rather obvious. We want to know what these human capital models explain, how they do so, and ultimately how well they work out, compared with other models in the case of Sweden. The assumptions ought to be specified in order to determine the extent to which the models are applicable.[2] What, e.g., is the potential of the approach - not only as an explanatory framework, but as an instrument of educational policy?

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[1] M.Blaug, op.cit., 1976, p. 829.
Sheng Cheng Hu, "Education and Economic Growth." Review of Economic Studies, 1976, pp. 509-518.

[2] Cf. the discussion of
T.C. Koopmans, Three Essays on the State of Economic Science. New York, 1957, p. 147.

Given the general restrictions by which we delineate "the human capital approach," our intention is to systematically determine its potentialities. At different points alternative assumptions are possible. We will explore some of these possibilities at given points, but will not necessarily accumulate them.

Each specific model addresses the same type of questions - namely, what factors explain educational choices, and what relations emerge between education and earnings? The complexity of the models varies, however. We begin our study with a rather simple set of models and successively add different aspects making the models more complex. Thus, in the first set of models - called Schooling Models - the only choice the individual is to make concerns the length of an initial period of schooling. In the models in Sections 3 and 4 - Life Cycle Models - not only the length of such an initial period of specialization but also the level of training, in the form of on-the-job training and recurrent education later in life, is decided by the individual. In Section 5 - Life Cycle Models Including Leisure Activities - the perspective is further extended, as these educational choices now have to be made with regard both to their effects on earning capacity and to their effects on other activities of various kinds. Finally, in Sections 6 and 7 an intergenerational perspective is introduced. Some of the activities of grown-up individuals will affect the initial opportunities of individuals in the next generation. In Section 6, intergenerational connections are discussed within the framework of Intergenerational Models of Individualistic Type, and in Section 7 within Household Models. The latter models are also investigated as possible alternatives or complements to the earlier Individualistic Life Cycle Models.

These human capital models will be presented in a straightforward manner without any a priori discussion of their pros and cons. Nonetheless, we try to state with care the bearing that the assumptions have, since this is essential for an understanding of the area of applicability of the models. In Section 8 we also return to this question, as we make a general characterization of the results obtainable from the models and suggest further empirical and theoretical research possibilities. The more detailed results in terms of implications, types of explanatory variables, etc. that emerge from the specific models are summarized at the end of each respective section.

The primary aim of this study is thus to determine which questions could be analyzed by using the human capital models described, and under what conditions. A discussion of the empirical relevance of these models in the case of Sweden and for Swedish educational policy will be taken up in a forthcoming report.

1. Development of ”The Human Capital Approach”

It is an old idea that the productivity of individuals varies and that human productive capacity can be increased by diverting resources from current production to, e.g., skill formation.[1] Passages referring to such considerations can be found throughout the development of economic theory - in the works of William Petty (1665), Adam Smith (1776), John Stuart Mill (1848), Alfred Marshall (1890), etc.[2] Nevertheless, this idea has not been dealt with at any great length by any of these authors. The subject only began to be treated more thoroughly in the late 1950s, when what might be called the "modern type" of human capital analysis appeared and its theoretical implications began to be explored.

As regards theoretical affiliation, the core of human capital theory belongs to the neoclassical school. It also owes much to the capital theory developed by Irving Fischer (1907, 1930).[2]

Preoccupation with certain problem areas has, at different times, inspired the development of human capital studies. These include:

- Studies in economic growth. When the subject area was being explored during the 1950s and 1960s, the studies tended to show that changes in technology and physical capital alone could not explain the observed increases in productivity. Qualitative changes in labor were then added as an explanatory variable.

- Studies of inequalities in individual earnings. It also became apparent that such studies required a better and more unified method of analyzing changes and differences in human qualities than was then available. This is especially relevant when a unified theory for the distribution of earnings is being sought. This field of study has come to enjoy renewed and increasing general interest since the middle of the 1960s. Investigations pertaining to differences in unemployment rates and studies of the labor supply can also be said to belong to this field.

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[1] Except where otherwise indicated, this survey is based on the following source materials:

M.Blaug, Economic Theory in Retrospect, Homewood, Illinois, USA, 1968.
J. Mincer, "The Distribution of Labor Incomes: A Survey." Journal of Economic Literature, March, 1970.
M.J. Bowman, "Learning and Earning in the Postschool Years." Review of Research in Education, 2. Ithaca, N.Y., USA, 1974.
G.S. Becker, Human Capital. New York, 1975.

[2] See M.J. Bowman, ibid., for further references.

Other interests - e.g., in educational planning and the discussion of the increasing amounts of resources devoted to education - have also inspired human capital theorists. On occasion, the human capital theory has also been applied in explaining international trade patterns.

G.S. Becker - one of the principal contributors to the development of human capital theory - has defined human capital as "resources in people." Activities that increase "resources in people" are thus investments in human capital. Examples of such activities are "schooling, on-the-job training, medical care, migration and searching for information." They "improve skills, knowledge, or health."1) Human capital is seen as consisting of resources related to productive power and being embodied in a person - in contrast to physical capital, which is productive power embodied in machines, tools and other equipment.2)

But let us see how "the human capital approach" has developed in connection with growth studies and distributional analysis.

The original "modern" work dealing with growth and human capital was done by T.W. Schultz. His first works were published in 1958, 1959, 1960, 1961 and 1962.3) Although he used the human capital concept in a very wide sense, Schultz's initial applications dealt with human capital only in the form of education. In his analyses, production at any given point in time was related to the existing stock of physical and human capital. The contribution of human capital to economic growth could be estimated from increases in the

---

1) G.S. Becker, Human Capital. New York, 1964, p.1.

2) B.R. Chiswick, Income Inequality. Regional Analysis within a Human Capital Framework. New York, 1974, p.17.

3) T.W. Schultz, "The Emerging Economic Scene and its Relation to High School Education," in F.S. Chase and H.A. Anderson (eds.), The High School in a New Era. University of Chicago Press, 1958.
T.W. Schultz, "Investment in Man; An Economist's View." Social Service Review, June, 1959.
T.W. Schultz,"Capital Formation by Education." Journal of Political Economy, December, 1960.
T.W. Schultz, Investment in Human Capital." American Economic Review, March ,1961.
T.W. Schultz, "Reflections on Investment in Man." Journal of Political Economy, Supplement on Investment in Human Beings, October, 1962.

stock of human capital - i.e., the net investments in human capital (schooling) and the returns on such investments. 1) The former could be approximated by measuring expenditures on education, the latter by calculating an "internal rate of return" derived by comparing the life earning streams of persons with varying amounts of schooling.

As for growth and macroeconomic studies, the tradition has been continued by various people - e.g., E.F. Denison (1962, 1967)2). His approach is somewhat different, however. Earnings differences are still used, but now as rental values to weigh years of schooling. However, Denison's quality-adjusted measure of the contribution of labor is not necessarily very unlike the corresponding measure based on the concept of a human capital stock.3)

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1) G. Psacharopoulos, Returns to Education. Amsterdam, 1973.
M.I. Nadiri, "Some Approaches to the Theory and Measurement of Total Factor Productivity:A Survey." Journal of Economic Literature, December, 1970, p 1164-1165.

2) For Denison's contributions, see, e.g.:
E.F. Denison, The Sources of Economic Growth in the United States and the Alternatives before us. New York, 1962.
E.F. Denison, Why Growth Rates Differ. Washington, 1967.

Other writers are:

C.C. Weizsäcker, Technischer Fortschritt und Ausbildung, Chapter 5 in Zur Ökonomischen Theorie des technischen Fortschritts.Göttingen, 1966.
C.C. Weizsäcker, "Training Policies under Conditions of Technical Progress: A Theoretical Treatment". Mathematical Models in Educational Planning. OECD, Paris, 1967.
F. Welch, "Measurement of the quality of schooling." American Economic Review, 1966, No.2, Papers and Proceedings.
Z. Griliches, "Production Functions in Manufacturing: Some Additional Results." Southern Economic Journal, October, 1968.
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E.R. Brubaker, "Embodied Technology, the Asymptotic Behaviour of Capital's Age, and Soviet Growth." Review of Economics and Statistics, August, 1968.
L.C. Thurow, Poverty and Discrimination. Washington, 1969.
J. Tinbergen, Income Distribution. Analysis and Policies. Amsterdam, 1975.

3) C.f. M.J. Bowman, "Schultz, Denison, and the Contribution of 'Eds' to National Income Growth." Journal of Political Economy, October, 1964.
M.J. Bowman, "On Theodore W. Schultz's Contribution to Economics." Journal of Scandinavian Economics, 1980,No.1.

The interpretation of the human capital concept used in these macroeconomic contexts is problematic due to the lack of a clearly defined microeconomic foundation (for an exception see Weizsäcker 1966, 1967,[1] who uses micro production functions to represent the individual creation of human capital). In these contexts human capital could be said to correspond to a monetary investment which may earn a return for a variety of reasons. A given sum of money invested in education could, for example, earn a return because it alters the individual's productive skills. But there could also be other reasons.

The effect of an increase in human skills and knowledge has in macroeconomic contexts also been discussed in other ways and in relation to other variables than labor. Sometimes, for example, a quality-adjusted measure of aggregate *physical capital* has been used; sometimes a quality-adjusted measure of labor, where the weights depend on the *education of people in decision-making positions* [3]; and sometimes *cumulative gross investments, inputs or outputs* are used as proxies for "learning." [4]

A characteristic of all these models at a macro level is the general lack of interpretational clarity. As many of them - at least formally - do not indicate that knowledge is embodied in individuals, and thus do not treat human resources as human capital - i.e., "resources in people" - they might not even be considered to be human capital models if judged according to Becker's definition of human capital. This may be a consequence of the level of aggregation. Of course, they could be given a micro basis with human resources as human capital, i.e., "resources in people." But this may be done in many different ways.

With respect to human capital, and in particular to education, another type of development should also be mentioned. Two-sector or multi-sector models have been constructed in which the educational system is one sector. Optimal

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1) C.C. Weizsäcker, *op.cit.*, 1966, 1967.

2) R. Solow, "Technical Progress, Capital Formation, and Economic Growth." *American Economic Review*, Proceedings, May, 1962.

3) R.R. Nelson, E.S. Phelps, "Investments in Humans, Technological Diffusion and Economic Growth." *American Economic Review*, Papers and Proceedings, May, 1966.

4) K.J. Arrow, "The Implications of Learning by Doing." *Review of Economic Studies*, June, 1962.
S. Rosen, "Learning by Experience as Joint Production." *Quarterly Journal of Economics*, August, 1972.

allocation of labor to the educational system and the rest of the economy has been discussed in terms of these models.1) These models often lie close to the so-called "manpower planning approach." The latter has been advanced as an alternative to "the human capital approach" as a basis for educational planning. 2) With a "manpower planning approach," educational policy would generally be based on the assumptions that economic growth demands a special development of branches and occupations , and that the development of branches and occupations in turn demands certain types of education. This approach tends to narrow the perspective on the ways that skills are created, on substitution possibilities among educational categories, and on individual differences. 3) A broader view is usually found among the authors who are identified with "the human capital approach"4) and who have worked out a micro basis for human capital theory.

Starting from the distributional side, J. Mincer (1957, 1958)5) presented a model relating the distribution of earnings to human capital in the form of schooling. Inspired by Mincer, but also by Schultz's work, G.S. Becker in 1962 6) incorporated schooling and post-school training into a human capital theory.7) From these works, a clear line of development extends to further

---

1) H. Usawa, "Optimum Technical Change in an Aggregative Model of Economic Growth." International Economic Review, No.1, 1965.
J. Bénard, "Modèle d'optimation générale: économie-éducation." Modèles mathématiques pour la planification de l'enseignement, OECD, Paris, 1969.
S. Bowles, Planning Educational Systems for Economic Growth. Cambridge, Mass., 1969.
C.R.S. Dougherty, "Optimal Allocation of Investment in Education," in H.B. Chenery (ed.) , Studies in Development Planning. Cambridge, Mass., 1971, pp. 270-292.
J. Aarestad, "On the Optimal Allocation of Labour to the Educational Sector." The Swedish Journal of Economics, No.3, 1975.

2) J.P. Jallade, "Educational Planning Methods." The Development of Educational Planning. Volume VI, OECD, Paris, 1971.

3) See, e.g.:
Å. Sohlman, Utbildning och arbetsmarknad. Nationalekonomiska institutionen, Stockholms universitet, Skriftserie 1976, No.1, p.118-120.
B. Ahamad, M. Blaug, The Practice of Manpower Forecasting. Washington, 1973.

4) J. Mincer, op.cit., 1970 ,
M. Blaug, op.cit., 1976.

5) J. Mincer, "A Study of Personal Income Distribution." Unpublished Ph.D. dissertation, Columbia University, 1957.
J. Mincer, "Investment in Human Capital and Personal Income Distribution." Journal of Political Economy, August, 1958.

6) G.S. Becker, "Investment in Human Capital: A Theoretical Analysis." Journal of Political Economy, October, 1962.

7) M.J. Bowman, op.cit., 1980.

work by Becker (1964, 1966 /with B.R. Chiswick/, 1967, 1975 /with G.R. Ghez/, 1975) and to Y. Ben-Porath (1967, 1970), W.J. Haley (1973), B.R. Chiswick (1974), J. Mincer (1974), 1) etc.

The focal point of interest in these studies has been in developing a theory explaining earnings differences, the distribution of earnings and individual decision-making with respect to human capital creation. 2) Differences in human capital are viewed as a major cause of income inequality. 3) The full implications of this approach have been pursued at more and more general levels of analysis.

Mincer's models from 1957 and 1958 have later come to be called the Schooling Model. It was originally developed within a competitive equilibrium framework. 2) According to Mincer, training (schooling) raises the productivity of the individual, 4) but is associated with costs - e.g., in the form of foregone earnings. In a competitive equilibrium the present values of different life earning streams associated with longer or shorter periods of schooling, discounted at the market rate of interest, are equal if one assumes that no differences exist among individuals.

Later came the development of a more comprehensive theory of investment in human capital (Becker 1964) 5), in which the individual was seen as having to decide at each moment during his entire lifetime whether to invest in human capital or not. The General Earnings Function was introduced, according to

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1) G.S. Becker, op.cit., 1964.
G.S. Becker, B.R. Chiswick, "Education and the Distribution of Earnings." American Economic Review, May, 1966.
G.S. Becker, Human Capital and the Personal Distribution of Income: An Analytic Approach. Ann Arbor, 1967.
G.R. Ghez, G.S. Becker, The Allocation of Time and Goods over the Life Cycle. New York, 1975.
G.S. Becker, op.cit., 1975.
Y. Ben-Porath, "The Production of Human Capital and the Life Cycle of Earnings." Journal of Political Economy, August 1967.
Y. Ben-Porath, "The Production of Human Capital over Time" in W.L.Hansen (ed.), op.cit., 1970.
W.J. Haley, "Human Capital: The Choice Between Investment and Income." American Economic Review, 1973, No.5.
B.R. Chiswick, op.cit.,1974.
J. Mincer, Schooling, Experience and Earnings. New York, 1974.

2) J. Mincer, op.cit., 1970, p.1, 6-23.

3) J. Mincer, op.cit., 1974, p. VIII.

4) J. Mincer, op.cit., 1970, p.7.

5) G.S. Becker, op.cit., 1964.

which the potential earnings of an individual at a given date would depend on:

- his initial endowments, or initial stock of human capital, and the returns on this stock;
- subsequent investments in human capital and returns on these investments.

Two categories of human capital are thus discerned: the initial endowments of the individual and the subsequent additions to his stock of human capital. The former concept is not further elaborated in this theory, and the latter is expressed in cost terms. For empirical work a specification of investments not only in terms of money costs but also in terms of time costs has often been used, in line with Becker and Chiswick (1966)[1].

The costs of investments in human capital are assumed to have a physical counterpart - the increase in "resources in the individual," or the productive power of the individual. However, this correspondence is not a simple one, as an investment of a given amount of money in human capital can result in different increases in productivity or "physical" human capital because of variations in individual "ability." Such differences in ability will be reflected in the rate of return. But, differences between individual rates of return can also result from market disequilibria.

The General Earnings Function as described above may be regarded as a general accounting identity. Some restricting assumptions must be added if it is to be turned into a theory. The hypothesis may, for example, be that individuals, maximizing the present value of their life earning streams, chose financial or time costs equal to their total potential earnings during their schooling period, and that these costs decline successively towards zero at the end of their working life. From such hypotheses, testable empirical implications can then be derived (Mincer 1974).[2]

So far nothing very precise has been said about the actual process of producing human capital. This is due to the lack of discussion of this aspect in previous works. However, an explicit production function for human capital was eventually introduced (Ben-Porath 1967, Becker 1967).[3] and a few more clarifications also appeared:

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[1] G.S. Becker, B.R. Chiswick, op.cit.

[2] Cf. J. Mincer, op.cit., 1974.

[3] Y. Ben-Porath, op.cit., 1967.
G.S. Becker, op.cit., 1967.

- Human capital is explicitly treated as a physical asset embodied in the individual and produced by him. Inputs in this production process are the already existing stock of human capital plus other purchased inputs. This physical human capital asset has a given rental value.
- The physical human capital asset is unidimensional or homogenous.
- With regard to the production function of human capital, certain efficiency parameters account for individual differences in "ability."

The explicit treatment of human capital as a "physical" asset reflecting human productive power affords many theoretical advantages. It enables an analytical distinction to be made, e.g., between earnings differences due to market disequilibria and those resulting from variations in productive capacity which was not possible with the General Earnings Function. There is, on the one hand, the "physical" amount of human capital (i.e. individual productive capacity) and, on the other, the rental price. Differences in individual productive capacity stem from varying amounts of resources having been devoted to human capital creation and ability. Eventual market disequilibria are reflected in a nonuniform level of rental prices. In earlier theories, differences in ability *and* rental price were reflected in the rate of return on monetary investments in human capital. Moreover, different explanatory factors in the technical production process for human capital can now be analyzed. A fuller description of the individual decision-making process was also made possible. This development may even be seen as a logical and necessary prerequisite for a theory that substantially explains differences in earnings.[1]

The distinctive feature of this human capital theory is *not only* its inclusion of *investment aspects*, such as the allocation of resources to education , which gives rise to future benefits, an aspect which is also found, e.g., in sorting theories. What is characteristic for human capital theory *in addition* is the specific *explanation* that it gives as to *why* these future benefits arise. They arise eventually because *human capital investments* (e.g. in terms of education) endow the individual with resources that *influence his productive aptitudes* - or, more affirmatively, because *human capital investments, education, etc., make people more productive*. This contrasts with pure sorting theories, which assume that the productive capacity of the individual is not changed by education and that, consequently, earning differences arise for other reasons.

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[1] B.C. Ysander, "Homogeneity in Education." *Learning and Earning: Three Essays in the Economics of Education.* Stockholm, 1978, p. 216.

The paragraph above can serve as a general definition of "the human capital approach." All the models that we are going to present are in line with this approach to human capital. Moreover, explicit production functions for human capital will be used in all of them, beacause our ultimate interest is in education and educational policy. We will also restrict our interest to educational human capital, leaving aside such aspects as investments in health and migration. However, as will be indicated in Sections 3 and 5 and further discussed in Section 8, another - weaker - interpretation of the human capital concept and human capital models is possible. For "unspecified" reasons a relation may exist between, e.g., earnings and schooling that induces an "investment type" of behavior on the part of individuals.

The general definition of "the human capital approach" given above sets very loose limits on the kinds of models that can be constructed. To facilitate a unified presentation and limit the potential dimensions of the models, they will be interpreted as economic equilibrium models 1) and all will use the concept of homogenous human capital. 2) The models will also be regarded as aiming at describing actual behavior and, e.g., realized earnings.

Given these restrictions, we shall endeavour to investigate the various development possibilities of "the human capital approach." This investigation is especially important if connections with other theories - e.g., in an interdisciplinary framework - are later to be elaborated. We do not propose to identify "the human capital approach" with any specific model. 3)

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1) The advantages of making this type of strong interpretation compared with weaker ones are also discussed in Section 8.

2) The latter means that our models are of the more affirmative character: Human capital investments make people more productive. For a further discussion of this point, see Section 8 and Appendix A.

3) Cf., e.g., the following authors who mainly identify the human capital approach with G.S. Becker's model from 1967 (G.S. Becker, *op.cit.*, 1967):
M. Blaug, *op.cit.*, 1976.
L. Pettersson, "Some Aspects of the Expansion of Education in Sweden." *Economy and History*, 1977, Vol. XX, No.2.
G.Persson, "Humankapital, olikhet och ideologi." *Nordisk tidskrift för politisk ekonomi*, No. 6, 1978.
B. Ekehammar, "Towards a psychological cost-benefit model for educational and vocational choice." *Scandinavian Journal of Psychology*, 1978, No.1.

Before proceeding to the analysis of the specific models, it is worthwhile to note that in equilibrium situations certain conditions are always fulfilled. In intertemporal equilibrium, for example, marginal costs for human capital investments would equal marginal returns. In a model, where the costs and benefits of human capital are related only to greater or lesser earnings, the marginal costs of human capital investments (foregone earnings, etc.) would equal the discounted value of the future marginal profits (increased earnings). Moreover, if the rate of interest is assumed to be constant over time, the marginal rate of return (internal rate of return) on human resource investments will equal this rate of interest, as for all other investments.1)

The fact that our models are taken to be equilibrium models will, of course, not prevent us from investigating the effects on individual choices of features characteristic of disequilibrium. We will, e.g., investigate how different individuals react if their preferences change; if they are discriminated against on the labor market - i.e., not paid the same rental price for human capital as others are; if they do not face the same price in other markets, among others the capital market; if they cannot make all the human capital investments they wish to, etc. Such investigations can then be interpreted as normal partial analysis. We think it more appropriate, however, to use the models as equilibrium models. They are intended to describe realized behavior over long periods of time, such as the life of an individual or, in some cases, over generations.

The agents in our models are individuals and firms. The firms are part of the demand side of the labor market - i.e., they rent human capital services. They may also provide services for the production of human capital, inter alia, schools. As the reader will notice, no mention is made in our models of the state or of taxes (except in partial analysis). This may be interpreted as meaning that the models are only applicable to situations where there is either no public sector or the public sector behaves as private enterprises do. Another interpretation compatible with the models is that of a public sector financed by taxes on rents. In other situations with, e.g., a public sector financed by income taxes and indirect taxes, the optimization problem of the individual will differ from that described in the models used here.

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1) B.C. Ysander, op.cit., p. 259.

2. Schooling Models

The common feature of the models to be discussed in this section is that they only contain investments in schooling. The individual only considers two possible allocations for his capacity: work or education. Models of this type were first advanced by J. Mincer (1957,1958).[1]

In Section 2.1 we study simple models in which individuals are assumed to be "identical". These *Simple Schooling Models* are inspired by J. Mincer (1958, 1970, 1974).[2] They are given a rather thorough presentation here, due to their importance for empirical research. While Mincer did not explicitly develop his model in terms of the human capital concept, we do so parallel with a presentation in terms of the relation between schooling and earnings studied by Mincer.[3] For each model we do not only establish equilibrium relations, that must necessarily hold if "identical" individuals are to choose different amounts of education, but also individual conditions for an optimization. Without changing the frame of reference, we can then extend our analysis to situations where individuals are not assumed to be "identical." The relation of the models for "identical" individuals to those in Section 2.2 for individuals who are "not identical" can then be seen more clearly.

We develop two versions of the Simple Schooling Models. Version A is for a given period of employment (irrespective of the length of schooling). Version B is for a given planning period (to include both the schooling and work phases). The resulting relations between earnings and education with our Simple Schooling Model Version A are identical to those of Mincer's Schooling Model. Version B, and its conditional extension to cases with individual variations in initial stocks of human capital and ability, does not seem to have been discussed in the literature.

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1) J. Mincer, *op.cit.*, 1957.
   J. Mincer, *op.cit.*, 1958.

2) J. Mincer, *op.cit.*, 1958.
   J. Mincer, *op.cit.*, 1970.
   J. Mincer, *op.cit.*, 1974.

3) The Simple Schooling Models have also been interpreted in terms of groups of individuals with skills that are not perfectly substitutable (cf. S. Rosen, *op.cit.*, 1977 /b/). As this formulation does not carry over to the more extended models, it is of less interest in this context.

The Simple Schooling Models are a very special case of the models analyzed in Section 2.2. In these Schooling Models with Individual Differences, optimizing individuals with differences in time horizon, rates of appreciation and depreciation, initial stock of human capital and ability are studied. With regard to ability, as a factor for the individual production of human capital in Schooling Models, it is noted in the works of Y.Weiss (1971), N. Oulton (1974), and S. Rosen (1977). [1] For the Schooling Models we made a systematic investigation of differences in results obtained for models with continuous and discrete time, respectively.

2.1 Simple Schooling Models – "Identical" Individuals Choosing Different Levels of Schooling

To arrive at these models we make a number of assumptions that are described and briefly commented on below.

In these models, individuals begin their lives with an initial endowment of human capital ($\bar{H}_0 > 0$) and with sufficient sophistication to make their own decisions about their future. At the outset they have an option to go to school or not to do so.[2] Schooling leads to an increased stock of human capital (H). Longer education results in a larger stock of human capital. The rental price of human capital - a - is assumed to be given. Thus, greater earnings are received when a larger human capital stock is rented on the labor market. This increase in earnings must be compared with the earnings lost during the prolonged period of schooling, as illustrated in Figure 1, where earnings Y_t^s and Y_t^0 are associated with s and zero years of schooling.

We assume the individual devotes his entire disposable capacity (H) to increasing his stock of human capital while at school. When not at school he is assumed to work, i.e., he rents his entire stock of human capital (H)

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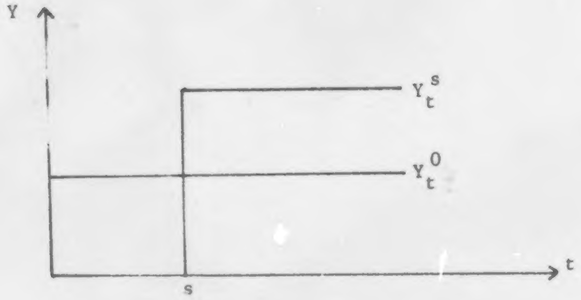
[1] Y.Weiss, "Ability and Investment in Schooling: A Theoretical Note on J.Mincer's 'Distributions of Labor Income'." Journal of Economic Literature, June ,1971.
N. Oulton, "The Distribution of Education and the Distribution of Income." Economica, November, 1974.
S. Rosen, op.cit., 1977 /a/.

[2] It can be shown that if the individual had an opportunity to choose the timing of this schooling period it would - given the other restriction of these schooling models - be optimal to place it at the beginning of life. Cf. T. Ishikawa, "A Note on the Optimal Spacing Properties in a Simple Jevonian Model of Educational Investment." Quarterly Journal of Economics, 1975, No.4.

Figure 1. The development of earnings with different amounts of schooling



in return for an income (Y) equalling the constant rental price - a - times his stock of human capital - aH. The fact that there are only two activities between which he can choose does not mean that other activities (e.g., sleep) are not assumed to take place. However, the model does imply that the amount of such activities is given.

The welfare of the individual is assumed to be determined by the consumption of goods and services. Education and work are not regarded as ends in themselves, nor does education influence what the individual gets out of goods and services. Since prices are assumed to be given, goods and services can be treated as a composite commodity, measured by outlays of money. The interest rate is also assumed to be given and the individual, by maximizing the present value of net earnings, assures himself of as much welfare as possible through the consumption of the type and time profile that he desires.

The individual plans for a certain period of time. We will consider two alternatives: a given finite time horizon and a given finite working life. We will further assume, for the time being, that human capital neither depreciates nor appreciates. Moreover, there are no direct schooling costs - only indirect ones that are measured in terms of foregone earnings. And most important, individuals are initially assumed to be "identical" in all aspects relevant to this maximization problem.

Based on these assumptions, the individual's problem of choosing an optimal schooling level can be expressed as follows:

$$(2{:}1) \quad \text{Max } J = \int_s^T Y_t^s \ e^{-rt} \, dt = \frac{1}{r} Y_t^s \ e^{-rs} \ (1 - e^{-r(T-s)})$$

subject to

$$Y_t^s = aH_t = aH(s) \qquad t > s \qquad \frac{\partial H}{\partial s} > 0$$

$$H_o = \bar{H}_o > 0.$$

Or, with a working life of given length

$$(2{:}2) \quad \text{Max } J = \int_s^{n+s} Y_t^s \ e^{-rt} \, dt = Y_t^s \ \frac{1}{r} e^{-rs} \ (1 - e^{-rn}).$$

Definition of symbols:

s = length of schooling period
T = planning horizon
n = length of working life
a = the constant rental price for human capital
r = the constant market rate of interest
H_t = level of human capital stock at date t
$Y_t^s = aH_t = aH(s)$ = actual earnings at date t for schooling level s $(t>s)$.

Human capital (H) and earnings (Y_t) are related by $aH_t = Y_t$, where a is the rental price for human capital. The rental price would not be an observable market magnitude. Neither is H itself directly observable. So while the models are obviously compatible with a human capital interpretation they can also be developed ad hoc.

With a given finite working life as in (2:2) the necessary condition for a maximum proves to be a very simple expression. Assuming that an interior solution exists, with optimal schooling period (s^x), the necessary condition for a maximum is:

$$(2{:}3) \quad \frac{d}{ds} \left\{ Y^{s^x} \ \frac{1}{r} \ e^{-rs^x} \ (1 - e^{-rn}) \right\} = 0, \text{ which is equivalent to}$$

$$\frac{d \ln Y^{s^x}}{ds} = r \ .$$

The individual should prolong his schooling period up to the point (s^x), where the increase in the logarithmic value of earnings associated with a marginal increase in schooling time (= the marginal rate of return - MRR) equals the interest rate. With a discrete time model we obtain approximately the same results.[1]

Figure 2 shows a situation where the logarithmic value of earnings (Y^s) for different levels of schooling increase at a declining rate with the number of school years (s); cf. Panel A. Thus, MRR is falling; cf. Panel B. At a certain point MRR equals the market rate of interest which would indicate the optimal schooling level (s^x). If MRR were to coincide with the rate of interest for different levels of schooling, the individual would, of course, be indifferent in choosing among them.

---

[1] $$\text{Max } J = \sum_{t=s}^{s+n-1} Y_t^s \, (1+r)^{-t} = Y_t^s \, (1+r)^{-s} \frac{1+r}{r} [1 - (1+r)^{-n}].$$

Assuming that an interior solution exists, the necessary condition for a maximum is:

$$\sum_{t=s^x-1}^{(s^x-1)+(n-1)} Y^{s^x-1} (1+r)^{-t} \leqq \sum_{t=s^x}^{s^x+(n-1)} Y^{s^x} (1+r)^{-t} \geqq \sum_{t=s^x+1}^{(s^x+1)+(n-1)} Y^{s^x+1} (1+r)^{-t}$$

$$Y^{s^x-1} (1+r)^{-(s^x-1)} \frac{1+r}{r} [1-(1+r)^{-n}] \leqq Y^{s^x} (1+r)^{-s^x} \frac{1+r}{r} [1-(1+r)^{-n}] \geqq Y^{s^x+1} (1+r)^{-(s^x+1)} \frac{1+r}{r} [1-(1+r)^{-n}]$$

$$Y^{s^x-1} (1+r) \leqq Y^{s^x} \geqq Y^{s^x+1} (1+r)^{-1}$$

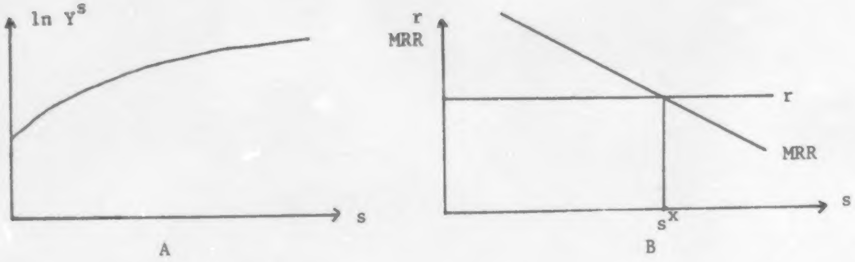
i.e., $\ln Y^{s^x} - \ln Y^{s^x-1} \geqq \ln(1+r) \approx r$

$\ln Y^{s^x+1} - \ln Y^{s^x} \leqq \ln(1+r) \approx r.$

However, by setting in plausible values for r, it becomes evident that the difference between the continuous and discrete time models is not quite negligible at higher interest levels:

r	r - ln (1+r)
0.02	0.0002
0.06	0.0017
0.10	0.0047
0.14	0.0090 .

Figures 2A and B The effect of schooling on earnings, marginal rate of return, and optimal schooling level



With a finite time horizon the simple expression (2:3) for the necessary condition is only approximately true. Assuming an interior solution, the necessary condition for a maximum is:

$$\frac{d}{ds}\{\frac{1}{r}\, Y^{s^x}\, e^{-rs^x}\, (1-e^{-r(T-s^x)})\} = 0$$

which is equivalent to

$$\frac{d \ln Y^{s^x}}{ds} = r - \frac{d \ln(1-e^{-r(T-s^x)})}{ds} > r \quad \text{or}$$

$$\text{(2:4)} \quad \frac{d \ln Y^{s^x}}{ds} = r\, \frac{1}{1-e^{r(s^x-T)}} > r.$$

If T is extended to infinity, the results are the same as in the case of the given working period.[1]

---

[1] With a discrete time model we again obtain approximately the same results:

$$\text{Max } J = \sum_{t=s}^{T-1} Y_t^s\, (1+r)^{-t} = Y_t^s\, \frac{1+r}{r}\, (1+r)^{-s}\, [1-(1+r)^{-(T-s)}]\ .$$

Assuming that an interior solution exists, the necessary condition for a maximum is:

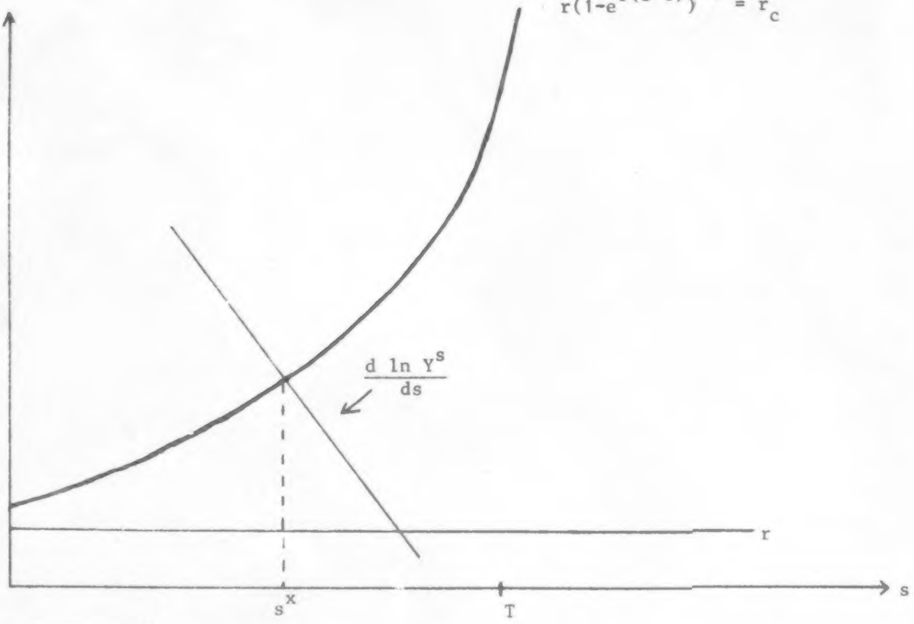
$$Y^{s^x-1}(1+r)^{-(s^x-1)}\frac{1+r}{r}\,[1-(1+r)^{-[T-(s^x-1)]}] \leqq Y^{s^x}\,(1+r)^{-s^x}\,\frac{1+r}{r}$$

$$[1-(1+r)^{-(T-s^x)}] \geqq Y^{s^x+1}(1+r)^{-(s^x+1)}\,\frac{1+r}{r}\,[1-(1+r)^{-[T-(s^x+1)]}]$$

Footnote continued on next page.

The optimal schooling level is no longer found where d ln Y^s/ds equals the rate of interest, but rather where it equals the rate of interest corrected for finite life time (r_c). This is illustrated in Figure 3 which shows that as the schooling period grows longer, a greater increase in earnings is required to cover the interest rate and the shorter recoupment

Figure 3 Optimal schooling level with a given planning horizon



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$$Y^{s^x-1}(1+r)[1-(1+r)^{s^x-T-1}] \leqq Y^{s^x}[1-(1+r)^{s^x-T}] \geqq Y^{s^x+1}(1+r)^{-1}[1-(1+r)^{s^x-T+1}]$$

$$\frac{Y^{s^x}}{Y^{s^x-1}} \geqq (1+r)\frac{[1-(1+r)^{s^x-T-1}]}{[1-(1+r)^{s^x-T}]} > (1+r)$$

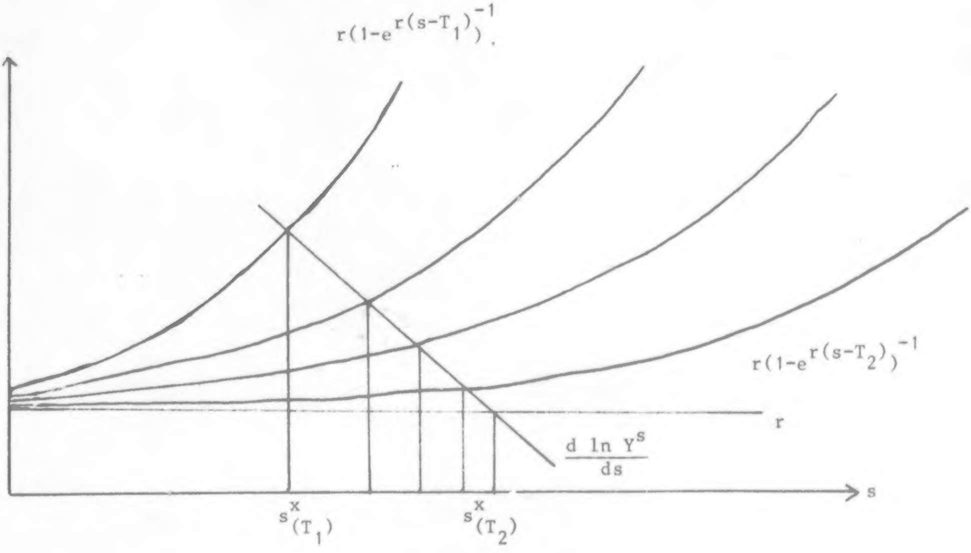
$$\frac{Y^{s^x+1}}{Y^{s^x}} \leqq (1+r)\frac{[1-(1+r)^{s^x-T}]}{[1-(1+r)^{s^x-T+1}]} > (1+r).$$

With $T_{\to\infty}$ the results reduce to those for a given working period.

period. In this case, the individual would be indifferent to the length of the schooling period only if - for some reason - $d \ln Y^s/ds$ should develop as the finite-life-corrected rate of interest. If $d \ln Y^s/ds$ never exceeds r_c, the optimal solution is a schooling period equal to zero. A corresponding corner solution with schooling period equal to T obtains only if $d \ln Y^s/ds$ is infinite.[1]

The ceteris paribus effect of extending the planning period (T) is illustrated in Figure 4. Each planning horizon gets its own schedule for finite-life-corrected rate of interest. A longer planning period ($T_2>T_1$) lengthens the optimal schooling period (s^x).

Figure 4 Optimal schooling levels with different planning horizons



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[1] $$\lim_{s \to T} r(1-e^{r(s-T)})^{-1} = \infty$$

We find that an assumption about a given period of employment, irrespective of the length of the schooling period, is unfortunate in that nothing in the model would then preclude an infinite period of schooling, and thus infinite life. It could be argued, of course, that such prolonged and profitable schooling would not, technically, be possible and thus would not be offered. Or, expressed differently: equilibrium does not exist when there is no maximum s^x. We assume that such an equilibrium exists. Thus, let us proceed as follows:

- use the above simple expression (2:3) assuming a finite working life and a maximum s^x (version A);
- use (2:3) as a approximation together with the assumption of a finite time horizon (version B_1); 1)
- use the more complex but exact expression (2:4) for a finite time horizon:

$$\frac{d \ln Y_t^{s^x}}{ds} \; A(s^x) = r$$

where $A(s^x)$ is a correction factor for finite life varying with s^x (version B_2).

Since individuals have been assumed to be alike, the present value of life earning streams connected with various periods of schooling must be identical if different educational alternatives are to be chosen. Let us compare two individuals with different amounts of schooling. With version A (and B_1) in equilibrium, for two individuals with s_1 and s_2 years of schooling, respectively, we have:

---

1) This would sometimes be a rather crude approximation, as the finite-life-corrected rate of interest would, e.g.

- instead of 4% be:
  - 4.48% with $s = 9$ and $T = 65$,
  - 5.72% with $s = 25$ and $T = 65$,
- instead of 10% be:
  - 10.04% with $s = 9$ and $T = 65$,
  - 10.52% with $s = 25$ and $T = 65$.

$$(2:5)\quad Y^{s_1}\int_{s_1}^{n+s} e^{-rt}\,dt - Y^{s_2}\int_{s_2}^{n+s} e^{-rt}\,dt = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{Y^{s_2}}{r} e^{-rs_2}(1-e^{rn}) - \frac{Y^{s_1}}{r} e^{-rs_1}(1-e^{rn}) = 0 \Leftrightarrow$$

$$\Leftrightarrow Y^{s_2} = Y^{s_1} e^{r(s_2-s_1)}$$

Thus, comparing an individual with s years of schooling and one with zero years, we have

$$(2:6)\quad Y^s = Y_0 e^{rs}$$

where Y_0 = earnings from initial stock of human capital ,or

$$(2:7)\quad \ln Y^s = \ln Y_0 + rs.$$

Therefore, an implication of the Simple Schooling Model version A (and B_1) is that the logarithmic value of earnings increases linearly (and the ordinary value of earnings exponentially) with the length of education. Moreover, the increase equals r - the market rate of interest.[1]

Figure 5 shows how the actual earnings develop (version A and B_1) for individuals with no schooling (Y_t^0), some schooling ($Y_t^{s_1}$) and long schooling ($Y_t^{s_2}$); the exponential relation between earnings and schooling is indicated by the dashed curve.

Relation (2:7) has been called the Schooling Model [2] and has been widely

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[1] With discrete time only approximately the same results are obtained:

$$\sum_{t=s_2}^{s_2+n} Y^{s_2}(1+r)^{-t} = \sum_{t=s_1}^{s_1+n} Y^{s_1}(1+r)^{-t}$$

and with $s_1=0$, $s_2=s$

$$Y^s = Y_0(1+r)^s$$

$$\ln Y^s = \ln Y_0 + s\ln(1+r) \approx \ln Y_0 + rs.$$

[2] J.Mincer, op.cit., 1970, pp.6-8.

applied [1] in empirical work. [2] It means that the marginal rate of return is constant and always equal to the market rate of interest for the relevant periods of schooling.

Furthermore, if costs and returns for an extra educational year were compared, they would always be found to be equal. Costs equal the earnings foregone. Returns are the present value of the increased stream of earnings caused by the addition to the stock of human capital. An individual with a longer period of schooling has an annual income that is greater than that of an individual with less education. However, among realized alternatives the choice always looks alike: the present value of the addition to the lifetime earning stream equals the earnings foregone.

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[1] See, e.g., J. Mincer, op.cit., 1974, B.R. Chiswick, op.cit., 1974, and S. Gustafsson, Lönebildning och lönestruktur inom den statliga sektorn. Uppsala, 1976.

[2] The Schooling Model and similar models (to be discussed in Section 4) are often not derived from individual welfare maximizing (cf., e.g., J. Mincer, op.cit., 1974 and G.S. Becker, op.cit., 1975). Instead, consider two individuals with equal ability and opportunities, one of them with no education and the other with some education. Comparing their actual earnings (Y_0 and $\hat{Y}$), we could calculate the internal rate of return (i) on school investments. Then, if we assume no investments prior to schooling and a constant rate of return on all investments equalling the internal rate of return (i), the investment costs (C_j) could be calculated successively as:

$$C_j = Y_o - \hat{Y}_j + i \sum_{t=0}^{j-1} C_t$$

Actual earnings equal: $\hat{Y}_j = Y_o + i \sum_{t=0}^{j-1} C_t - C_j$.

If C_j is assumed to be equal to potential earnings (E_t) during the period of schooling and no investments are made after schooling, we have:

$$Y^s = Y_o + i \sum_{t=0}^{s-1} E_t = Y_o (1+i)^s$$

since

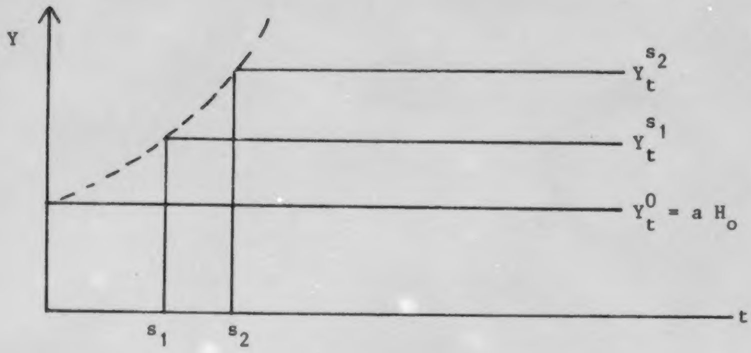
$$E_t = E_{t-1} - i E_{t-1} = E_{t-1} (1+i)$$

$$Y^s = Y_o \prod_{j=0}^{s-1} (1+i) = Y_o (1+i)^s.$$

and $\ln Y^s \approx \ln Y_o + is$.

The latter expression looks rather similar, compared to the one we derived above. The main problem with the present approach is that it lacks an underlying analytic framework into which more aspects can be explicitly introduced (cf. S. Rosen, op.cit., 1977 /b/).

Figure 5. Relations between earnings and schooling



In the case of a finite time horizon (version B_2), the result would be somewhat different. If identical individuals are to choose different schooling periods, the result in terms of human capital production in later school years must compensate for the shorter period of recoupment. We have:

$$Y^{s_2}\int_{s_2}^{T} e^{-rt}\,dt - Y^{s_1}\int_{s_1}^{T} e^{-rt}\,dt = 0$$

$$\frac{1}{r}\,Y_t^{s_2}\,(e^{-rs_2} - e^{-rT}) - \frac{1}{r}\,Y_t^{s_1}\,(e^{-rs_1} - e^{-rT}) = 0$$

$$Y_t^{s_2} = \frac{Y_t^{s_1}(e^{-rs_1} - e^{-rT})}{(e^{-rs_2} - e^{-rT})}$$

or, accordingly

$$Y^s = Y_0\,\frac{(1-e^{-rT})}{(e^{-rs}-e^{-rT})} = Y_0\,e^{rs}(1-e^{-rT})(1-e^{r(s-T)})^{-1}$$

and

(2:8) $$\ln Y^s = \ln Y_o + \ln(1-e^{-rT}) + rs - \ln(1-e^{r(s-T)}).$$

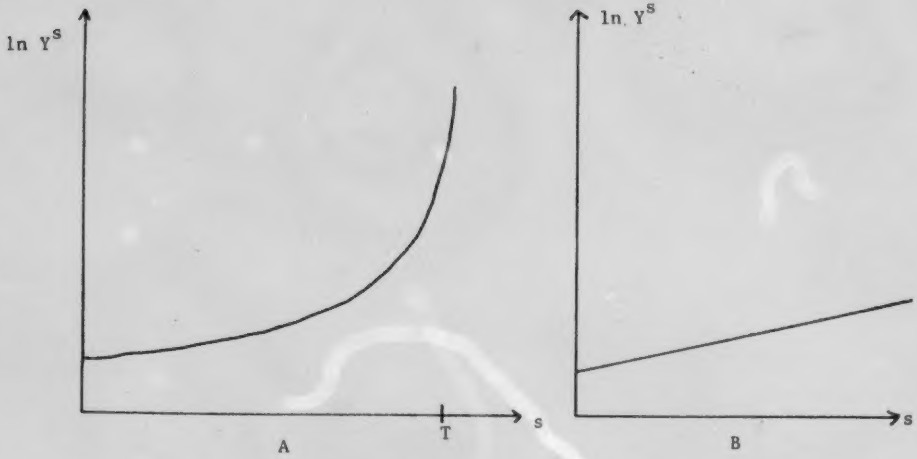
Thus the requirement is:

(2:9) $$\frac{d\ln Y^s}{ds} = r\,\frac{1}{1-e^{r(s-T)}}\ .$$

Extending T to infinity, the results are the same as for a given working period.

This implies an increasing slope (always greater than r) of the curve relating the logarithmic value of earnings and schooling, as shown in Figure 6, panel A. (Panel B contrasts the results for a given period of employment.) At T, the slope tends towards infinity - in other words, infinite increases in earnings are required if individuals are to pursue a really extended period of education (up to the age of retirement).

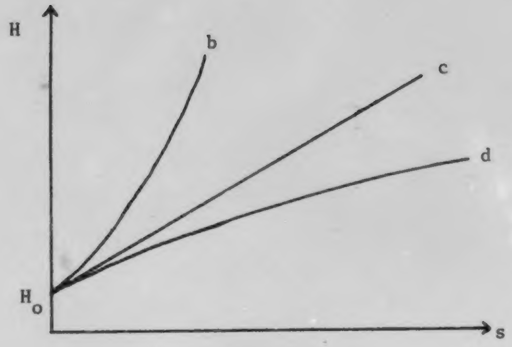
Figures 6 A and B The relation between earnings and schooling if "identical" individuals are to choose different schooling levels.



Thus far, the model offers no explanation as to why different periods of schooling and, consequently, different annual earnings are chosen. This holds true irrespective of the assumptions used about planning horizons. The Simple Schooling Models only say that if different schooling periods are chosen by "identical" individuals, certain specific differences in human capital stock and earnings must be associated with these schooling levels.

It should be noted that this has implications for the educational process. An educational process may be illustrated by learning curves. Figure 7 shows three such curves. According to curve c , every year of schooling adds an equal amount to the stock of human capital. According to curve d it becomes more and more difficult to acquire human capital - and, according to curve b, easier. Curve b might be interpreted as representing those pedagogical theories that stress that each new element "learned" is not simply an addition to but also an enhancer of previous knowledge.

Figure 7. Learning curves in terms of human capital units and schooling



Now, what curve fits in with an equilibrium with different schooling levels of the models presented above? As we saw above (2:6) we should (with version A och B_1) have an exponential relation between earnings and schooling: $Y^s=Y_o\ e^{rs}$. In terms of human capital, this means that the stock of such capital also increases exponentially with years of schooling ($Y_o\ e^{rs}=aH_o e^{rs}$). Only those educational levels that make the human capital stock grow at an exponential rate would survive in equilibrium. With version B_2 this tendency would be still more pronounced.

Each firm renting human capital will decide how many units of human capital it will hire, given its technology and the prices of outputs and inputs (including the rental price for human capital). The earnings hierarchy within a firm, or society at large, will depend on the amount of human capital possessed and placed on the market by individuals. The characteristics of the individuals (the supply side) rather than those of the firms (the demand side), will determine the earnings structure. This feature is characteristic of all the human capital models in this report.

With homogeneous human capital there is no need to discuss the demand of firms in terms of different types of educated labor or occupations. Demand can be expressed in units of human capital. For every unit of human capital, that is profitably supplied and rented at the prevailing market price, there is, so-to-speak, a job.

Can we get some more results from these human capital models if, for example, we reformulate them in monetary terms or time costs? When using the Simple Schooling Model (irrespective of version), it is no problem to translate investment costs into earnings. As we have seen, one prerequisite for the choice of different periods of schooling by "identical" individuals was a marginal rate of return that was constant and equal to the market rate of interest.[1] This means that the average rate of return is also constant and equal to the given market rate of interest. It would always be possible to take the investment costs - be they large or small - and multiply by the market rate of interest to obtain the earnings of the individual if the earnings on initial endowments are added.[2]

Thus we would have:

$$Y^s = Y_o + r \sum_{t=0}^{s-1} C_t \quad \text{and} \quad Y^{s_2} - Y^{s_1} = r \sum_{t=s_1}^{s_2-1} C_t \, .$$

Or, $Y^s = Y_o + r \sum_{t=0}^{s-1} E_t$

since investment costs (C) are only earnings foregone (E_t).

However, this does not add anything new to what has already been said. Investment costs - except for initial schooling - can only be estimated when we know how earnings are influenced by education. Foregone earnings are always included among the costs. Thus, no independent result can be obtained by using costs until the relation between earnings and schooling is known. Once known, however, it may then be practical to work with costs. The same is true for "time costs", i.e. investment costs expressed as a fraction of earnings.

We have, e.g.:[3] $E_{ti} = E_{0i} + r \sum_{j=0}^{t-1} C_{ji}$

---

1) For discrete time and continuous time with a finite planning horizon this is only approximately the case.

2) For discrete time and continuous time with a finite planning horizon the correspondingly corrected market rate of return should be used instead.

3) This is only approximately true for discrete time and continuous time with a finite planning horizon.

Let us express the investment costs as a proportion (v) of the potential earnings (E):

$$C_{ji} = v_{ji} E_{ji}.$$

Thus,(omitting the individual subscript) we have:

$$E_0 = E_0$$
$$E_1 = E_0 + rv_0 E_0 = E_0 (1+rv_0)$$
$$E_2 = E_1 + rv_1 E_1 = E_0 (1+rv_0)(1+rv_1)$$
etc.

and generally

$$E_t = E_0 \prod_{j=0}^{t-1} (1+rv_j)$$

which means that:

$$\ln E_t \approx \ln E_0 + \sum_{j=0}^{t-1} rv_j \text{ 1)}.$$

For actual earnings we then have:

$$\ln Y_t \approx \ln E_0 + \sum_{j=0}^{t-1} rv_j + \ln (1-v_t) \text{ as } Y_t = E_t(1-v_t).$$

Summing up the results of the Simple Schooling Models, the reason why "identical" individuals would choose different levels of schooling is that they are indifferent to the educational level. From this follows a number of necessary relations between earnings, human capital production, schooling, etc.

With these models we would get a completely elastic supply of human capital. After having supplied their initial stock of human capital ($\Sigma\bar{H}_0$), without costs to themselves, the individuals can increase the supply by produced human capital. If education is profitable at the going rate of interest, every individual is indifferent to the amount of schooling. He earns just enough to make the investments worthwhile. The amount actually rented would be determined by the demand of the firms. And with the elastic supply curve, the individuals would not earn any rents on the human capital produced. However, all earnings on initial human capital are rents.

---

1) Where rv_j is the same type of approximation of $\ln(1+rv_j)$ as r for $\ln(1+r)$ numerically exemplified in the footnote on page 23.

An increase in the rate of interest would, however, take away all investment incentives and a reduction in the rate of interest would raise them infinitely.

The situation on the supply side might thus be described as unstable. Disequilibria may often be expected to occur and equilibrium may be difficult to reestablish. Analytically, the model is therefore less satisfactory, and, as noted above, it cannot explain why different levels of schooling are chosen. Let us therefore turn the question around and instead investigate how individuals who are not identical would behave in a schooling model.

2.2 Schooling Models with Individual Differences as Explanatory Factors

The factors that could vary between individuals in the Simple Schooling Models include initial stock of human capital, time horizon and "ability" - i.e., how much individuals can profit from a given education. Before analyzing the effects of individual variations in these respects, we shall discuss the addition of two features to the Schooling Models - direct investment outlays and depreciation and appreciation. We will also say a bit more about ability and the ways by which it could be treated somewhat more explicitly in Schooling Models.

The effects of certain factors with individual variations are discussed by groups. Time horizon, depreciation and appreciation are dealt with in Section 2.2.4 and initial endowments and ability in Section 2.2.5. Variations in prices, i.e., the rental price of human capital and the rate of interest, are analyzed in Section 2.2.6.

2.2.1 Direct Investment Outlays

In the Simple Schooling Models the only resource going into the human capital production process was the capacity of the individual. The pursuit of education can then be visualized as being organized by the individual himself. The direct investment outlays that we are now going to consider cover other types of resources - e.g., teacher services, books, and materials. While it is still possible to think of the educational process as being organized by the individual, it seems more natural to view it as taking place at special institutions. The payment for the services then provided by the educational institution would reflect not only the quantity but also the quality of material and services. However, assuming competitive markets, more expensive resources would also mean more efficient resources and a greater production of human capital. Investment outlays for higher quality are therefore part of the direct investment outlays and do not form any special category.

Variations in direct investment outlays between individuals imply - as do variations in schooling period - that greater or smaller personal investments are made in human capital. Direct investment outlays mean that, instead of having zero earnings during the schooling period, the individual has negative earnings equal to the direct investment outlays. Individuals with "identical" characteristics may choose to make different amounts of direct investments. However, if "identical" individuals, e.g., having the same level of schooling, combine it with more or less direct investment outlays, the present value of their income streams must still be equal. If we could keep account of these differences in direct investment outlays, they - and differences in schooling - could explain the differences in annual earnings. For continuous time and a given working period (n) we then get the following expression:

$$E^s = E_0 + r \int_0^s (E_t + D_t)\, dt = E_0\, e^{rs} + r \int_0^s D_t\, dt$$

where E^s = potential earnings from schooling level s

E_0 = potential earnings from initial human capital stock

D_t = direct investment outlay at date t

and since $E^s = Y^s$ = actual earnings $(t>s)$, we have

$$Y^s = Y_0\, e^{rs} + r \int_0^s D_t\, dt.$$

Or, expressing the total investment costs as a fraction of potential earnings:

$$v_t = \frac{E_t + D_t}{E_t}$$

$$E^s = E_0 + r \int_0^s v_t\, E_t\, dt = E_0 + r \int_0^s v_t\, E_0\, e^{rt} dt.$$

Assuming that v_t is constant, we could get

$$E^s = E_0\, e^{rvs}, \text{ or}$$

$$\ln Y^s = \ln Y_0 + rvs.$$

With a given planning horizon (T) we would have to add the correction term for finite lifetime.

For discrete time models we could, for example, use the following approximate formula (as derived below[1]):

$$\ln Y^s = \ln Y_0 + r \sum_{j=0}^{s-1} v_j$$

where v_j is the fraction of potential earnings during the years of schooling invested in human capital. With positive direct investment outlays $v_j>1$.

Since in these cases involving "identical" individuals we have an hypothesis about how earnings develop with human capital investments, these approaches are empirically applicable. However, we cannot use them for individuals who differ, e.g., with respect to ability. If in the latter case we want to relate earnings to schooling period and to direct investment outlays, we must have a more precise specification of the production function of human capital. Then we could also explain why individuals with different characteristics choose to make different direct investment outlays. However, as we can pursue the development of the Schooling Models and obtain interesting results without specifying the exact form of the production function of human capital, we shall leave these questions for Section 3 where we study different production functions.

---

1) In the Simple Schooling Model, both with a given working period and a given planning horizon not involving direct investment outlays, we have at least approximately

$$E^s = E_0 + r \sum_{t=0}^{s-1} E_t .$$

In a similar situation with direct investment outlays we would have:

$$E^s \approx E_0 + r \sum_{t=0}^{s-1} (E_t + D_t).$$

Or, expressing the total investment costs as a fraction of potential earnings:

$E^s \approx E_0 + r \sum_{t=0}^{s-1} v_t E_t$, using the same notations as above.

Since $E_t \approx E_{t-1} + rv_{t-1} E_{t-1} = E_{t-1}(1+rv_{t-1})$, we obtain

$$E^s \approx E_0 \prod_{j=0}^{s-1} (1+rv_j), \text{ or } \ln E^s \approx \ln E_0 + \sum_{j=0}^{s-1} \ln (1+rv_j) \approx \ln E_0 + r \sum_{j=0}^{s-1} v_j .$$

After the schooling period ($t>s$) we then have $\ln Y^s \approx \ln Y_0 + r \sum_{j=0}^{s-1} v_j$.

2.2.2 Depreciation and Appreciation

Depreciation means that the human capital stock - or the earning capacity acquired by schooling - deteriorates over time and that productivity diminishes. One reason may be that the individual forgets skills that he once learned. Appreciation is the opposite. Due to such factors as biological maturing, the productivity of the individual's human capital stock could increase.

We shall assume a constant rate of depreciation (δ) and appreciation (α), and call the net effect d:

$$d = \delta - \alpha.$$

A greater d thus means a faster deterioration of the stock of human capital and lower returns from education. The prerequisite for choosing an optimal schooling period now becomes the level of education at which the increase in the logarithmic value of earnings for a marginal increase in schooling equals the rate of interest plus the net rate of depreciation (approximately or exactly with continuous time models, depending on version with respect to planning horizon.) This can be shown as follows. Assume that the human capital stock depreciates after the schooling period according to

$$H_t = H(s)\, e^{-dt} \qquad t > s.$$

The individual would now be assumed to maximize

$$J_1 = \int_s^{n+s} aH(s)\, e^{-dt}\, e^{-rt}\, dt = Y_t^s \frac{1}{r}\, e^{-(r+d)s}\, (1-e^{-rn})$$

alternatively

$$J_2 = \int_s^{T} aH(s)e^{-dt}\, e^{-rt}\, dt = Y_t^s \frac{1}{r}\, e^{-(r+d)s}\, (1-e^{-r(T-s)}).$$

Assuming an interior solution, the necessary conditions for a maximum would for J_1 be:

$$\frac{d \ln aH(s^x)}{ds} = (r+d)$$

and for J_2:

$$\frac{d \ln aH(s^x)}{ds}\, (1-e^{-r(T-s^x)}) = (r+d).$$

For discrete time the results would only be approximately the same.[1)]

In a situation with declining effects of schooling on earnings the higher the net rate of depreciation the shorter would the optimal schooling period be.

2.2.3 Ability

Thus far, "ability" has not been explicitly treated in our presentation of Schooling Models. To prepare for the discussion of individual variations in "ability", this must be done. Let us do it as generally as possible and instead of writing H=H(s) we write:

$$H = g(s,A)$$

where A is the "ability" efficiency parameter.

For a given period of employment the individual would now be assumed to maximize

$$J_1 = \int_s^{n+s} Y_t^{s,A} e^{-rt} dt = Y_t^{s,A} \frac{1}{r} e^{-rs} (1-e^{-rn})$$

where $Y_t^{s,A} = a H_t = a g(s,A)$

with the necessary condition for a maximum:

$$\frac{d \ln a g(s^x,A)}{ds} = r.$$

---

1) Let us assume that the human capital stock depreciates after schooling as follows:

$$H_{t+1} = H_t(1+d)^{-1}.$$

With a given period of employment, the individual would now be assumed to maximize

$$J = \sum_{t=s}^{s+n-1} [a H(s)(1+d)^{-t}](1+r)^{-t} = a H(s)(1+r)^{-s}(1+d)^{-s} \frac{(1+r)(1+d)}{r+d+rd}\left(1-[(1+r)(1+d)]^{-n}\right)$$

Assuming the existence of an interior solution, the necessary condition for maximum is:

$$\ln Y^{s^x} - \ln Y^{s^x-1} \geqq \ln(1+r) + \ln(1+d) \approx r + d$$

$$\ln Y^{s^x+1} - \ln Y^{s^x} \leqq \ln(1+r) + \ln(1+d) \approx r + d.$$

Alternatively, with a given time horizon

$$\text{Max } J_2 = \int_s^T Y_t^{s,A} e^{-rt}\, dt = Y_t^{s,A} \frac{1}{r} e^{-rs} (1-e^{-r(T-s)})$$

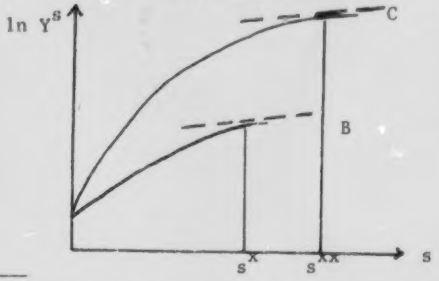
with the necessary condition for a maximum:

$$\frac{d \ln a\, g(s^x,A)}{ds} = r\, (1-e^{-r(T-s^x)})^{-1} \approx r.$$

For discrete time the results are approximately the same.[1]

Let us assume that for a given individual the ln a g(s,A) curve - i.e., the logarithmic value of earnings, which is equal to $\ln a\, g(s,A) = \ln Y^{s,A}$ develops with s as curve B in Figure 8. The optimality condition then means that the individual should follow his curve to the point where its slope equals r and stop schooling at that point. In the figure, r is indicated by the slope of the dashed line and the optimal schooling level with s^x. The figure also shows how the optimal schooling level for a more able individual with a more quickly rising curve (C) is determined (s^{xx}), i.e., where the slope of his curve is equal to the rate of interest.

Figure 8. The choice of optimal schooling level for individuals with differences in ability



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[1] The individual would, e.g., with a given period of employment, be assumed to maximize

$$J = \sum_{t=s}^{s+n-1} a\, g(s,A)(1+r)^{-t} = a\, g(s,A)(1+r)^{-s} \frac{1+r}{r} [1 - (1+r)^{-n}].$$

Assuming that an interior solution exists, the necessary condition for a maximum is:

$$\ln [a\, g(s^x,A)] - \ln [a\, g(s^x-1,A)] \geqq \ln(1+r) \approx r$$

$$\ln [a\, g(s^x+1,A)] - \ln [a\, g(s^x,A)] \leqq \ln(1+r) \approx r.$$

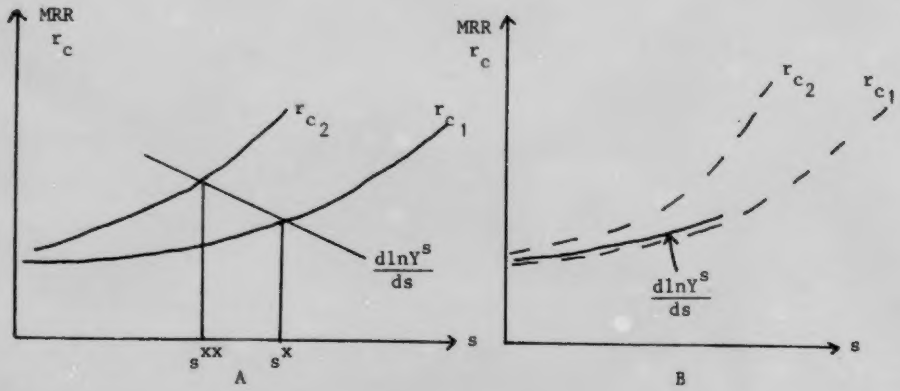
2.2.4 Individual Variations in Time Horizon, Depreciation and Appreciation

The effects of the individual variations to be discussed in this section refer only to continuous time models. The results as regards discrete time models can be inferred from what has been said above.

Let us discuss the effects of individual differences with respect to time horizon in relation to Figure 9.

A shorter time horizon for otherwise "identical" individuals would mean a shorter schooling period, as $d \ln Y^s/ds$ has to equal a lifetime-corrected rate of interest (r_c) that increases with a shorter planning period (s^{xx} compared with s^x in figure 9 A where $r_{c_2} > r_{c_1}$ because $T_2 < T_1$). If at a given time horizon all schooling periods are equally profitable (figure 9 B, r_{c_1}), none would be profitable for an individual with a shorter time horizon, (r_{c_2}).

Figures 9 A and B The choice of optimal schooling level for individuals with different planning horizons.



In the case described by figure 9 A, we would according to (2:1) have

$$\frac{1}{r} Y^{s_2} e^{-rs_2} (1-e^{-r(T_2-s_2)}) > \frac{1}{r} Y^{s_1} e^{-rs_1} (1-e^{-r(T_2-s_1)})$$

and

$$\frac{1}{r} Y^{s_1} e^{-rs_1} (1-e^{-r(T_1-s_1)}) > \frac{1}{r} Y^{s_2} e^{-rs_2} (1-e^{-r(T_1-s_2)})$$

where s_2 would correspond to s^{xx} and s_1 to s^x.

Accordingly, we have

$$e^{r(s_1-s_2)} \frac{1-e^{-r(T_1-s_2)}}{1-e^{-r(T_1-s_1)}} < \frac{Y^{s_1}}{Y^{s_2}} < e^{r(s_1-s_2)} \frac{1-e^{-r(T_2-s_2)}}{1-e^{-r(T_2-s_1)}}$$

or, comparing individuals with s and zero years of schooling, where T_2 optimally implies zero years of schooling:

$$\ln Y_0+\ln(1-e^{-rT_1})+rs-\ln(1-e^{r(s-T_1)})<\ln Y^s<\ln Y_0+\ln(1-e^{-rT_2})+rs-\ln(1-e^{r(s-T_2)}).$$

The effect of the introduction of a constant rate of depreciation is that the schooling period at which $\frac{d\ln Y^s}{ds}$ equals the market rate of interest (or lifetime-corrected rate of interest) plus the rate of depreciation will be chosen. In Figure 10 we illustrate the situation with varying rates of depreciation for the model with a given working period. (The qualitative results are the same with finite planning; cf. Section 2.2.2.)
A longer period of schooling is seen to be optimal for an individual with a lower rate of depreciation (panel A). If at a given market rate of interest and no depreciation all schooling periods are equally profitable (panel B), none would be profitable for individuals with a positive rate of depreciation.

The effect of a constant rate of appreciation would be similar to that of depreciation but with opposite sign. In terms of Figure 10 a shorter period of schooling would be chosen by an individual having a lower rate of appreciation. If all schooling levels were equally profitable without appreciation, they would all remain so for an individual with a positive rate of appreciation.

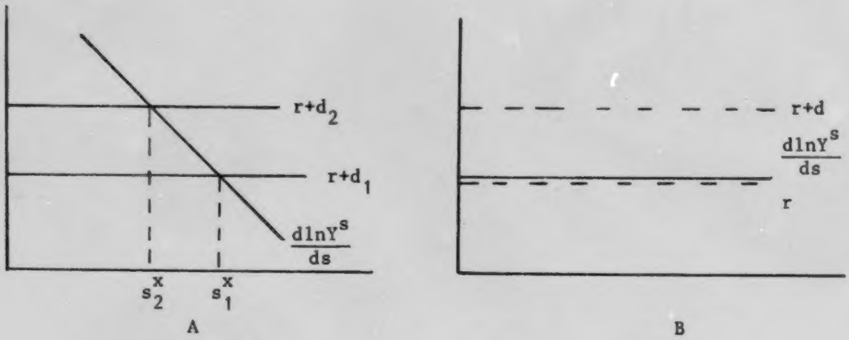
According to Section 2.2.2, the relation between earnings, with a given period of employment, would be:

$$Y^{s_1} \frac{1}{r} e^{-(r+d_1)s_1}(1-e^{-rn}) > Y^{s_2} \frac{1}{r} e^{-(r+d_1)s_2} (1-e^{-rn})$$

$$Y^{s_2} \frac{1}{r} e^{-(r+d_2)s_2}(1-e^{-rn}) > Y^{s_1} \frac{1}{r} e^{-(r+d_2)s_1} (1-e^{-rn})$$

where s_1 and s_2 correspond to s_1^x and s_2^x in Figure 10 A.

Figures 10 A and B The choice of optimal schooling level by individuals with different rates of depreciation



Accordingly, when comparing individuals with s and zero years of schooling, and where d_2 optimally implies no schooling and d_1 s years, we have 1)

$$\ln Y_0 + (r+d_1)\, s < \ln Y^s < \ln Y_0 + (r+d_2)\, s.$$

2.2.5 Individual Variations in Initial Endowments and Ability

The effect of individual variations with respect to initial endowments and ability will be discussed in relation to Figure 11. Panel A depicts individual variations in initial human capital stock within the Simple Schooling Model (version A and B_1). The present value of earnings by individuals with equal initial endowments would be the same regardless of the period of schooling. However, the present value of earnings of individuals with different initial endowments would not be equalized. This is due to the rents on initial endowments that the market cannot equalize. For each group with equal initial endowments the Simple Schooling Model would still be exact. 2)

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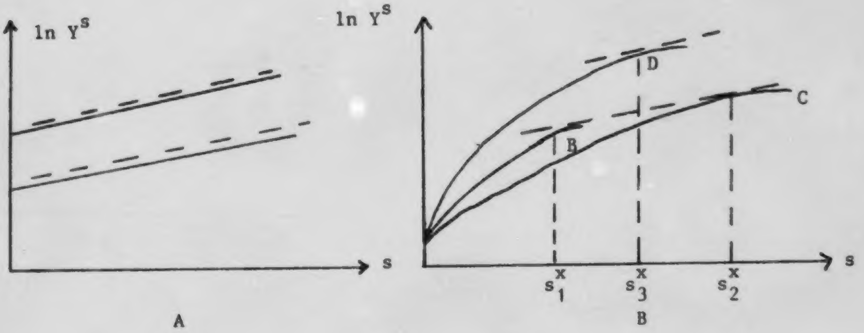
1) With a finite planning horizon the correction factors for finite life would have to be added.

2) Diagrammatically version B_2 of the Simple Schooling Model looks like Figure 11 B, where differences in initial endowments would be illustrated by parallel shifts in the $\ln Y^s$ curve, which is thus only approximated by 11 A.

Figures 11 A and B The choice of schooling level by individuals with different initial endowments and abilities



Individual variations in ability could have the same effects as individual variations in initial endowments if the only factor involved was a parallel shift in the ln Y^s curve, as illustrated in Figure 11 A. In Figure 11 B the ability factor is shown as giving a different curvature to the effect of schooling on the logarithmic value of earnings. In this case the individual would no longer be indifferent as to the length of the schooling period, but there is one optimal level of schooling. If all individuals were alike, they would choose the same period of schooling. If individuals are to choose different schooling levels they must now be assumed to have different abilities. However, the result could still be that individuals with different levels of ability would choose different amounts of schooling but would have the same present value of earnings (with curves B and C). Generally, of course, we could expect to find different schooling periods associated with different present values of earnings (with ,e.g., curve B and D), or the same schooling period associated with different present values of earnings.

Not until we have made more definite assumptions about the effect of ability on earnings via schooling can any precise implications be drawn in terms of choice of schooling level and the present value of earnings. In Figure 12 A, ability is only shown as influencing the present value of earnings, not the optimal level of schooling (s^x). This can be seen as a less extreme version of the ability interpretation of Figure 11 A. In Figure 12 B ability is shown to influence both the optimal level of schooling (s^x contra s^{xx}) and the present value of earnings.

If the ability factor influences the present value of earnings so that

$$PVY(A_2) > PVY(A_1) \text{ with } s_2 > s_1$$

then according to Section 2.2.3, for a given working period we have

$$\ln Y^{s_2} > \ln Y^{s_1} + r\,(s_2 - s_1)$$

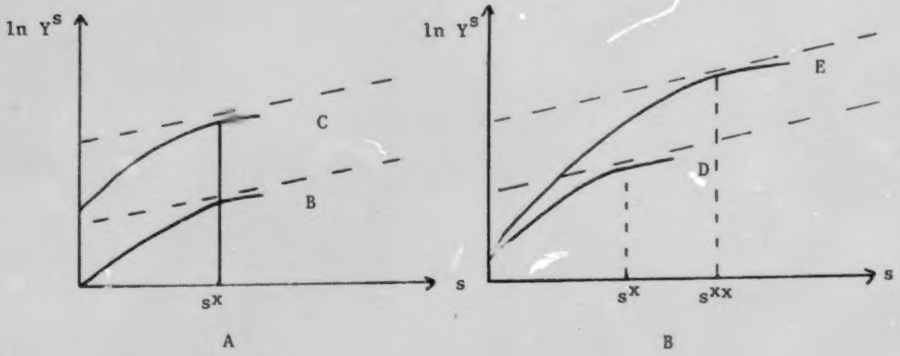
or comparing an individual with s years of schooling and one with zero years

$$\ln Y^{s} > \ln Y_0 + rs.$$

With a given planning horizon we have

$$\ln Y^{s} > \ln Y_0 + rs + \ln(1-e^{-rT}) - \ln(1-e^{r(s-T)}).$$

Figures 12 A and B The choice of levels of schooling by individuals with different levels of ability



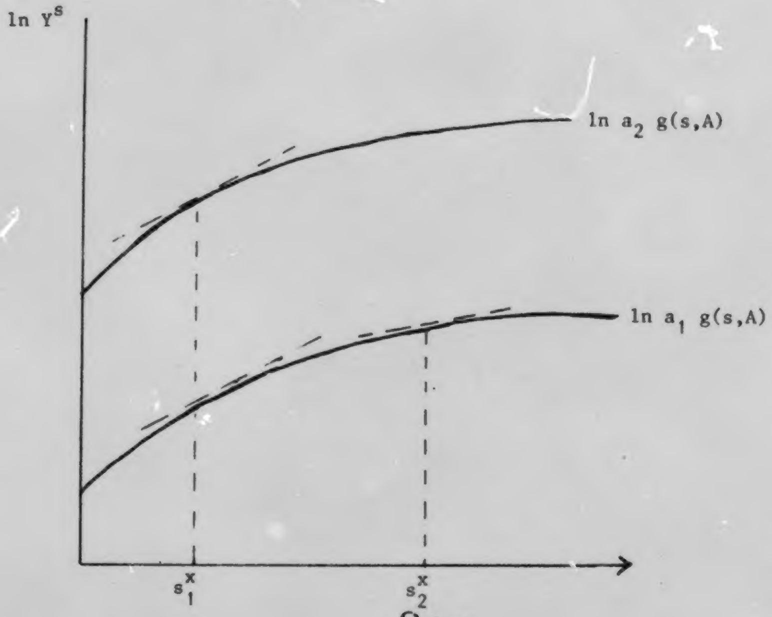
2.2.6 Variations in Prices

So far, we have worked under the assumption that all prices - and particularly those pertaining to human capital production - are equal for all individuals. This follows if we assume a competitive equilibrium. We could, however, use the models partially to indicate how an investment decision would be affected if individuals should meet different conditions with respect to such factors as rental prices and interest rates.

In the models in which we permit individual variations in ability we saw that we would normally expect only one schooling period to be optimal for the individual. What would happen to that level if the individual had to calculate with another a or r ? As shown in Figure 13 a lowering of the rental price (from a_2 to a_1) does not change the optimal level of schooling - only the present value of earnings. This is due to the fact that the rental price affects costs and benefits in the same way. A reduction of initial endowments or ability, if ability acts as a shift factor, produces the same kind of effects. This means that an empirical separation of the effects of variations in the rental price requires both precise knowledge of the disequilibrium situation (e.g., that only a varies and not r) and possibilities to identify initial endowments and "ability."

If, instead, the level of the rate of interest differs among individuals, each individual should follow his ln a g(s,A)-curve up to the point where its slope equals his rate of interest. A lower rate of interest would thus lead to a

Figure 13 The choice of schooling level by individuals with different rental prices for human capital and different rates of interest.



longer period of schooling. This is also seen in Figure 13 as the change from s_1^x to s_2^x, due to a lower r level. For a given period of employment, and extending the results in Section 2.2.4 on depreciation, the relation between earnings would be

$$\ln Y_0 + r_1 s < \ln Y^s < \ln Y_0 + r_2 s$$

where r_2 implies zero years of education and r_1 s years.

If we could identify groups possessing equal abilities (and equal $T, \bar{H}_0$, etc.) but calculate with different levels of r, it would be possible to find these ln a g(s,A)-curves. A negative relation between s and r would then be observed. This would also be the case not distinguishing different ability groups if "ability" variations were relatively unimportant.

2.2.7 Rents, Supply and Demand for Human Capital

In a situation with a decreasing marginal rate of return to education and one optimal level of schooling for each individual, the total supply of human capital can no longer be perfectly elastic. In this situation, rents are earned on initial stocks as well as on intramarginal units of human capital.

Given the characteristics of the individual, his optimal schooling level is determined by the rate of interest. Since the rental price does not influence that level, the supply is inelastic in the rental market for human capital and consists of initial stocks and the human capital produced during schooling. The rental price will be determined by demand, i.e., by the demand by firms for human capital. In competitive equilibrium a single rental price will be established for human capital. This price - a - will remain constant as long as supply and demand are stable or expand (contract) at the same rate.

2.3 Special Features of the Schooling Models

In the Schooling Models the exogenous factors of special interest in the three categories - initial endowments, production possibilities and preferences - are the following:

$\bar{H}_0$ - initial stock of human capital,

A - ability,

δ , α - rate of human capital depreciation or appreciation,

$Y=f(s)$ or $H=g(s,A)$ - earnings as a function of schooling or more explicitly the production function for human capital with inputs: ability and the length of the schooling period,

T(n) - planning horizon (working life period),

preferences - utility derived from goods (not leisure and schooling) which results in a maximization of the present value of net earnings (PVY).

The exogenously given factors together with the behavioral assumptions for consumers and firms lead to an equilibrium situation characterized, e.g., by:

a - the common rental price for human capital,

r - the rate of interest,

s_i - individual levels of schooling,

$Y_i=aH_i$ - individual levels of actual earnings.

Let us now make a few observations about the assumptions common to both the Simple Schooling Models and the alternatives considered, after which we will indicate separately some of the implications for the Simple Schooling Models and the alternatives.

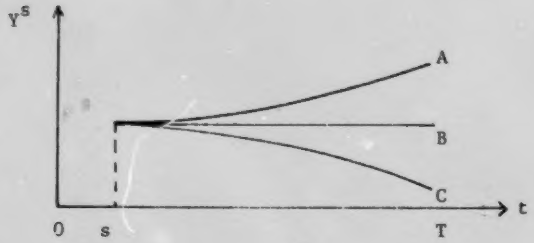
There are three types of activities in which the capacity (H) of the individual can be used: in work in the labor market, in human capital production and "other activities." In the Schooling Models "other activities" were not considered as varying between the individuals, or over time for the same individual, as they will be in later models.

In the Schooling Models the period of education means full-time schooling, and it is followed by a period of full-time work. We did not consider the possibility of individual variations in the intensity with which the stock of human capital is utilized either for training or work. Nor will this distinction be made in later models.

The theoretical concept of education in the Schooling Models has certain implications for empirical work with respect to actual or formal schooling. In principle these models not only exclude part-time work during schooling, but also on-the-job training. In other words, one test of their applicability is the extent to which full-time schooling is followed by a period of full-time work, without on-the-job training. Alternatively, they should be reserved for individuals exhibiting such a pattern, or for situations where variations in work-time and the effects of on-the-job training could be eliminated. The latter point will be discussed in relation to the models in the following sections.

As no new human capital is created after schooling the implication is, accordingly, that age earning profiles (Y) would be flat - if it were not for depreciation and appreciation. Depending on the net value of these factors, the age earnings profile could alternatively rise or decline at a constant rate,[1] as illustrated in Figure 14 by curves A and C, respectively.

Figure 14 Age earning profiles after schooling according to the schooling models



As to the individual planning period it would appear most reasonable to assume that it goes from 0 to T, which means, of course, that we assume that the individual has such a definite planning period from the beginning.

In all human capital models the result will be that the individual pursues human capital investments up to the point where marginal returns equal marginal costs. However, as will be seen, marginal returns and costs mean different things depending on the model specification.

Let us now turn to the special hypotheses and implications of the Simple Schooling Models. Here we have considered "identical" individuals who choose different amounts of education. This approach led to certain specific results: [1]

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[1] Assuming both rates to be constant over time.

- All choices must be equally good, i.e., produce as much PVY; if not, "identical" individuals would not consider them. But the model cannot explain why "identical" individuals choose one level of schooling rather than another.

- The stopping rule for human capital production - i.e., the optimal schooling level - is the date when the effect of schooling on the relative increase in earnings associated with that level of schooling equals the rate of interest $(\frac{d \ln Y^s}{ds} = r)$; or, with a finite planning horizon, that increase in earnings corrected for a finite life $[(\frac{d \ln Y^s}{ds}(1-e^{-r(T-s)}) = r]$. With the former version, nothing in the model would exclude eternal schooling.

- In this model we obtain a relation between earnings and schooling, but it should be kept in mind that they both are endogenous variables. The figure below illustrates this relation. Figure 15 A is applicable for a given working period and 15 B for a finite planning horizon. [2]

- The educational process implied by these models is a type of human capital production in which the human capital units produced per school year (and the related work productivity) increase exponentially.

- Every given distribution of schooling implies a certain distribution of annual earnings. But all present values of life earning streams are equal.

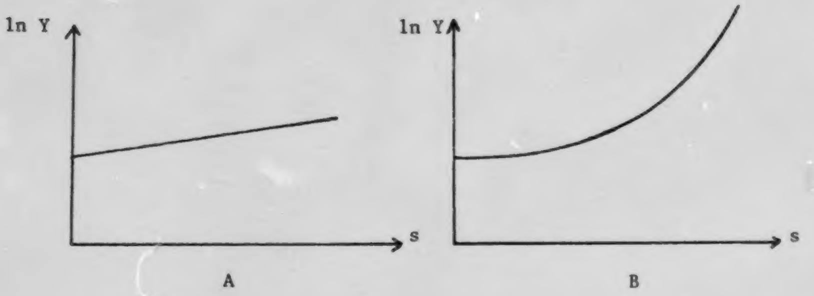
- With a given working period, the marginal and average rate of return (2:7) is constant and equal to the rate of interest. According to the relation $\ln Y^s = \ln Y_0 + rs$, if educational years were normally distributed for people on the labor market, ln Y would also be normally distributed, but not Y. A skewed distribution of school years is needed if the distribution of earnings is to be distributed normally.

---

[1] Here we only indicate the results for continuous time models which are only approximately true for the discrete time models.

[2] The risks of obtaining biased results when applying this and the other relations from the Simple Schooling Models to individuals who are not identical are further discussed below and in Sections 4 and 8.

Figures 15 A and B The relation between schooling and earnings in the Simple Schooling Models



- With finite planning - where we have an exponential increase in $\ln Y^s$ with schooling years (2:9) - a normal distribution of schooling means a skewed distribution for both ln Y and Y.

- For the variance of annual earnings, from $\ln Y^s = \ln Y_0 + rs$ and the fact that we do not consider any individual variations in Y_0 or r and the fact that r and s are not correlated, we get(with respect to a given working period):

  $$\text{Var}(\ln Y) = r^2 \,\text{Var}(s)$$

The variance in relative earnings depends on the absolute variance in schooling. The higher the rate of return on investments and the greater the variance in s, the greater the variance in relative earnings. With a given planning horizon the variance of the logarithmic value of annual earnings would be more complicated, as we have:

$$\ln Y^s = \ln Y_0 + \ln(1-e^{-rT}) + rs - \ln(1-e^{r(s-T)}).$$

The Simple Schooling Models are very special as they only apply to identical individuals. For this reason we also studied Schooling Models with Individual Differences. The varying charcteristics were: time horizon (T), rate of depreciation (δ), rate of appreciation (α), ability (A) and initial endowments ($\bar{H}_0$). The effects of differences in the rental price of human capital (a) and rate of interest (r) were also considered. The introduction of these

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individual variations generally means that it will no longer be possible to discuss the relation between schooling and earnings. Instead, one has to concentrate on the factors varying between individuals which will determine how much schooling they will undertake in order to maximize the present value of earnings. These factors determine both schooling and annual earnings. Only for individuals "identical" in all respects considered could the level of schooling and the present value of earnings be expected to be the same.

When the ability factor acts not only as a shift parameter but also turns the ln a g(s,A)-curve (i.e., the curve that shows the relation between the logarithmic value of earnings and schooling) into a strictly concave function, only one level of schooling would be optimal for the individual. However, that level also depends on his time horizon, net rate of depreciation, and the rate of interest. As a result, there will no longer be any simple common relation between earnings and schooling.

With respect to the concave ln a g(s,A)-function, we may consider three possibilities for individual variations: parallel shifts, non-parallel shifts for non-crossing curves, and non-parallel shifts for crossing curves. In the first two cases we may speak of "hierarchical" ability since individuals could be ranked unambiguously as to ability. In the case of "hierarchical" ability, in equilibrium increased ability always means a greater present value of earnings, but more schooling only in the case of a non-parallel shift.[1)]

With crossing ln a g(s,A)-curves , some individuals would have a comparative advantage at a higher educational level, and some at a lower one.[2)] However, in equilibrium more schooling does not necessarily imply greater earnings.

While there will no longer be any common earnings-schooling relation, such relations may be identified for different ability groups. If it were possible to distinguish groups of individuals who are identical with respect to A, T, r, and $\bar{H}_0$, we could identify the $\ln Y^s = \ln a\, g(s,A)$-curve for variations

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[1)] If more ability means $\frac{d \ln a g_1(s',A_1)}{ds} < \frac{d \ln a\, g_2(s',A_2)}{ds}$ for all s'>0, then more ability($A_2>A_1$) increases the investment incentives. The logaritmic function might, e.g., be of the form: $\kappa + A s^{1/b}$ where k and b are constants and b>1.

[2)] Cf. R.J. Willis, S. Rosen, "Education and Self-Selection." Journal of Political Economy, 1979, No.5, Part 2.

in $(\delta-\alpha)$. The same holds true for groups that are identical with respect to A, T, $(\delta-\alpha)$, and $\bar{H}_o$ given variations in r, and for groups identical with respect to A, $(\delta-\alpha)$, r, and $\bar{H}_0$ given variations in T.

As we have shown, certain relations between earnings and schooling were also implied for variations in only one of the explanatory variables at a time.

We have seen that:

- a longer planning period means more schooling and higher earnings;
- a lower net rate of depreciation $(\delta-\alpha)$ means more schooling and higher earnings;
- as regards hierarchical ability, more ability always means a greater present value of earnings and larger annual earnings, but not necessarily a longer period of schooling;
- greater initial endowments will also mean a greater present value of earnings and larger annual earnings, but will not affect schooling; (We only considered initial endowments acting as a shift factor not influencing the choice of schooling.)
- individual differences in the rental price of human capital did not change the optimal level of schooling - only the level of earnings;
- a lower rate of interest makes a longer schooling period optimal and earnings larger.

These results are summarized in Table I.

However, results parallel to those of the Simple Schooling Models may be obtained even in models in which individual characteristics vary. If only initial endowments and/or ability (assuming ability to act as the same type of shift parameter as initial endowments) should vary, the following relation would still hold for groups with equal initial endowments and ability: 1)

$$\ln Y^s = \ln Y_0 + r\ s.$$

However, correctly identifying these groups of equal endowments and ability is less acute as long as a linear specification - as discussed here and illustrated by Figure 16 - holds. As there is no correlation between s and

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1) Models with continuous time and a given working period.

Table I. Relations between dependent and independent variables in Schooling Models with Individual Differences

	S	Y	PVY
T	+	+	+
$\delta-\alpha$	-	-	-
A_{hp}	0	+	+
A_{np}	+	+	+
A_{nh}	+	?	?
$\bar{H}_0$	0	+	+
a	0	+	+
r	-	-	-

A_{hp} = Ability hierarchical parallel shifts
A_{np} = Ability hierarchical non-parallel shifts
A_{nh} = Ability non-hierarchical (where more A_{nh} = more S)
+ positive relation
- negative relation
0 no relation
? ambiguous effect

initial endowments according to this model, an empirical estimation of

$$\ln Y^s = a + b\ s + u$$

where u is the residual
by ordinary regression analysis would give a correct general picture of the relations. Of course, the share of earnings variations explained could be small if variations in Y_0 are not considered, but the estimated parameters a and b could be interpreted in terms of the model.

In this case the variance in earnings would be:

$$\mathrm{Var}(\ln Y) = r^2\ \mathrm{Var}(s) + \mathrm{Var}(\ln Y_0) + 2\ r\ \mathrm{cov}(\ln Y_0, s).$$

As $\ln Y_0$ and s are assumed to be uncorrelated, the last term would equal zero.1) To the variations caused by s are added the variations in $\ln Y_0$.

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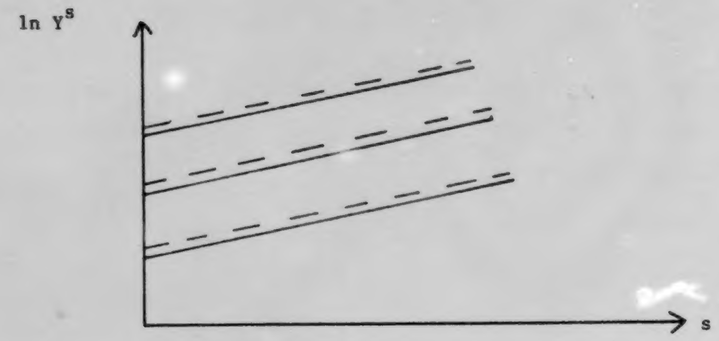
1) This expression could also be seen as an approximation for a non-linear case. In this case the $\mathrm{cov}(\ln Y_0, s)$ would necessarily be positive. Those with more ability choose a longer schooling period, which increases the variance.

Figure 16. The relation between the logarithmic value of earnings and schooling for groups with different initial endowments and abilities, where ability is only a shift factor



However, problems may arise if we apply that model (still assuming that the only individual differences are in initial endowments and ability) when the linear specification does not hold - as described above (see, e.g., Figure 12). If we estimate

$$\ln Y = a + b\ s + u$$

b then represents the average return, and the rest of the interpersonal differences end up in the residual. However, s and the A variable which was omitted may be correlated, and thus the residual and s would not be independently distributed. As is clear from our discussion above, the mere addition of an explanatory variable A may not improve the situation very much.

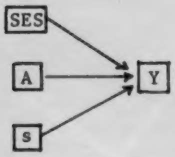
A priori, we cannot know whether a linear specification is correct or not. And, if it is incorrect we would like to know what effects the ability variable has. Therefore, working empirically with these models, the best strategy would seem to be to concentrate initially on identifying the $\ln Y^s = \ln a\ g(s,A)$ curves for different ability groups. This could be done for certain homogenous groupings as described above. One might either use equilibrium data or data from disequilibrium situations that have been clearly defined as to their character.

Another approach is to work with the respective relations between S, Y, and PVY and the explanatory variables as given in Table I separately or recursively. Once again, the ambiguity as to the effect of the A variable should be observed and empirically identified.[1)]

In addition, direct investment outlays should be kept track of, as should the assumptions about full-time schooling and work.[2)] Both continuous and discrete time models might be tested. In the following sections we return to these questions and discuss ways by which the more complete models could be used to improve empirical work with the Schooling Models. It may be of particular interest from the aspect of educational policy to be able to concentrate on Schooling Models.

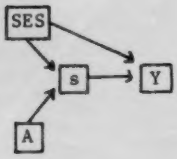
Additional light can be shed on the Schooling Models by comparing them with some other models proposed in the literature. Sometimes earnings (Y) have been assumed to depend directly on socioeconomic background (SES), individual ability (A), and schooling (s) or recursively as sketched below:

1.



$Y = f(SES, A, s)$

2.



$Y = g\,[SES, s(SES, A)]$

The first model depicted above is compatible with the Simple Schooling Models with individual variations in initial endowments and ability added as shift parameters (cf. Figure 16). It is contrary to the Schooling Models with Individual Differences where schooling could not be considered an exogenous

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[1)] For a survey of estimation problems in relation to similar Schooling Models cf.:

Z. Griliches,"Estimating the Returns to Schooling: Some Econometric Problems." Econometrica, January, 1977.

E.A. Hanushek,"Conceptual and Empirical Issues in the Estimation of Educational Production Functions." Journal of Human Resources, 1979, No.3.

[2)] A. Leibowitz, in "Years of Intensity of Schooling and Investment," American Economic Review, 1976, No.3, points out the econometrical risks of applying the Simple Schooling Models when the share of the individual capacity devoted to education on average does not equal one and varies individually.

variable. The second model is more in line with the latter models. SES may be taken as a proxy for $\bar{H}_0$ or r. With the Schooling Models with Individual Differences we would thus expect certain aspects of the socioeconomic backtround (r) to influence schooling but not all aspects (not $\bar{H}_0$). Ability (A) might be expected to influence s, but not necessarily so. However, our Schooling Models suggest two other variables that might influence s: $(\delta-\alpha)$ and T. While the Schooling Models with Individual Differences may be formulated so that A influences both s and Y or not, they definitely expect an influence from $\bar{H}_0$ and the rental price of human capital on Y. Moreover, an important feature with the latter models is the simultaneous determination of education and earnings.

3. Life Cycle Models

In this section a further step will be taken to elaborate the human capital models. The individual's investment choice will no longer be restricted to the length of the period of full-time schooling. His investment opportunities will persist during his whole lifetime. Moreover, he will not necessarily have to devote all of his capacity to training. Part-time education and on-the-job training are possible alternatives.

The questions that can now be analyzed include the following:

- how much full-time education do different individuals choose?
- how much part-time or on-the-job training?
- what does the pattern of part-time or on-the-job training look like?
- what factors determine the choice of full-time and part-time schooling, and what earnings result?

We will begin by presenting a basic model, a simplified version of the very influential model presented by Y. Ben-Porath (1967) and reworked by different followers.1) Our Basic Model has purposely been stripped of all but the fundamental features. Thus, it does not, e.g., contain "other resources" in the production function for human capital or depreciation. Thus, we can classify the effect of additional features - to be discussed later - as being either of the same type as the different explanatory factors in the Basic Model, or of a different type.

The Basic Model includes the specific assumption that the production of human capital is subjected to diminishing returns to scale, and that the input is the existing stock of human capital. And we use the same type of

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1) See, e.g.:
Y. Ben-Porath, op.cit., 1967.
Y. Ben-Porath, op.cit., 1970.
C. Brown, "A Model of Optimal Human Capital Accumulation and the Wages of Young High School Graduates." Journal of Political Economy, 1976, No.2.
W.J. Haley, "A Generalization of the Path of Human Capital Accumulation." Workshop Paper 7204, Michigan State University, 1972.
W.J. Haley, op.cit., 1973.
W.J. Haley, "Estimation of the Earnings Profile from Optimal Human Capital Accumulation." Econometrica, 1976, No.6.
L.A. Lillard, "Human Capital Life Cycle of Earnings Models: A Specific Solution and Estimation." Working Paper No.4, Center for Economic Analysis of Human Behavior and Social Institutions, N.B.E.R. New York, 1973.

production function that appears in Ben-Porath's analysis (1967).[1] Without some such specification it would no longer be possible to gain a firm grip on the results of this more complex setting. However, the assumptions concerning the production function are fundamental for the results. When we discuss alternative formulations of the Basic Model we will devote special attention to variations in these assumptions.[2]

3.1 Decisions on Full-time, Part-time and On-the-job Training: A Basic Model

The question the individual must consider at each date is how great an investment of human capital he should make in order to maximize lifetime earnings. As much as physically possible? None? A modest investment? Full-time investment in human capital production at an early age, combined with smaller and smaller investment at older ages, is one possible life cycle outcome in these models.

3.1.1 Assumptions of the Basic Model and its Solution

Let us enumerate the special assumptions we use in this model and comment on those elements that are new, compared with the Schooling Models:

- Individuals are assumed to maximize the present value of earnings.
- They make plans for a period extending from 0 to T. At the beginning of this period the individuals possess an initial stock of human capital $\bar{H}_0$ ($\bar{H}_0>0$).
- Human capital is homogeneous and neither depreciates nor appreciates.
- There are no direct costs for training, only indirect ones (=foregone earnings).
- The individual can devote his capacity, i.e., his existing stock of human capital (H) to work or human capital creation. The share of capacity devoted to human capital creation (v) can vary from zero to one ($0\leq v\leq 1$). In the Schooling Models the capacity of individual was assumed to be devoted wholly to schooling or to work.

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[1] Y. Ben-Porath, op.cit., 1967.

[2] The neglect of the education process has been denounced as one of the weak points of human capital theory. Cf. J. Karabel (ed.), Power and Ideology in Education. New York, 1977.

- The production function of human capital is assumed to appear as follows:

$$Q(t) = \beta_0 [v(t)\ H(t)]^{\beta_1} \qquad \begin{array}{l} 0 < \beta_0 \\ 0 < \beta_1 < 1 \end{array}$$

  where $Q(t)$ = quantity of human capital produced at date t

  β_0, β_1 = efficiency parameters.

  At each date the individual possesses a given stock of human capital [$H(t)$] which also represents his potential earning capacity. It is the result of previous "learning." In the Basic Model we assume that this earning capacity [$H(t)$] is also an input factor in the production of greater earning capacity. By $v(t)$ we are told what share of the existing stock of human capital is devoted to human capital production. The remaining share [$(1-v(t)$] is used on the labor market. Considering a longer period of time, $v(t)$ can alternatively be interpreted as the share of time the individual capacity is devoted to "learning."

- There is a constant rental price for human capital (a). The market rate of interest (r) is also constant.

- Individual variations are assumed to exist as regards $\bar{H}_0$, T, β_0 and β_1. We will also investigate the effect of changes in a and r on individual behavior. In disequilibrium situations individuals can experience differences in a and r which will cause differences in behavior.

Either no human capital or, alternatively, a constant amount of human capital is assumed to be reserved for leisure activities.1) Thus, as the only alternative use of human capital is work, and no other inputs are assumed to be necessary in this Basic Model, the costs of human capital investments are simply foregone earnings. The costs (C) can be expressed as:

$$C(t) = a\ v(t)\ H(t).$$

---

1) In Section 3.2.3 we shall discuss the effects of individual differences with respect to the given amount of human capital set aside for leisure activities. In Section 5, models with variable inputs of individual capacity in leisure activities are analyzed.

Actual and potential earnings $^{1)}$ at date t $Y(t)$ and $E(t)$ are respectively:

$$Y(t) = [1-v(t)]\, aH(t)$$
$$E(t) = a\, H\, (t).$$

$E(t)$ and $Y(t)$ may differ, however, throughout the individual's lifetime, as investment activities may always be pursued. Only when $v(t) = 0$ is $Y(t) = E(t)$. When $v(t) = 1$ then $Y(t) = 0$.

These assumptions permit the maximizing problem of an individual to be written as follows:

maximize the present value of earnings

$$J = \int_0^T e^{-rt}\, [aH(t) - a\, v(t)\, H(t)]\, dt = \int_0^T e^{-rt}\, [1 - v(t)]\, a\, H(t)\, dt$$

subject to the restrictions:

$$\begin{array}{ll} \dot{H}(t) = Q(t) = \beta_0 [v(t)\, H(t)]^{\beta_1} & 0 < \beta_0 \\ & 0 < \beta_1 < 1 \\ H(0) = \bar{H}_0 & 0 < \bar{H}_0 \\ v(t) - 1 \leq 0 & 0 \leq v. \end{array}$$

The remainder of this section will be devoted to a presentation of some results derived by solving this maximization problem. They will be further investigated and interpreted in Sections 3.1.2 and 3.1.3.

The necessary conditions for a solution $\{v\}$ are that a costate trajectory $\{p_1\}$ exists by which $\{v\}$ maximizes:

(3:1) $M = e^{-rt}\, (1-v)aH + p_1 \beta_0 (vH)^{\beta 1}$

subject to

$$0 \leq v \leq 1$$

and

(3:2) $\dot{p}_1 = -[a(1-v)e^{-rt} + p_1 \beta_0 \beta_1\, (vH)^{\beta_1 - 1}\, v]$

(3:3) $p_1(T) = 0$

(3:4) $\dot{H}(t) = \beta_0\, [v(t)\, H(t)]^{\beta_1}.$

---

1) Earnings if no investments in human capital were undertaken.

If $\{v\}$ is to maximize M subject to the given restrictions, the necessary conditions are:

(3:5) $-e^{-rt} aH + p_1\beta_0\beta_1 (vH)^{\beta_1-1} \cdot H - p_2 \leq 0$

(3:6) $[-e^{-rt}aH + p_1\beta_0\beta_1 (vH)^{\beta_1-1} \cdot H - p_2]\, v = 0 \quad v \geq 0$

(3:7) $-(v - 1) \geq 0$

(3:8) $p_2(v - 1) = 0 \quad p_2 \geq 0$

where p_2 is the shadow value of the capacity restriction.

If we substitute $p_1 = \Pi_1 e^{-rt}$, $p_2 = \Pi_2 e^{-rt}$ and $\dot{p}_1 = \dot{\Pi}_1 e^{-rt} - \Pi_1 r e^{-rt}$, the conditions (3:2)-(3:8) can be written:

(3:9) $\dot{\Pi}_1 = -(1-v)a - \Pi_1 [\beta_0\beta_1 (vH)^{\beta_1-1} \cdot v - r]$

(3:10) $\Pi_1(T) = 0$

(3:11) $\dot{H}(t) = \beta_0[v(t)\, H(t)]^{\beta_1}$

(3:12) $-aH + \Pi_1\beta_0\beta_1 (vH)^{\beta_1-1} \cdot H - \Pi_2 \leq 0$

(3:13) $[-aH + \Pi_1\beta_0\beta_1 (vH)^{\beta_1-1} \cdot H - \Pi_2]\, v = 0 \quad v \geq 0$

(3:14) $-(v - 1) \geq 0$

(3:15) $\Pi_2(v-1) = 0 \quad \Pi_2 \geq 0.$

Three solution phases are possible for the optimal trajectory $\{v\}$ that we can call:

phase 1 $v = 1$
phase 2 $0 < v < 1$
phase 3 $v = 0.$

$\Pi_1(t)$ is the shadow value at date t of marginally having more human capital. (3:10) shows that it is zero at T. As shown below, it will otherwise be positive. Π_2 is the shadow value of the capacity limit v=1. According to (3:15), only in phase 1 can it be positive but will not necessarily be so.

Before investigating the features of the respective phases, let us consider the ordering of phases and show that:

- phase 3 only occurs at $t = T$
- phase 3 is immediately preceded by phase 2.

If phase 1 occurs at all, it must therefore precede phase 2.

From (3:15) it follows that during phase 3 $\Pi_2 = 0$. (3:12) then gives

$$(3{:}16) \quad a \geq \Pi_1 \beta_0\beta_1(B)^{\beta_1-1}$$

where $B = vH$.

Since $\lim_{B\to 0} \beta_0\beta_1 B^{\beta_1-1} = \infty$

(3:16) can only be satisfied when $\Pi_1 = 0$.

Suppose $\Pi_1(t) = 0 \qquad t \neq T$

according to (3:8) we would then have

$$(3{:}17) \quad \dot{\Pi}_1 = -a.$$

Allow $I_t = [\delta - t, t + \delta]$, $\delta > 0$, be an interval, during which phase 3 occurs. (3:15) implies that a $\tau \in I_t$ exists, for which $\Pi_1(\tau) \neq 0$ which is a contradiction. Thus, $v = 0$ implies that $t = T$.

$$H(T) = e^{-rT}(1-v)aH$$

and it is maximized at $v = 0$. Therefore $t = T$ implies that $v = 0$. Thus, $v = 0$ if and only if $t = T$.

If v is continuous, phase 3 must be preceded by phase 2. (3:1) is a concave function of v. Since the v that maximizes (3:1) is unique, and all the other variables are continuous, v is also continuous. Thus, phase 3 is immediately preceded by phase 2.

Let us now consider phase 2. During that phase we have $0 < v < 1$.

If $0 < v < 1$, it follows from (3:15) and (3:13) that

(3:18) $$\Pi_1 = \frac{a}{\beta_0\beta_1(vH)^{\beta_1-1}} \; .$$

From (3:18) and 3:9) it follows that

(3:19) $\dot{\Pi}_1 - r\,\Pi = -a$. Thus,

(3:20) $\Pi_1 = \frac{a}{r}\,[1 - e^{r(t-T)}]$ and

(3:21) $\dot{\Pi}_1 < 0$.

From (3:18) and (3:20) it follows that during this period

(3:22) $$\frac{a}{\beta_0\beta_1 B^{\beta_1-1}} = \frac{a}{r}\,[1-e^{r(t-T)}] \; .$$

The RHS equals Π_1 and Π_1 definitionally equals

$$\Pi_1(t_i) \equiv \frac{d\int_{t_i}^{T} e^{-rt+rt_i}\,[(1-v)(t)]\;a\;H(t)\;dt}{dH(t_i)}$$

which can be identified with marginal return (MR) at date t [MR(t)]. This can be further clarified by noting that it equals the present value at t of a from t to T 1), i.e., the present value of a unit of human capital at date t with rental price a. MR(t) > 0 and monotonically decreasing.

The LHS can be interpreted as representing the marginal cost for the investment (MC). Since

$$\Delta J = e^{-rt}\,(-dv)\;a\;H\Delta t$$

and

$$\Delta H = \beta_0\beta_1\; v^{\beta_1-1}\; dv\;\Delta t\; H^{\beta_1}$$

---

1) $$e^{rt}a\int_t^T e^{-r\tau}d\tau = a\int_0^{T-t} e^{-r\tau}\,d\tau = \frac{a}{r}\,[1-e^{r(t-T)}]$$

We can identify undiscounted marginal costs at date t [MC(t)] with

$$-\frac{\Delta J}{\Delta H}e^{rt}.$$

As

$$-\frac{\Delta J}{\Delta H}=\frac{e^{-rT}\ a\ H^{1-\beta_1}}{\beta_0\beta_1\ v^{\beta_1-1}}$$

it follows that

$$-\frac{\Delta J}{\Delta H}e^{rt}=\frac{aH^{1-\beta_1}}{\beta_0\beta_1 v^{\beta_1-1}}=\frac{a}{\beta_0\beta_1 B^{\beta_1-1}}$$

which equals LHS of (3:22).1)

Thus, (3:22) indicates the marginal return which should equal the marginal cost during phase 2.

Inspection of LHS of (3:22) shows that the MC-curve is independent of the level of H and is invariant over time. During phase 2 we should thus have MR(t)=MC. This condition determines B(t) and v(t) as B(t)=v(t)H(t). Given H(t) and the production function $Q(t)=\beta_0[v(t)H(t)]^{\beta_1}$, Q(t) can be determined. MC is constant, but since MR declines with t, optimal human capital production and v(t) and B(t) also decline. In v this decline must occur faster than in B, as H increases up to T.

The marginal return is always positive. In competitive equilibrium individual variations in MR(t) will only be due to differences in T. A smaller T means a lower marginal return.

Individual variations in r influence MR(t). A higher r means a lower MR(t).2) Changes in the rental price do not effect the investment pattern as MR and MC - according to (3:22) - are affected to the same extent by changes in a.

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1) This can also be illustrated in another way. From the investment costs (C=avH) and the production function we have

$$\frac{\partial C}{\partial Q}=a\,\frac{1}{\beta_1}\left(\frac{Q}{\beta_0}\right)^{\frac{1}{\beta_1}-1}\cdot\frac{1}{\beta_0}$$

which reduces to the LHS in (3:22) by the substitution $Q=\beta_0(B)^{\beta_1}$.

2) As $e^{-r_1t}<e^{-r_2t}$ $r_1>r_2$

$$\int e^{-r_1t}<\int e^{-r_2t}$$

Let us also examine the MC-curve - LHS of (3:22) - a bit more closely. It begins at zero, for zero human capital production, and rises with the level of production. We have:

$$MC = \frac{a}{\beta_1} \left(\frac{1}{\beta_0} \right)^{\frac{1}{\beta_1}} Q^{\frac{1-\beta_1}{\beta_1}} \geq 0$$

$$\frac{\partial MC}{\partial Q} = \frac{a}{\beta_1} \left(\frac{1}{\beta_0} \right)^{\frac{1}{\beta_1}} \frac{1-\beta_1}{\beta_1} Q^{\frac{1-2\beta_1}{\beta_1}} \geq 0$$

$$\frac{\partial^2 MC}{\partial Q^2} = \frac{a}{\beta_1} \left(\frac{1}{\beta_0} \right)^{\frac{1}{\beta_1}} \frac{1-\beta_1}{\beta_1} \frac{1-2\beta_1}{\beta_1} Q^{\frac{1-3\beta_1}{\beta_1}} \gtreqless 0 \text{ for } \beta_1 \lesseqgtr \tfrac{1}{2}.$$

As Q approaches zero, MC also approaches zero. For any given quantity produced ($\bar{Q}$) MC will be lower the higher β_0 (ceteris paribus). The effect of changes in β_1 on the MC-curve is ambiguous.

B can be solved for. (3:22) gives

$$(3{:}23) \quad B(t) = \left(1-e^{r(t-T)}\right)^{\frac{1}{1-\beta_1}} \left(\frac{\beta_0\beta_1}{r} \right)^{\frac{1}{1-\beta_1}} > 0$$

and

$$\dot{B} = \left(\frac{\beta_0\beta_1}{r} \right)^{\frac{1}{1-\beta_1}} \frac{1}{1-\beta_1} \left(1 - e^{r(t-T)}\right)^{\frac{1}{1-\beta_1}-1} (-1) r\, e^{r(t-T)} < 0.$$

Since investment costs equal a·B, it can be interesting to study B a bit further:

$$(3{:}24) \quad \ddot{B} = \left(\frac{\beta_0\beta_1}{r} \right)^{\frac{1}{1-\beta_1}} \left(\frac{1}{1-\beta_1}\right) \left(\frac{1}{1-\beta_1}-1\right) \left(1-e^{r(t-T)}\right)^{\left(\frac{1}{1-\beta_1}-1\right)-1} \cdot$$

$$\cdot (-1) r e^{r(t-T)} + \left(\frac{\beta_0\beta_1}{r} \right)^{\frac{1}{1-\beta_1}} \left(\frac{1}{1-\beta_1}\right) \left(1-e^{r(t-T)}\right)^{\frac{1}{1-\beta_1}-1} (-1) r^2 e^{r(t-T)} =$$

$$= \dot{B} \left[\left(\frac{\beta_1}{1-\beta_1}\right) \left(1 - e^{r(t-T)}\right)^{-1} + r \right] < 0.$$

The relative growth of investment costs is equal to:

$$(3{:}25) \quad \frac{\dot{B}}{B} = \frac{\frac{1}{1-\beta_1} \left(1-e^{r(t-T)}\right)^{\frac{1}{1-\beta_1}-1} \cdot (-1) r e^{r(t-T)}}{\left(1-e^{r(t-T)}\right)^{\frac{1}{1-\beta_1}}} = \frac{-r}{1-\beta_1} \cdot \frac{e^{r(t-T)}}{1-e^{r(t-T)}} < 0$$

and

$$(3{:}26) \quad \frac{\partial \left(\frac{\dot{B}}{B}\right)}{\partial t} \Big/ \frac{\dot{B}}{B} = \frac{r}{1-e^{r(t-T)}} > 0.$$

Investment costs (avH = aB) are positive in phase 2, but decline; and the absolut decline becomes more pronounced. The negative growth rate (3:25) increases (3:26).

For the development of potential earnings $E(t)$ - i.e., earnings when no human capital investments are made - we have:

$$E(t) = a\,H(t) = a\,[H(t^x) + \int_{t^x}^{t} \dot{H}(\tau)\,d\tau] > 0$$

where t^x = date of entry into phase 2.

$$(3{:}27) \quad E(t) = a\,\left(H(t^x) + \int_{t^x}^{t} \beta_0 \left[(1-e^{r(\tau-T)})^{\frac{1}{1-\beta_1}} \left(\frac{\beta_0\beta_1}{r} \right)^{\frac{1}{1-\beta_1}}\right]^{\beta_1} d\tau\right) > 0$$

$$\dot{E} > 0$$
$$\ddot{E} < 0$$

For the development of actual earnings $Y(t)$ we have:

$$Y(t) = E(t) - a\,B(t) > 0$$

$$\dot{Y} = \dot{E} - a\,\dot{B} > 0 \qquad \dot{Y} > \dot{E}$$

$$(3{:}28) \quad \ddot{Y} = \ddot{E} - a\,\ddot{B} \substack{>\\=\\<} 0$$

as

$$\ddot{E} \substack{>\\=\\<} a\,\ddot{B}$$

or

$$a\,\beta_0\beta_1[B(t)]^{\beta_1-1}\,\dot{B} \substack{>\\=\\<} a\dot{B}\,[(\frac{\beta_1}{1-\beta_1})\,(1-e^{r(t-T)})^{-1} + r\,]$$

or

$$\beta_0[B(t)]^{\beta_1-1} \substack{<\\=\\>} (\frac{1}{1-\beta_1})\,(1-e^{r(t-T)})^{-1} + \frac{r}{\beta_1}$$

or

$$\frac{\beta_1}{1-\beta_1} \substack{>\\=\\<} re^{r(t-T)} .$$

This means that the smaller β_1 is, the greater the likelihood of a concave Y profile. A $\beta_1 < \frac{1}{2}$ implies a MC-curve with an increasing slope. Given $\beta_1 \geq \frac{1}{1+r}$, the Y trajectory will increase progressively. With $re^{r(t^x-T)} < \frac{\beta_1}{1-\beta_1} < r$, where t^x denotes the date at which phase 2 begins, the Y trajectory will first have a convex and later a concave slope.

During phase 1 we have $v = 1 \Rightarrow B = H \Rightarrow$

$$(3{:}29) \quad \Rightarrow H(t) = [(1-\beta_1)\,\beta_0 t + H_0^{(1-\beta_1)}]^{\frac{1}{1-\beta_1}}.$$[1]

From condition (3:15) we then see that $\Pi_2 \geq 0$.

From (3:13) we then have

$$(3{:}30) \quad a\,H = \Pi_1\,\beta_0\,\beta_1\,B^{\beta_1} - \Pi_2,$$

and from (3:9)

$$(3{:}31) \quad \dot{\Pi}_1 = -\Pi_1(\beta_0\,\beta_1\,B^{\beta_1-1} - r).$$

Rearranging (3:30) we have

$$(3{:}32) \quad \Pi_1 = \frac{a}{\beta_0\,\beta_1\,B^{\beta_1-1}} + \frac{\Pi_2}{B\,\beta_0\,\beta_1 B^{\beta_1-1}}.$$

The first part on the right-hand side of (3:32) equals the phase 2 MC. In phase 1 the shadow price of human capital (Π_1) should equal this marginal cost plus a second term that can be regarded as the cost of a capacity restriction.

The date at which the individual leaves phase 1 (t^x) is determined by the equations (3:29) and (3:23) as illustrated in Figure 17. During phase 1 the stock of human capital starts growing from $\bar{H}_0$ according to the curve H (3:29). When it reaches a level that is optimal for use of human capital

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[1] $\dot{H} = \beta_0 H^{\beta_1}$

$H(0) = H_0$

a. $H^{-\beta_1}\,\dot{H} = \beta_0$

Let $z = H^{1-\beta_1}$

$\frac{dz}{dt} = (1-\beta_1)\,H^{-\beta_1}\,\frac{dH}{dt} = (1-\beta_1)\beta_0$

$z(t) = (1-\beta_1)\,\beta_0\,t + C$

$H = z^{(\frac{1}{1-\beta_1})}$

$H(t) = [(1-\beta_1)\beta_0 t + C]^{\frac{1}{1-\beta_1}}$

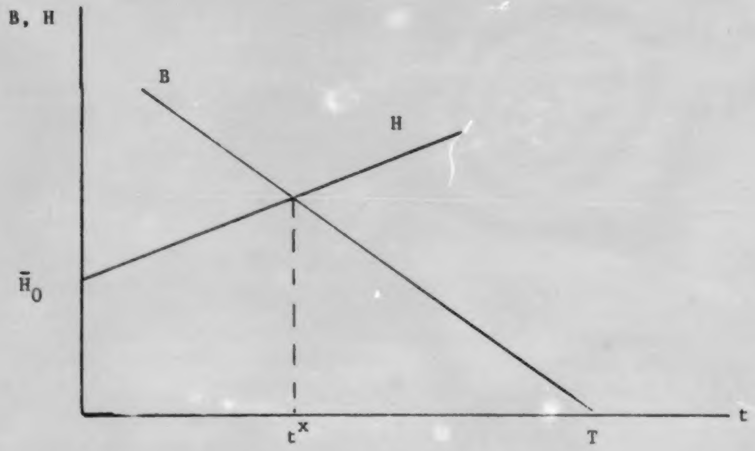
b. $t = 0$

$H(0) = [C]^{\frac{1}{1-\beta_1}}$

$\therefore C = H(0)^{1-\beta_1}$

$H(t) = [(1-\beta_1)\beta_0\,t + H_0^{1-\beta_1}]^{\frac{1}{1-\beta_1}}$

Figure 17 Determination of length of phase one



according to curve B (3:23), the individual shifts over to phase 2. If B always remains below H, the individual will never enter phase 1.

The effects of individual characteristics and prices on t^x are:

$$\frac{\partial t^x}{\partial H_0} < 0$$

$$\frac{\partial t^x}{\partial T} > 0$$

$$\frac{\partial t^x}{\partial r} < 0$$

$$\frac{\partial t^x}{\partial a} = 0$$

$$\frac{\partial t^x}{\partial \beta_0} > 0 \quad 1)$$

---

1) At t^x we have:

$$(1) \quad (1-e^{r(t^x-T)})\frac{\beta_0\beta_1}{r} = (1-\beta_1)\beta_0 \; t^x + H_0^{\;1-\beta_1}$$

and

$$(2) \quad \frac{\partial t^x}{\partial \beta_0} = \frac{(1-e^{r(t^x-T)})\frac{\beta_1}{r} - (1-\beta_1)t^x}{re^{r(t^x-T)}\frac{\beta_0\beta_1}{r} + (1-\beta_1)\beta_0} > 0$$

as at t^x

$$[1-e^{r(t^x-T)}]\frac{d\,\beta_0\beta_1}{r} - re^{r(t^x-T)}dt^x\,\frac{\beta_0\beta_1}{r} = (1-\beta_1)\,d\beta_0 t^x + (1-\beta_1)\beta_0 dt^x$$

i.e.

$$[(1-e^{r(t^x-T)})\frac{\beta_1}{r} - (1-\beta_1)t^x]\,d\beta_0 = [re^{r(t^x-T)}\frac{\beta_0\beta_1}{r} + (1-\beta_1)\beta_0]dt^x$$

Footnote continued on next page.

The net effect of β_1 on t^x is not clear as

$$(3{:}33)\quad \frac{\partial t^x}{\partial \beta_1} = \frac{\overbrace{[(1 - e^{r(t^x-T)}]\frac{\beta_0}{r} + \beta_0 t^x}^{+} + \overbrace{H_0^{(1-\beta_1)}(-\ln H_0)}^{-}}{\underbrace{re^{r(t^x-T)}\frac{\beta_0\beta_1}{r} + (1-\beta_1)\beta_0}_{+}}$$

The level of H_{t^x} will ceteris paribus:

- be higher the higher T ;
- be higher the higher H_0 ;
- not be influenced by a;
- be lower the higher r;
- be higher the higher β_0.

3.1.2 The Production of Human Capital and the Investment Costs Involved

According to the production function additional human capital is produced by channeling existing human capital into a production process.[2] The parameters β_0 and β_1 represent different aspects of ability. The elasticity of

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1) Continued from previous page.

and (1) implies

$$(1-e^{r(t^x-T)})\frac{\beta_1}{r} - (1-\beta_1)t^x = \frac{1}{\beta_0}H_0^{1-\beta_1} > 0$$

and

$$re^{r(t^x-T)}\ \frac{\beta_0\beta_1}{r} + (1-\beta_1)\beta_0 > 0.$$

2) During the period of specialization and education it is rather natural to think of the creation of human capital as a production process. For the period after specialization, a formulation in terms of individuals shifting from jobs with different wages and learning possibilities has also been suggested (cf. S. Rosen, op.cit., 1977 /b/). An interpretation in terms of a general trade-off on the labor market between current and future earnings has also been given (cf. A.S. Blinder, Y. Weiss, "Human Capital and Labor Supply: A Synthesis." Journal of Political Economy, 1976, No.3 and the footnote on page 168). As long as all these approaches result in models of the same general (or specific) kind:

$$Y = f(H,\dot{H}) \quad \frac{\partial Y}{\partial H} > 0,\ \frac{\partial Y}{\partial \dot{H}} < 0$$

or as with our Basic Model

$$Y = aH(1-v)$$
$$\dot{H} = g(v,H)$$
$$v = h(H,\dot{H})$$

which gives

$$Y = aH[1-h(H,\dot{H})]$$

they are theoretically undistinguishable.

scale, β_1, indicates how efficiently the individual can expand human capital production. The effects of all other factors that are relevant, but fixed and given from the point of view of the individual are summed up by β_0. Both β_0 and β_1 could be inherited traits, subject to or independant from influences of the family and society. The same is true of $\bar{H}_0$. The assumption that $\beta_1 < 1$ means that the production process is subject to diminishing returns to scale. A one percent increase in (v H) augments Q by less than one percent.

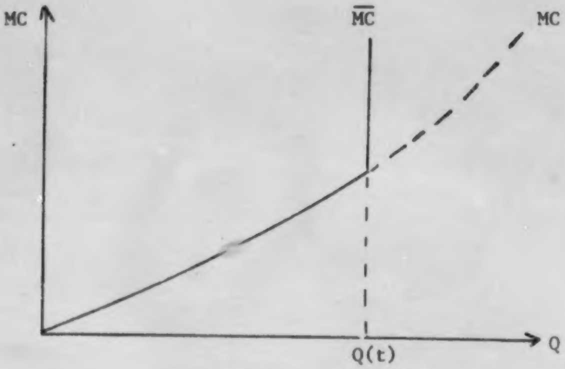
At each date the maximum amount of input equals H(t) with v(t) = 1. The existing human capital stock thus sets an upper limit to the amount of human capital that can be produced. At that level of human capital production the marginal cost (MC) of producing human capital becomes infinite. This is illustrated in Figure 18, where, given the stock of human capital, at most Q(t) units of human capital can be produced and $\overline{MC}$ is the relevant cost curve. However, as long as human capital production takes place and the stock of human capital thus increases, this limit is pushed to the right. During the next period the individual will be able to follow the unrestrained MC-curve (MC) a bit further up before the capacity constraint sets in.

As explained in the previous section in connection with (3:22), we obtain for the unrestrained marginal costs:

$$MC = a \frac{1}{\beta_1} \left(\frac{Q}{\beta_0}\right)^{\frac{1}{\beta_1}-1} \frac{1}{\beta_0} = \frac{a}{\beta_1 \beta_0 (vH)^{\beta_1 - 1}}$$

Thus, the unrestrained part of the MC-curve starts at zero and increases monotonically. The rate of increase depends on the value of β_1. Another interesting feature of the assumed production function is that the MC-curve remains the same at all dates. As human capital accumulates on the one hand, it will be more expensive to use a given share (v) of the total stock for purposes of human capital production; on the other hand, the efficiency of v in human capital production also increases, due to the greater stock of human capital. With the given specification of the production function, the individual increases his efficiency in human capital production and in earning money on the labor market to the same extent during each period. Therefore, his costs for producing human capital do not change. In the literature this has been called the "neutrality hypothesis."

Figure 18 The marginal cost curve



3.1.3 Three Phases in the Accumulation Process

In the solution to the individual maximizing problem there may be three types of phases with respect to the extent of human capital investment: one with maximum investments ($v=1$), one with no investments ($v=0$) and one with investment activities in between ($0<v<1$). We call these phase 1($v=1$), phase 2 ($0<v<1$) and phase 3 ($v=0$). The date at which phase 1 - a period of specialization in schooling - ends is called t^x.

For some individuals the optimal accumulation program may include a successive passage through phases 1, 2 and 3. Others whose initial endowment of human capital is adequate, or who are not sufficiently productive in human capital production, would start out in phase 2 and stop in phase 3. At date T, all individuals will be in phase 3. They will make no human capital investments at date T, simply because there are no incentives to do so, since in his planning the individual only considers what will happen up to that date, which can be interpreted as representing the end of his presumptive active life, his death, or date of retirement.

However, with the assumed production function, phase 3 occurs only at T. In all prior periods there would be an investment incentive. Marginal costs at a given date approach zero when Q approaches zero, but marginal returns are always positive. The marginal return (MR) to the creation of a unit of human capital at a certain date, is the value of earnings to be had from that unit during the remainder of the planning period discounted back to that date. At the least, date T will always remain for earning a return, and marginal return will be positive (cf. 3:22):

$$MR(t) = \int_0^{T-t} a\, e^{-rt}\, dt = \frac{a}{r}\left[1 - e^{r(t-T)}\right] > 0 \quad t < T$$

In phases 2 and 3 the human capital stock does not constitute a restriction on the individual choice. Therefore, at each date he should (as always) invest up to the point where marginal return equals marginal cost. As noted earlier, in this case the MC-curve does not change over time, but MR declines as the individual ages and the period during which he can reap the benefits of producing a marginal unit of human capital grows shorter. Figure 19 illustrates the situation at date date t and $t+1$. At date t, $Q(t)$ new units of human capital would be produced, this being the level of production where MC = MR is found. At date $t+1$ somewhat less than $Q(t)$ would be produced as $MR(t+1)$ would be lower than $MR(t)$ and so on up to date T when marginal return is zero and no more human capital is produced.

During phase 1 the individual makes as many human capital investments as possible. He successively will be able to make - and does make - an increasing amount of human capital investments. The human capital stock, potential earnings and investment costs grow at an increasing rate. The investment costs reflect increasing foregone "potential" earnings.

Differences among individuals with respect to, e.g., $\bar{H}_0$ and β_0 influence the development of the human capital stock. As $\bar{H}_0$ and β_0 have positive effects on human capital production, the higher they are the more rapid the human capital accumulation.

Two aspects of phase 1 are especially interesting - namely when does it end, and how much human capital do individuals possess when they leave it?

Phase 1 will be shorter (t^x comes earlier) the larger the initial human capital stock. If one starts out with substantial human capital he does not have to specialize for as long as one who begins with less. More ability (the parameters β_0 and β_1 in the production function) has a different effect. It increases investment incentives, reduces investment costs, and stimulates a longer period of specialization. 1) An increase in T has the same effect. The longer the period planned for, the more rewarding it is

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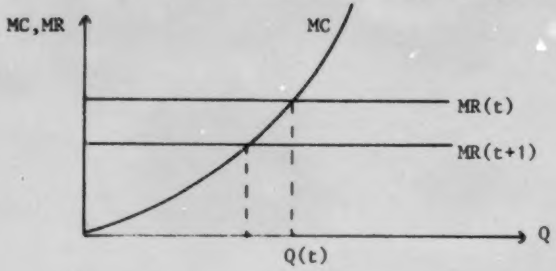
1) For β_1 this is not always true cf. (3:33).

Figure 19 Marginal costs and marginal returns determine the optimal level of human capital production



to have a greater abundance of human capital (cf. 3:23 - 3:33).

When people emerge from the specialization phase they will possess varying amounts of human capital. The longer the period of specialization, the more human capital has - *ceteris paribus* - been accumulated. Individuals who plan for a longer period (larger T) or have greater ability (β_0 and β_1) therefore start the period of non-specialization with a greater human capital stock than individuals with shorter planning periods or less ability.1) A greater initial human capital stock ($\bar{H}_0$) on the one hand shortens the period of specialization, but on the other during that period the human capital stock increases more rapidly. However, on balance an individual with a greater initial endowment will enter the non-specialization phase with a greater stock of human capital than one with less initial endowments (cf. 3:23 - 3:33).

During *phase 2* the production of human capital decreases successively. Since production is always positive, potential earnings (E) increase to T but at a declining rate (3:27). Investment costs (C) decline and the decline increases (3:23,3:24). Because actual earnings (Y) are potential earnings minus investment costs, they will also not only grow but grow faster than potential earnings. However, they need not necessarily increase at a declining rate as is true of potential earnings (cf. 3:28). Concave and convex profiles as well as convex plus concave profiles for actual earnings are both possible. The smaller the value of β_1, the greater the chance of having a concave earnings profile throughout. (A small β_1 $[<\frac{1}{2}]$ also means a convex MC-curve.) Figure 20 illustrates such a development.

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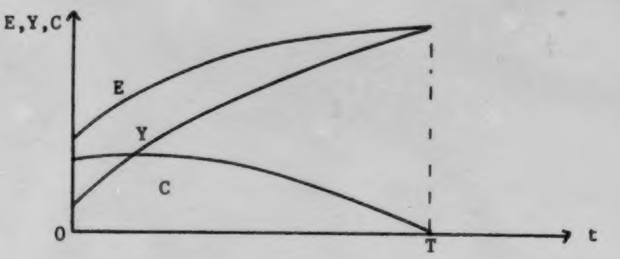
1) For β_1 this is not always true cf. (3:33).

Figure 20 Development of potential and actual earnings and investment costs during phase 2



For the sake of simplicity, the following comparisons of the actual earnings of individuals with different values for certain parameters will apply to individuals having concave earning profiles.

As diminishing amounts of human capital will be produced by an increasing stock of human capital, a smaller and smaller share must be set aside for this activity. This will be reflected in the amount of capacity devoted to labor market activities, which then increases.

Differences among individuals during this phase will produce effects via MR or MC. A reduction in MR or an increase in MC lowers investment incentives and human capital production (Q), resulting in a smaller increase in potential earnings.

Differences in MR resulting from variations in individual characteristics at a given date will only be attributable to differences in T. A shorter period for reaping the benefits of investments will lead to smaller investments of human capital. Similar differences in MC will result solely from differences in ability (β_0 and β_1). Higher ability (β_0 or β_1) lowers MC and leads to greater human capital investments (cf. 3:22). 1)

The effect that individual differences will have on earnings can now be summarized and illustrated as follows:

Individual differences in $\bar{H}_0$ do not affect the MC- or MR-curves that determine investment behavior during phase 2. Once two individuals who differ

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1) For β_1 this is not always true. Cf. the discussion in connection with (3:22).

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only with respect to $\bar{H}_0$ and confront the same a and r are on the labor market, they make the same investments and have equal growth in potential earnings. However, the period of non-specialization for an individual with a lower $\bar{H}_0$ starts later, and potential and actual earnings are always lower, as illustrated in Figure 21. (E_1 and Y_1 apply for the individual having greater initial endowments.) Annual absolute differences in earnings are constant. The present value of lifetime earnings will thus be larger for the individual with a higher $\bar{H}_0$.

For the person who possesses a larger human capital stock a given investment activity requires a smaller share fo that stock. Thus, a higher (1-v) at each date is implied during phase 2 for the individual with greater initial endowments. Empirically a higher (1-v) could be expected to be reflected by an increased amount of time being devoted to the labor market.

Individual differences in T have the following effects: A lower T lowers MR at each date. Thus, smaller investments will be made and actual earnings will grow more slowly. An individual with a lower T also enters the labor market earlier and with a smaller human capital stock as illustrated in Figure 22 (where E_2 and Y_2 apply for the individual with the larger T).

Figure 21 Development of potential and actual earnings for individuals having different initial stocks of human capital

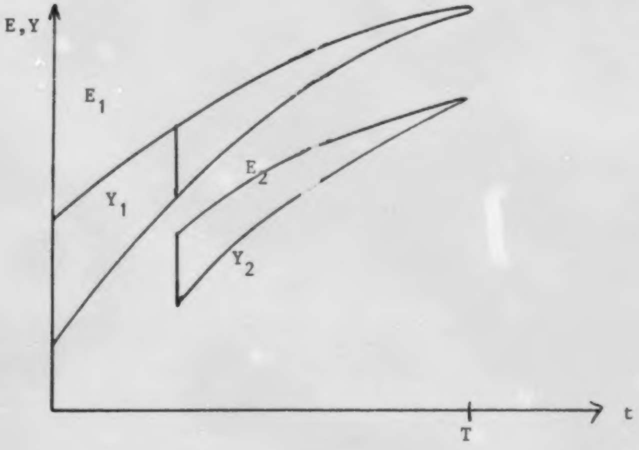
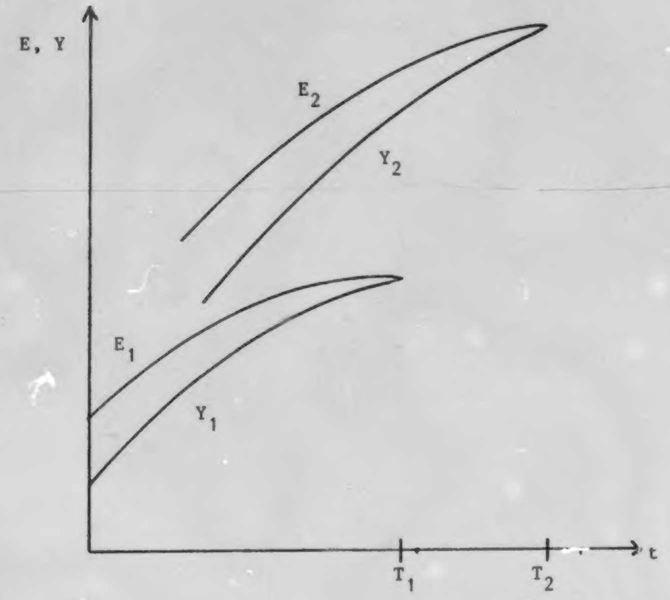


Figure 22 Development of potential and actual earnings for individuals with different planning periods



The Y_2 and Y_1 curves may cross but the increases in Y_2 will be larger and will continue for a longer period of time. The present value of earnings from Y_2 will also be larger. In the absence of further specifications, the implication for labor market time is ambiguous.

Individual differences in ability (β_0 and β_1) mean that the MC curve will be lower for individuals with more ability 1), that investments at each date will be larger, and that earnings will therefore grow more rapidly. The widening gap in earnings is a result of the greater efficiency in learning which characterizes the more able individuals. Moreover, more able individuals will start their labor market experience with more human capital and at a later date than others, as illustrated in Figure 23 (where E_2 and Y_2 represent the more able individual). The present value of earnings will be greater for the more able individual. Differences in labor market time (1-v) cannot be established without further specification of the differences in ability.

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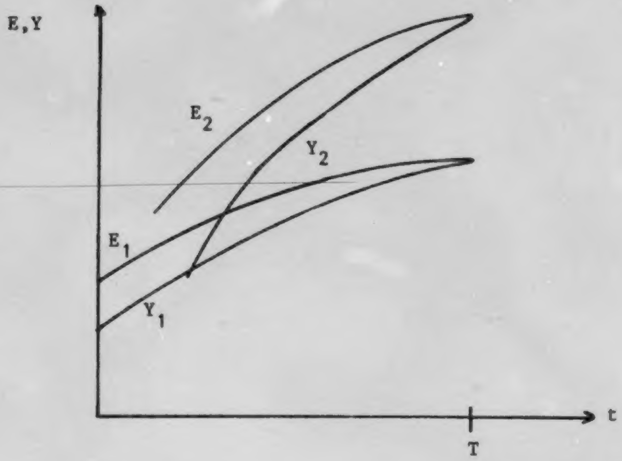
1) For β_1 this is not always true, cf. (3:22).

Figure 23 Development of potential and actual earnings of individuals with different levels of ability



Special combinations of individual differences may also be investigated. An individual who possesses more initial endowments and ability would, e.g., have higher earnings (or higher potential earnings) and his earnings would increase more rapidly than for one with less.

In a disequilibrium situation individuals may experience differences in the rental price of human capital (a) and the market rate of interest (r). In this model, differences in a would neither influence t^x nor the investment pattern during phase 2 (cf. 3:22, 3:23 and 3:29). A higher rental price does not change the individual's behavior, as both MR and MC are influenced equally by changes in a.

Variations in r , however, result in different individual behavior. A higher r means heavier discounting of the future, and not only lowers MR and investment incentives during phase 2, but also shortens the period of specialization (cf. 3:22, 3:23 and 3:29).[1]

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[1] A proportional tax that turns the individual maximization problem into

$$\text{Max}(1-\Theta)\int_0^T e^{-r(1-\Theta)t}\, a\, H(1-v)dt$$

where Θ is the tax rate
would have the same effect as a reduction of the rate of interest.

For testing purposes the expression for earnings could also be elaborated in order to obtain empirically applicable relations. During phase 2, e.g., we have: 1)

$$E_t = aH_t$$

$$Y_t = a(H_t - vH_t)$$

$$\dot{H} = Q_t = (vH_t)^{\beta_1} \quad \text{assuming } \beta_0 = 1$$

$$MC = \frac{a}{\beta_1} Q^{\frac{1-\beta_1}{\beta_1}} = MR_t = \frac{a}{r} [1 - e^{-r(T-t)}].$$

Thus

$$Q_t = [MR_t \frac{\beta_1}{a}]^{\frac{\beta_1}{1-\beta_1}},$$

$$vH_t = (MR_t \frac{\beta_1}{a})^{\frac{1}{1-\beta_1}},$$

and

$$Y_t = a\{H_t - (\frac{\beta_1}{r}(1-e^{-r(T-t)})^{\frac{1}{1-\beta_1}}\} \quad .$$

H_t can be computed recursively, given H_t^x. Using non-linear and iterative techniques, e.g., a, H_{tx}, r, and β_1 could be estimated, if T is given.

3.2 Life Cycle Models with Alternative Assumptions

In this section the effects of changing some assumptions in the Basic Model will be studied. The shape of the production function for human capital is of special interest in this connection. We concentrate on the differences that arise during phase 2 due to variations in the assumptions.

3.2.1 Alternative Production Costs for Human Capital

Changes in the assumptions concerning the production function of human capital influence both the number and length of the possible phases and the investment incentives during each phase.

We begin by considering a production function with constant - instead of decreasing - returns to scale. This model is of special interest, as it pro-

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1) This example is an adaptation of the approach used by J. Moreh, "Optimal human capital accumulation." Applied Economics, 1979, No.12.

duces results similar to those of the Schooling Models and thus indicates the sense in which they can be regarded as simplified Life Cycle Models. Technically, this change could be obtained by setting $\beta_1 = 1$ in the production function for human capital. Y. Ben-Porath (1967)[1] has briefly commented on this version.

The production function in the Basic Model can be written:

$$Q = \beta_0 \, v^{\beta_1} \, H^{\beta_1} \, .$$

In Section 3.2.1.2 we shall investigate what happens if the coefficients for v and H are not assumed to be the same but, e.g., γ_1 and $\gamma_2 (\gamma_1 \neq \gamma_2)$. We discuss a few different versions of this type, all of which result in changing costs over time. Our approach here is inspired by Y. Ben-Porath (1967), G.S. Becker (1975) (who in another setting uses a production function in which $\gamma_2=0$), S. Rosen (1976) (who in his analysis uses one in which $\gamma_1=\frac{1}{2}$ and $\gamma_2=0$), and P.E. Stephan (1976)(who investigates production functions in which $\gamma_1 \neq \gamma_2$).2)

Finally, we devote a special section to studying the effects of adding "other resources" to the production function for human capital. "Other resources" appear as inputs, in the works of, e.g., Y. Ben-Porath (1967) and T.D. Wallace and L.A. Ihnen (1975). 3)

3.2.1.1 Constant Costs

If we have a production function with constant returns to scale, marginal costs will be constant up to the capacity limit, as illustrated in Figure 24. And the production function can be expressed as:

$$Q = \beta_0 (vH)^{\beta_1} \qquad \beta_1 = 1$$

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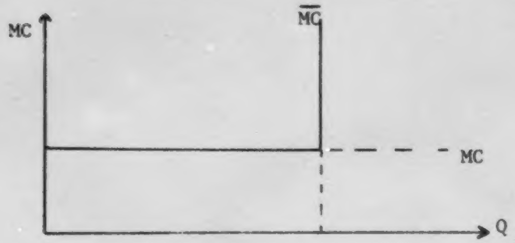
1) Y. Ben-Porath, op.cit., 1967.

2) Y. Ben-Porath, op.cit., 1967.
G.S. Becker, op.cit., 1975.
S. Rosen, "A Theory of Life Earnings." Journal of Political Economy, 1976, No. 4 II.
P.E. Stephan, "Human Capital Production: Life Cycle Production with Different Learning Technologies." Economic Inquiry, April, 1977.

3) Y. Ben-Porath, op.cit., 1967.
T.D. Wallace, L.A. Ihnen, "Full Time Schooling in Life Cycle Models of Human Capital Accumulation." Journal of Political Economy, 1975, No. 1.

Figure 24 Marginal costs for human capital production with a constant returns-to-scale production function



The maximization problem now turns into:

$$\text{Max } J = \int_0^T e^{-rt}[(1-v(t)]\ aH(t)\ dt$$

subject to

$$\dot{H}(t) = Q(t) = \beta_0 v(t)\ H(t)$$

$$0 \leq v \leq 1$$

or

(3:2:1) $\text{Max } M = e^{-rt}(1-v)\ aH + p_1\beta_0\ vH = e^{-rt}aH + [p_1\beta_0 - a\ e^{-rt}]\ vH$

subject to $0 \leq v \leq 1$

and

(3:2:2) $\dot{p}_1 = -e^{-rt}(1-v)a - p_1\beta_0\ v$

(3:2:3) $p_1(T) = 0$

(3:2:4) $\dot{H}(t) = \beta_0\ v(t)\ H(t)$.

If v is to maximize M subject to the given restrictions, the necessary conditions are:

(3:2:5) $-e^{-rt}aH + p_1\beta_0\ H - p_2 \leq 0$

(3:2:6) $(-e^{-rt}aH + p_1\beta_0\ H - p_2)\ v = 0 \quad v \geq 0$

(3:2:7) $-(v-1) \geq 0$

(3:2:8) $p_2(v-1) = 0 \quad p_2 \geq 0$

where p_2 is the shadow value of the capacity restriction.

Let $p_1 = \Pi_1 e^{-rt}$, $p_2 = \Pi_2 e^{-rt}$ and $\dot{p}_1 = \dot{\Pi}_1 e^{-rt} - \Pi_1 r e^{-rt}$. (3:2:1-8) can then be written:

(3:2:1') $\text{Max } M = aH + [\Pi_1 \beta_0 - a] vH$

(3:2:9) $\dot{\Pi}_1 = -(1-v)\, a - \Pi_1(\beta_0 v - r)$

(3:2:10) $\Pi_1(T) = 0$

(3:2:11) $-e^{-rt} aH + \Pi_1 e^{-rt} \beta_0 H - \Pi_2 e^{-rt} \leq 0$

(3:2:12) $(-e^{-rt} aH + \Pi_1 e^{-rt} \beta_0 H - \Pi_2 e^{-rt})\, v = 0 \quad v \geq 0$

(3:2:13) $-(v-1) > 0$

(3:2:14) $\Pi_2(v-1) = 0 \quad \Pi_2 > 0.$

From (3:2:1') is seen that

(3:2:15) $\Pi_1 \beta_0 > a$ implies $v = 1$

(3:2:16) $\Pi_1 \beta_0 < a$ implies $v = 0$

i.e., we get a "bang-bang" solution.

In phase 1 with $v = 1$ (3:2:9) implies that

(3:2:17) $\dot{\Pi}_1 = - \Pi_1 (\beta_0 - r)$

If we assume $\beta_0 > r$, then

(3:2:18) $\dot{\Pi}_1 < 0.$

In phase 3, with $v = 0$ it follows from (3:2:9) that

(3:2:19) $\dot{\Pi}_1 = -a + \Pi_1 r.$

With $\beta_0 > r$ and according to (3:2:16) then $\Pi_1 r < \Pi_1 \beta_0 < a$. Thus,

(3:2:20) $\dot{\Pi}_1 < 0.$

(3:2:15, 16, 18 and 20) imply that phase 1 comes first (if at all), followed by phase 3. Therefore, during phase 3 from (3:2:10 and 19) it follows that:

(3:2:21) $\Pi_1 = \frac{a}{r}(1-e^{r(t-T)})$

for $t_0 < T$ where t_0 is identified by

(3:2:22) $\Pi_1(t_0)\beta_0 = a$ if $t_0 > 0$, otherwise $v = 0$ for $0 \leq t \leq T$.

As in the Basic Model, (3:2:21) can be identified with marginal return (MR), and $\frac{a}{\beta_0}$ in (3:2:22) can be interpreted as marginal cost (MC).

With this model we have

$$\Delta J = e^{-rt}(-dv)\, a\, H\Delta t$$

and

$$\Delta H = \beta_0\, dv\, \Delta t\, H$$

and, accordingly,

$$-\frac{\Delta J}{\Delta H}e^{rt} = \frac{a}{\beta_0}.$$

Thus, during phase 3 we have MR = MC when it starts, and thereafter MR < MC. The opposite would be true for phase 1.

With these MC- and MR-curves there would thus only be two phases: one with specialization in human capital production when MR is greater than MC. If the MC-curve is too high the first phase would never be entered. All earnings profiles would be flat.1)

This situation resembles that of the Schooling Models. However, the individual is not indifferent as to the length of the period of specialization as in the Simple Schooling Models. Rather, as in the Schooling Models with Individual Differences, there is one optimal level of full-time

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1) In E. Sheshinski, "On the Individual's Lifetime Allocation between Education and Work." Metroeconomica, April/June, 1968, a similar situation is investigated, though an explicit human capital concept is not used:

$Y(t) = f[E(t)][1-v(t)]$
$\dot{E}(t) = v(t) - dE(t)$
where $E(t)$ = education
d = rate of depreciation

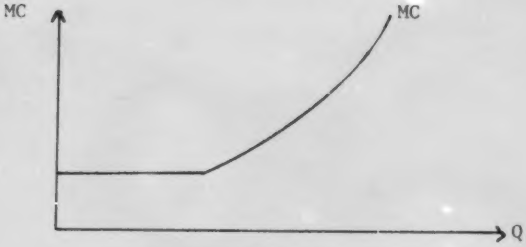
There are thus constant returns in the production of E though with this formulation E is not a production factor for E. Results similar to ours are obtained. However, with depreciation added the possibility exists for a phase 2 with $0<v<1$, a constant human capital stock and constant earnings. With $v=0$ the human capital stock and earnings diminish.

training. When marginal return falls below marginal cost, specialization will be stopped. In the Schooling Models, however, the individual could only choose between full-time training or no training. Here, by contrast, intermediate choices are permitted but not used, the reason being the constant costs. Such costs can thus be one motive for concentrating exclusively on Schooling Models instead of Life Cycle Models.

As in Schooling Models with Individual Differences as explanatory factors, and also in the Basic Life Cycle Model, individuals with a longer planning period (T) and a lower rate of interest would choose more specialization, while investment incentives would remain independent of changes in the rental price of human capital. Given the specific form of influence of ability (β_0) that is assumed here, higher ability always increases investment incentives and makes for higher annual earnings.

Another possibility would be an MC-curve combining the features of constant costs with those of the Basic Model. With the MC-curve illustrated in Figure 25 we could expect to find one period with specialization in human capital production, one with declining human capital investments, and a prolonged period with no investments at all.

Figure 25 A marginal cost curve that could produce extended periods of complete, declining, or no interest in human capital production

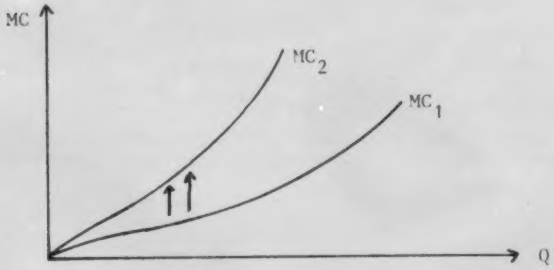


3.2.1.2 Changing Costs over Time

Up to this point, only MC-curves that remain constant over time (except for changes in the capacity restriction) have been considered. If, e.g., the costs for human capital investment should increase over time, this increase would be an incentive (in addition to lower MR) to reduce investments or vice versa.

One question that could be raised is why the costs should increase when prices remain unchanged? One reason may be exogenous changes in the production function of human capital. The β_0 coefficient may, e.g., fall over time. Another reason may be that the "neutrality hypothesis" does not hold - i.e., the individual becomes relatively more productive in the labor market than in human capital production during the accumulation process, and thus the costs in terms of foregone earnings increase as illustrated in Figure 26, where MC_2 holds for a larger stock of human capital than MC_1.

Figure 26 A marginal cost curve increasing with the stock of human capital



Marginal costs may change over time for production functions with given coefficients. The way this is related to the production function can easily be understood by considering the following production function:

$$Q = \beta_0 v^{\gamma_1} H^0 = \beta_0 v^{\gamma_1} .$$

Here, the level of the human capital stock does not influence the production of human capital. A greater earning capacity does not facilitate learning - i.e., it does not facilitate further increases in earning capacity. In human capital production what one has learned earlier (H) is of no help; the only thing that counts is the time (v) one applies to the task. In this case it is obvious that costs must rise with H. With a greater stock of human capital, any given share means more foregone earnings but not more human capital production.

This can also be shown as follows. With the production function $Q=\beta_0 v^{\gamma_1}$, we should, with the same approach as in Section 3.2.1.1, seek

(3:2:23) $\text{Max } M = e^{-rt}(1-v)a\,H + p_1 \beta_0\, v^{\gamma_1}$

subject to $0 \leq v \leq 1$

and

(3:2:24) $\dot{p}_1 = - e^{-rt}(1-v)\, a$

(3:2:25) $p_1(T) = 0$

(3:2:26) $\dot{H}(t) = \beta_0\, v^{\gamma_1}.$

During phase 2, and substituting $p_1 = \Pi_1\, e^{-rt}$ we would have

(3:2:27) $\Pi_1 = \frac{a}{\beta_0 \gamma_1}\, v^{1-\gamma_1} H.$

Π_1 can again be interpreted as MC. With this production function we have

$$\Delta H = \beta_0\, \gamma_1\, v^{\gamma_1 - 1}\, dv \;\; \Delta t\,.$$

Thus

$$MC = - \frac{\Delta J}{\Delta H}\, e^{rt} = \frac{a}{\beta_0 \gamma_1}\, v^{1-\gamma_1} H.^{1)}$$

Similar results are obtained when we have a production function:

$$Q = \beta_0\, v^{\gamma_1}\, H^{\gamma_2}$$

and $\gamma_1 > \gamma_2$ (not $\gamma_2 = \gamma_1$ as in the Basic Model). Marginal costs rise with H and thus over time as long as human capital is accumulating. When the effect of v and H on the production of human capital is not the same, it is not solely the number of human capital units put into the production process that counts. It also makes a difference whether a large share of a small stock of human capital is used, or a small share of a big stock. In "learning", compared with earning, it is not simply the number of units of earnings capacity that counts.

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1) This could also be regarded in another way. From the investment costs ($C = avH$) and the production function $Q = \beta_0\, v^{\gamma_1}$ we have

$v \quad \frac{Q}{\beta_0}^{\frac{1}{\gamma_1}}$ and accordingly

$$\frac{\partial C}{\partial Q} = a\left(\frac{1}{\beta_0}\right)^{\frac{1}{\gamma_1}} \frac{1}{\gamma_1}\, (Q)^{\frac{1}{\gamma_1} - 1} \;\; H(t),$$

which reduces to the expression (3:2:27) for Π_1 by the substitution $Q = \beta_0 v^{\gamma_1}$. And marginal costs increase with H.

When $\gamma_1 < \gamma_2$ the MC-curve would fall with H. Marginal costs decline as the individual becomes relatively more productive in learning. This can be shown as follows. With the production function

$$Q = \beta_0 \, v(t)^{\gamma_1} \, H(t)^{\gamma_2} ,$$

and using the same approach as above we should seek

$$\text{Max } M = e^{-rt}(1-v)aH + p_1 \beta_0 v^{\gamma_1} \, H^{\gamma_2}$$

subject to $0 \leq v \leq 1$

and

(3:2:29) $\dot{p}_1 = - e^{-rt}(1-v) \, a - p_1 \beta_0 v^{\gamma_1} \, \gamma_2 \, H^{\gamma_2 - 1}$

(3:2:30) $p_1(T) = 0$

(3:2:31) $\dot{H}(t) = \beta_0 v(t)^{\gamma_1} \, H(t)^{\gamma_2}$.

During phase 2, substituting $p_1 = \Pi_1 \, e^{-rt}$ we would have

(3:2:32) $\Pi_1 = \dfrac{aH}{\beta_0 \gamma_1 v^{\gamma_1 - 1} H^{\gamma_2}} = \dfrac{a}{\beta_0 \gamma_1} v^{1-\gamma_1} \, H^{1-\gamma_2}$.

Π_1 can again be interpreted as representing marginal costs (MC). With this production function we have

$$\Delta J = e^{-rt}(-dv) \; a \; H \;\; \Delta t$$

and

$$\Delta H = \beta_0 \gamma_1 \; v^{\gamma_1 - 1} \; dv \; \Delta t \; H^{\gamma_2} .$$

Thus,

$$MC = - \frac{\Delta J}{\Delta H} e^{rt} = \frac{aH^{1-\gamma_2}}{\beta_0 \gamma_1 v^{\gamma_1 - 1}} . \quad 1)$$

---

1) This could also be stated in the following way. From the investment costs (C=avH) and the production function $Q=\beta_0 v^{\gamma_1} H^{\gamma_2}$, we have

$$v = \frac{Q}{\beta_0}^{\frac{1}{\gamma_1}} \; (H)^{-\frac{\gamma_2}{\gamma_1}} \text{ and, accordingly,}$$

$$\frac{\partial C}{\partial Q} = MC = a\left(\frac{1}{\beta_0}\right)^{\frac{1}{\gamma_1}} \; (H_t)^{1-\frac{\gamma_2}{\gamma_1}} \; \frac{1}{\gamma_1} \; (Q)^{\frac{1}{\gamma_1}-1}$$

which reduces to RHS above by the substitution $Q = \beta_0 \; v^{\gamma_1} \; H^{\gamma_2}$.

MC is not independent of H(t) unless $\gamma_1 = \gamma_2$. When $\gamma_1 > \gamma_2$ the MC-curve rises with the level of H. However, with Q = 0, MC still equals zero. When $\gamma_1 < \gamma_2$ the MC-curve falls with the level H.

Introducing production functions with $\gamma_1 \neq \gamma_2$ into our Basic Model makes its solution more complicated, since time then enters the production function as an independent argument. Current actions influence decisions in the future. With a "rising MC-curve" ($\gamma_1 > \gamma_2$), the qualitative results of the Basic Model would, however, still hold. During phase 1 human capital production is at a maximum and human capital production increasing except for $\gamma_2 = 0$. During phase 2 human capital production decreases as the recoupment period shortens and marginal costs increase.1) And phase 3 still only occurs at T.

If the MC-curve falls over time ($\gamma_1 < \gamma_2$) this will make a difference, especially during phase 2. Then, at least initially, human capital production may be stimulated and increasing. 1) During such a period the increases in potential earnings will grow and be greater than those of actual earnings. Later in life human capital production would, as usual, be expected to fall. Then, as in the Basic Model, actual earnings grow faster in absolute terms than potential earnings.1)

Besides education technology considerations implying $\gamma_1 \neq \gamma_2$, *a priori* arguments against the "neutrality hypothesis" could also be related to arguments against the homogeneity of human capital. If a distinction should be made between investments to increase the *capacity to learn* and investments to increase the *capacity to produce*, it could be rational to begin with the first type of investments if one is planning an important learning program. Marginal costs for *learning* could then decline during this period. When the time comes to be more market oriented, and more investments of the second type are made, the MC-curve for *learning* could rise. The opposite could be true for the marginal costs for investments in productive capacity.

If schooling can be described as the first type of investment, then the productive capacity of individuals with different amounts of schooling need not differ, although their learning capacity might. The development of earnings for individuals with different levels of education could then be illustrated as in Figure 27. Both individuals leave school with equal potential

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1) Cf. P.E. Stephan, *op.cit.*

earnings ($E_1=E_2$). The individual with the longer schooling period, is better equipped, however, for on-the-job investments. They are more profitable to him, and therefore he makes more investments of that type and gets more rapidly increasing actual earnings (Y_2). However, with models of this type we are outside the traditional human capital approach where learning and productive capacity are inherently integrated in the same concept of human capital.

As we saw with regard to the "neutrality hypothesis", the MC-curve for human capital production is stock independent. This implied that individuals who at a given date differed only as to the level of human capital stock would make equal human capital investments during phase 2. They would have the same absolute growth of actual earnings. The production functions we have been discussing in this section - with $\gamma_1 \neq \gamma_2$ - do not imply such a stock independency. They might, however, have other stationary properties, e.g., as to the rate of growth of human capital and earnings. Relative annual earnings differences would then remain the same, but absolute annual earning differences would not.[1] Other individual differences - besides H - could then be analyzed in terms of variations in the rate of growth of earnings.

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[1] All production functions

$$Q = \beta_0 v^{\gamma_1} H^{\gamma_2} \qquad 0 < \gamma_1 < 1, \quad \gamma_2 = 1$$

are of the general form: $Q = f(v)H$.

With $\dot{H} = Q$ we have

$$\dot{H}/H = f(v).$$

The rate of growth of the stock of human capital ($\dot{H}/H$) would thus be stock independent during phase 2 as long as v is independent of H.

A production function of that type, i.e. (simplifying and using our notation):

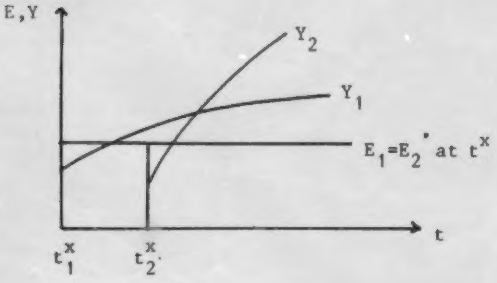
$$Q = \beta_0 H[1 - (1 - v)^{\gamma_1}]$$

has been used in Y. Weiss, L.A. Lillard, "Experience, Vintage, and Time Effects in the Growth of Earnings: American Scientists, 1960-1970." Journal of Political Economy, 1978, No.3. Cf. also A. Razin , "Optimum Investment in Human Capital." Review of Economic Studies, 1972, Vol.89, p 455.

Such a production function results in a relation $Y = H\gamma(\dot{H}/H)$ compared with our Basic Model $Y = f(H) - g(\dot{H})$.

In the literature the former has been called the multiplicative model and the latter the additive model. The former is stock independent in growth rates and naturally leads to earnings relationsships in logs, and the second is stock independent in absolute growth and naturally leads to earnings relationships in absolute levels of earnings.

Figure 27 Development of actual earnings for individuals with equal potential earnings after different periods of specialization



A few words should finally be said about exogenously shifting production functions. If the MC-curve of the Basic Model shifts up or down over time exogenously, the effects are straightforward. This would, e.g., strengthen, respectively weaken the decline in investment incentives during phase 2. The MR-curve is still applicable and should in each period be compared with the given MC-curve to determine the optimal human capital production. If marginal costs, initially fall, e.g., due to biological maturing, human capital production might increase. Investments can also become constant or declining.

3.2.1.3 Direct Outlays in the Production of Human Capital

Let us also add a few words about what happens when the investment costs include not only foregone earnings but also direct outlays for such things as books and teacher services. Formally, we can illustrate this by adding an item (D) for "other resources" and for the production function of the Basic Model write:

$$Q = \beta_0 (vH)^{\beta_1} D^{\beta_2}.$$

From the perspective of policy, the addition of these "other resources" is, of course, of great interest. Educational policy is often focused on the availability of such resources. Let us therefore see what difference they make.

Suppose we have the somewhat more general production function:

$$Q = \beta_0 v^{\gamma_1} H^{\gamma_2} D^{\beta_2} \qquad 0 < \gamma_1, \gamma_2, \beta_2 < 1$$
$$\gamma_1 + \gamma_2 + \beta_2 < 1$$

where D = inputs of "other resources".

Investment costs are then $C = avH + P_d D$

where P_d = price of "other resources".

The maximization problem can now be written

$$\text{Max } J = \int_0^T [a(1-v)\, H - P_d\, D]\, e^{-rt}\, dt$$

$$\begin{aligned} \text{subject to } \dot{H} &= \beta_0 v^{\gamma_1} H^{\gamma_2} D^{\beta_2} \\ H(0) &= \bar{H}_0 > 0 \\ 0 &\leq v \leq 1 \\ D &\geq 0 \end{aligned}$$

or

$$\text{(3:2:33)} \quad \text{Max } M = e^{-rt}[(1-v)\, aH - P_d\, D] + p_1\, \beta_0\, v^{\gamma_1} H^{\gamma_2} D^{\beta_2}$$

$$\begin{aligned} \text{subject to } 0 &\leq v \leq 1 \\ D &\geq 0 \end{aligned}$$

$$\text{(3:2:34)} \quad \dot{p}_1 = - e^{-rt}(1-v)\, a - p_1 \beta_0\, v^{\gamma_1}\, \gamma_2\, H^{\gamma_2 - 1}\, D^{\beta_2}$$

$$\text{(3:2:35)} \quad p_1(T) = 0$$

$$\text{(3:2:36)} \quad \dot{H} = \beta_0\, v(t)^{\gamma_1}\, H(t)^{\gamma_2}\, D(t)^{\beta_2} .$$

During phase 2 and substituting $p_1 = \Pi_1\, e^{-rt}$ we would have

$$\text{(3:2:37)} \quad \Pi_1 = \frac{a}{\beta_0 \gamma_1}\, v^{1-\gamma_1} H^{1-\gamma_2} D^{-\beta_2}$$

which as before could be interpreted as marginal cost.

Moreover, we should have:

$$\text{(3:2:38)} \quad \frac{a}{P_d} = \frac{\gamma_1 D}{\beta_2 vH} .$$

This could also be seen in the following way. If investment costs ($C = avH + P_d D$) are to be minimized subject to the production function, a necessary condition is:

$$\frac{avH}{P_d D} = \frac{\gamma_1}{\beta_2} \quad \text{and we have}$$

$$\frac{\partial C}{\partial Q} = \frac{a}{\beta_0 \gamma_1} \left(\frac{\gamma_1 P_d}{\beta_2 \; a}\right)^{\beta_2/(\gamma_1+\beta_2)} \left(\frac{Q_t}{\beta_0}\right)^{[1/(\gamma_1+\beta_2)]-1} (H_t)^{\frac{\gamma_1-\gamma_2}{\gamma_1+\beta_2}} \tag{3:2:39}$$

This expression reduces to the one for MC above upon substituting $Q = \beta_0 \, v^{\gamma_1} \, H^{\gamma_2} \, D^{\beta_2}$ and $\frac{\gamma_1 P_d}{\beta_2 \; a} = \frac{vH}{D}$.

If $\gamma_1 = \gamma_2$ the MC-curve is independent of H. With $\gamma_1 > \gamma_2$ it rises with H, and with $\gamma_1 < \gamma_2$ it falls with H. With a constant MC-curve the solution to the maximization problem is similar to that in the Basic Model. With $\gamma_1 \neq \gamma_2$ the same qualifications as discussed in Section 3.2.1.2 apply.[1]

With $\beta_1 + \beta_2 < 1$ or $\gamma_1 + \gamma_2 + \beta_2 < 1$ we have decreasing returns to scale, and with $\beta_1 + \beta_2 = 1$ or $\gamma_1 + \gamma_2 + \beta_2 = 1$ constant returns to scale. Differences in this respect have the same influence on the number and types of phases as before.

During phase 2, with $\gamma_1 = \gamma_2$, and decreasing returns to scale, marginal return would be the same as in the Basic Model and the main features of the marginal cost curve for human capital production would also be preserved. The implications for investment and earnings would thus be the same as in the Basic Model. The same holds true when we make a distinction between the effects of v and H ($\gamma_1 \neq \gamma_2$) except for the possibility of initially increasing investments ($\gamma_1 < \gamma_2$).[1]

However, during the period of specialization there would be a difference. At $v = 1$ there would not be any physical stop to human capital production, as more of the other resources could be brought into the production process. But this extra production would be more expensive, of course, than if more human capital were available. This is illustrated in Figure 28.

The marginal cost curve labelled MC is the unrestrained cost curve. With human capital stock H_t it can only be followed up to Q_t, the capacity limit (v=1). Thereafter production cannot be increased by means of the cheapest combination of own resources and purchased resources, but only by increasing the latter along $\overline{MC}$.

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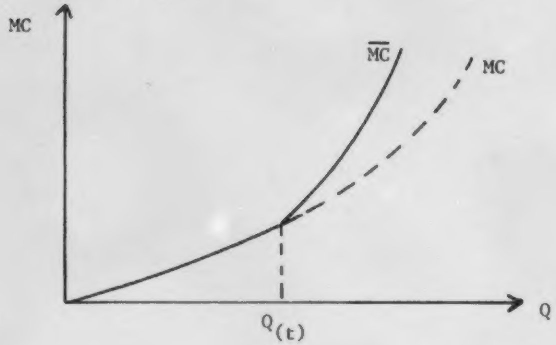
[1] Cf. P.E. Stephan, op.cit.

Figure 28 The marginal cost curve in a model with direct investment outlays



The main effects of introducing "other resources" in the model can thus be summarized as follows:

- the individual's choice will now also be influenced by his efficiency in using these other inputs (β_2) and by the relative price of these inputs (P_d);
- individual variations in β_2 (and P_d) will create further individual differences in investments and earnings;
- with constant returns to scale, only two phases are possible; with decreasing returns to scale, three;
- with $\gamma_1 = \gamma_2$, investments during phase 2 will be determined by the same type of condition MR = MC as before, leading to positive but decreasing investments; with $\gamma_1 \neq \gamma_2$, investments might initially increase and then fall;
- if "other resources" are added to the Basic Model, individuals with lower P_d will have greater investment incentives (cf. 3:2:39), make more investments, have more rapidly increasing earnings and a higher present value of lifetime earnings;
- when "other inputs" are added to the production function of human capital of the Basic Model, the level of the rental price for human capital (a) is also of importance for the behavior of the individual. A higher a means greater investments during phase 2. In the Basic Model it did not, as it increased marginal costs and returns to the same extent. In this case, an increase in a would increase marginal return more than marginal cost, as the latter comprises both foregone earnings and direct investment outlays (cf. 3:2:39).

These are the results when "other resources" are sold to the individual, e.g., by educational institutions, at a market price P_d. But let us also consider a regulated assignment of "other resources" (D) to different individuals. If these resources are then regarded as being placed at the disposal of the individual free of charge, but in a quantity that he cannot influence, he would then have to take this constraint into consideration when deciding how to use his capacity. However, during the phase of specialization there is no choice as to the use of capacity. The effect of then giving, e.g., the same amount of D to individuals initially "identical" except for β_2 would be to permit the individual who can better use these resources to produce a greater amount of human capital. As seen from the production function, a fixed D and β_2 has the same effect as ability (β_0), and thus in the Basic Model would also prolong the period of specialization for individuals with higher β_2.

An equal absolute increase in D to individuals "identical" except for β_2 would afford greater benefits to the more "able" individuals (with higher β_2).

3.2.2 Depreciation and Appreciation

Depreciation has been treated by, e.g., S. Rosen (1973, 1975, 1976), W.J. Haley (1973), and T.D. Wallace and L.A. Ihnen (1975).1) Appreciation may be expected to have a symmetrical effect with opposite sign, though apparently this aspect has not been considered in the literature.

Let us consider the effects of introducing a constant net rate of depreciation (d) for investment behavior and earnings during phase 2. The net rate of depreciation (d) would consist of two elements: the rate of depreciation (δ) that lowers the human capital stock, if, e.g., the individual forgets skills he has acquired; and the rate of appreciation (α) that increases the stock of human capital. Such increases could be due to biological maturing at no cost to the individual. Let us define d as $d = \delta - \alpha$.

---

1) S. Rosen, Income Generating Functions and Capital Accumulation. University of Rochester, 1973.
S. Rosen, "Measuring the Obsolence of Knowledge." In T.F. Juster (ed.), Education, Income and Human Behavior. New York, 1975.
S. Rosen, op.cit., 1976.
W.J. Haley, op.cit., 1973.
T.D. Wallace, L.A. Ihnen, op.cit.

The maximization problem can now be formulated:

$$\text{Max } J = \int_0^T a(1-v)\, H\, e^{-rt} dt$$

subject to:

$$\dot{H} = H(-d) + Q$$

$$Q = \beta_0 (vH)^{\beta_1}$$

$$H(0) = \overline{H}_0 > 0$$

$$0 \leq v \leq 1$$

or

$$\text{(3:2:40)} \quad \text{Max } M = e^{-rt} (1-v)\, a\, H + p_1 [H_t(-d) + \beta_0 (vH)^{\beta_1}]$$

subject to:

$$0 \leq v \leq 1$$

$$\text{(3:2:41)} \quad \dot{p}_1 = -[e^{-rt}(1-v)a + p_1\beta_0\beta_1 (vH)^{\beta_1 -1} v - p_1 d]$$

$$\text{(3:2:42)} \quad p_1(T) = 0$$

$$\text{(3:2:43)} \quad \dot{H} = H(-d) + \beta_0 (vH)^{\beta_1} .$$

During phase 2 and substituting $p_1 = \Pi_1 e^{-rt}$ we would have

$$\text{(3:2:44)} \quad \Pi_1 = \frac{a}{\beta_0\beta_1 (vH)^{\beta_1 - 1}}$$

and

$$\text{(3:2:45)} \quad \Pi_1 = \frac{a}{r+d} (1-e^{(r+d)(t-T)}).$$

If human capital decreases (depreciation outweighing appreciation) the returns that can be had from a unit of human capital will be affected, but not the costs of producing human capital (cf. 3:2:44 and 45). Thus it is sufficient to identify the effects on returns to establish what happens to investment incentives. Such a depreciation will lower the returns from each unit of human capital produced at a given date, including the marginal one. At date t the marginal return can be written:

$$MR(t) = a \int_0^{T-t} e^{-(r+d)}\, d\tau = \frac{a}{r+d} [1-e^{(r+d)(t-T)}].$$

The introduction of depreciation has the same effect as varying r. A higher rate of depreciation would reduce MR.[1] Heavier discounting makes the present value of each a earned at future dates smaller. The total effect will be more important the longer the planning period. The need for a period of specialization may thus become smaller. During phase 2 investment incentives are smaller at each date, but otherwise the pattern will be the same as before. With a stable MC-curve human capital production falls continually over time up to T. At T it is zero.

---

[1] As $e^{-(r+d_1)} > e^{-(r+d_2)}$ $d_2 > d_1$
$\int e^{-(r+d_1)} > \int e^{-(r+d_2)}$

If appreciation outweighs depreciation, investment incentives are greater compared with no net depreciation. This may lead to a need for a period of specialization. Furthermore, human capital production during phase 2 will be greater at each date. Over time, however, the pattern will be the same as above.[1]

With the introduction of depreciation, a difference in gross and net investment in human capital will arise. The gross investment equals Q, the newly produced human capital. However, the increase in the stock of human capital ($\dot{H}$) equals:

$$\dot{H} = Q - dH.$$

For the development of earnings we have (including "other resources"):

$$E = a\,H \qquad \dot{E} = a\,\dot{H} = a[Q - dH]$$

$$\hat{Y} = a\,H - C \qquad \dot{\hat{Y}} = \dot{E} - \dot{C}$$

$$Y = a\,H - b\,C \qquad \dot{Y} = \dot{E} - b\,\dot{C}$$

where E = potential earnings
C = total investment costs
$\hat{Y}$ = net earnings
bC = investment costs due to foregone earnings
Y = actual earnings.

Investment costs due to foregone earnings will always be a constant share (b) of total investments costs, and b will be less than one if there are direct investment outlays.1) Therefore, Y will always be greater than (or equal to) $\hat{Y}$. Furthermore, as during phase 2, $\dot{C} < 0$, we have:

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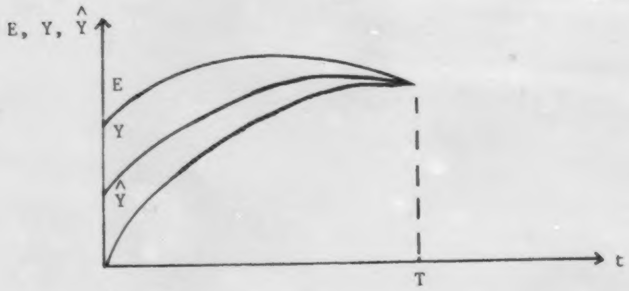
[1] Assuming $r > \alpha$.

$$\left|\frac{dE}{dt}\right| < \left|\frac{dY}{dt}\right| \leqq \left|\frac{d\hat{Y}}{dt}\right| .$$

With depreciation dominating, potential earnings will increase as long as depreciation does not outweigh gross investment. After that point they will decline. With a certain lag, actual earnings (Y) will also be affected (and net earnings $\hat{Y}$ if they differ from actual earnings - i.e., with positive direct investment outlays), as illustrated in Figure 29. Y (and $\hat{Y}$) will dip later than E because Y grows for two reasons. One is that E grows; the other is that investment costs decline. Even if the first effect should turn negative, the second would be positive enough to outweigh the first negative effect for a while.

Figure 29 Potential, actual and net earnings with positive net depreciation



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1) From what was said in Section 3.2.1.3 it follows that with a production function $Q = \beta_0(vH)^{\beta_1} D^{\beta_2}$ we should have:

$$\frac{Q}{v} / \frac{\partial Q}{\partial D} = \frac{\beta_1 v^{\beta_1 - 1} H^{\beta_1} D^{\beta_2}}{v^{\beta_1} H^{\beta_1} \beta_2 D^{\beta_2 - 1}} = \frac{\beta_1 D}{\beta_2 v} = \frac{aH}{P_d}$$

and $\frac{avH}{P_d D} = \frac{\beta_1}{\beta_2}$.

Moreover, $\frac{avH}{avH + P_d D} = \frac{\beta_1}{\beta_1 + \beta_2} < 1$.

Thus, b either equals 1 (no direct investment outlays) or: $b = \frac{\beta_1}{\beta_1 + \beta_2} < 1$.

With appreciation dominating, the human capital stock would grow, both due to production of human capital (Q) and appreciation of the previous stock of human capital. As a result, earnings profiles may grow at an increasing rate.

The most important effect of introducing depreciation into the model is that earnings profiles can have a negative slope - i.e., actual earnings could decrease towards the end of the active working life. As depreciation lowers investments during phase 2, it also makes for flatter earnings profiles than would otherwise be the case. The need for a period of specialization is also reduced.

When appreciation is dominating, the opposite holds true for investment incentives and the need for a period of specialization. Moreover, the likelihood that earnings profiles will grow at an increasing rate is strengthened.

Depreciation also introduces a new element where individual differences could result in a different type of investment behavior. Furthermore, the numerical value of d for different individuals could possibly be influenced by educational or other types of policy.

3.2.3 Variations in the Given Amount of Leisure and Labor Force Participation

Thus far, we have assumed that each individual at each date reserves a constant absolute amount of human capital for activities other than work and human capital production, and we have not regarded this amount as variable among individuals. These other activities are leisure activities of various kinds, work at home, etc. The numerical value of the equilibrium rental price for human capital - $\underline{a}$ - would, inter alia, depend on the level of this amount of capacity that individuals set aside.

Individual differences in the exogenously given amount of the capacity devoted to other activities ("leisure") than work and human capital production will have different effects depending on how they are assumed to enter the decision-making problem. Two versions will be discussed here: absolute and proportional variations. In the literature, the capacity set aside for "other activities" is discussed by S.W. Polacheck (1975) 1) in the form:

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1) S.W. Polacheck, "Differences in Expected Post-school Investment as a Determinant of Market Wage Differentials." International Economic Review, 1975, No. 2.

$$\text{Max } J = \int_0^T (n - v)\; aHe^{-rt}dt$$

which is less easily interpretable. It is a variant of the proportional reservation of resources, which will be analyzed below. But let us first investigate what the difference would be if individuals set aside different absolute amounts of resources.

Assume that the individual always sets aside a given number of human capital units ($\bar{H}$, $\bar{H} < \bar{H}_0$) for these other activities. Disposable human capital (H_d) will then initially be $H_d(0) = \bar{H}_0 - \bar{H}$ and generally

$$H_d(t) = (\bar{H}_0 - \bar{H}) + \int_0^t \dot{H}\, dt.$$

With the same production function as in the Basic Model, the individual will now be assumed to maximize:

$$\text{Max } J = \int_0^T [1-v(t)]aH_d(t)\; e^{-rt}dt$$

subject to $0 \leq v \leq 1$

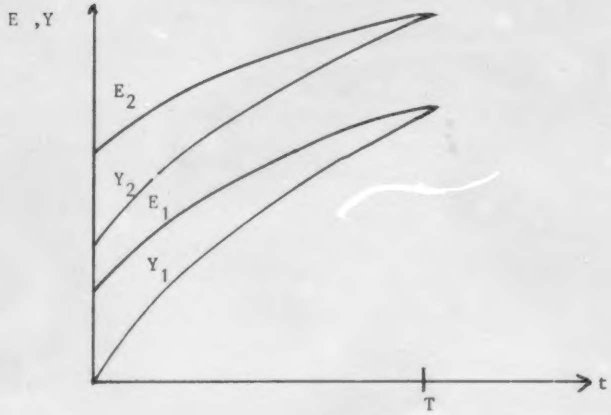
$$\dot{H} = Q = \beta_0(vH)^{\beta_1}.$$

During phase 2 MR(t) will as before be $MR(t) = \frac{a}{r}[1-e^{r(t-T)}]$ and

$$MC = \frac{a}{\beta_0\beta_1(vH)^{\beta_1-1}}\ .$$

Consequently, different values for $\bar{H}$ do not influence the investment pattern during phase 2, since the differences in the absolute amount of human capital set aside have exactly the same effects as differences in initial stocks of human capital ($\bar{H}_0$). For individuals reserving different amounts of human capital but who are otherwise "identical", earnings would still increase to the same extent. But, of course, the individual who sets aside less human capital would have larger earnings. Figure 30 illustrates such a situation for otherwise "identical" individuals who are assumed to enter phase 2 directly. E_2 and Y_2 represent the earnings of the individual who sets aside less human capital. In practical terms, Figure 30 can be thought of as representing two individuals with the same skills, but where one uses more of them at home.

Figure 30 Earnings profiles for individuals differing only with respect to the amount of human capital set aside for activities outside the labor market



The situation would be different, however, if at each date individuals set aside a given share of their stock of human capital and this share could vary among individuals. Another way of expressing this situation is to say that the maximum "labor force participation" then differs among individuals. Some could, e.g., be prepared to work full-time, others only part-time.

This problem could be formulated in various ways. Assume first that the individual always sets aside a given share $(1-n,\ 0<n\leq 1)$of his human capital stock for other activities. Maximum "labor force participation" will then be n. An alternative would then be to assume the individual to maximize:

$$\text{Max } J = \int_0^T [1-v(t)]\ a\ nH(t)\ e^{-rt}dt = n\int_0^T [1-v(t)]\ a\ H(t)\ e^{-rt}\ dt.$$

subject to $0 \leq v \leq 1$

$$\dot{H} = Q = \beta_0\ (vH)^{\beta_1}$$

If $n = 1$ we have the same situation as in the Basic Model, but with $n \neq 1$ this is not so. However, the solution to this problem will be identical to the one in the Basic Model. Since n does not enter the restrictions and is constant over time, it is evident that

$$\text{Max } n\int_0^T [1-v(t)]\ a\ H(t)e^{-rt}\ dt \quad \text{gives the same solution as}$$

$$\text{Max } \int_0^T [1-v(t)]\ a\ H(t)e^{-rt}\ dt.$$

During phase 2, e.g., investments would be independent of the level of n.

However, this formulation of the problem is very special. It means that when the labor market participation $(1-v)$ is changed, the individual loses the corresponding return on nH units of human capital ; however, when he then increases v in human capital production he puts in the corresponding share of H units. To make the situation more symmetrical, let us reformulate the problem:

$$\text{Max } J = \int_0^T [1-v(t)]\ a\ nH(t)\ e^{-rt}\ dt$$

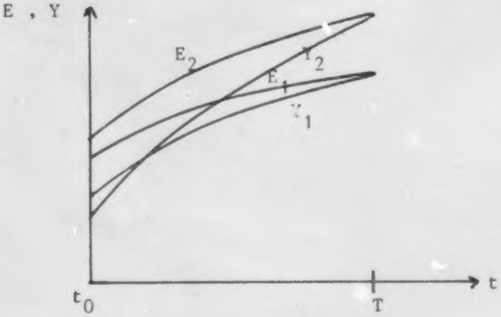
subject to

$$0 \leq v \leq 1$$
$$\dot{H} = Q = \beta_0 (vnH)^{\beta_1}.$$

We will still have largely the same solution as in the Basic Model - but with the difference that it holds for (nH) instead of H and, moreover, that β_0/n [1) is substituted for the "old" β_0.

Figure 31 depicts the situation of two individuals who are "identical" except for the amount of capacity or "time" they expend for work and human capital production. They are both assumed to enter phase 2 directly. The one who has more "time" has higher potential earnings from the outset $(E_2(0)>E_1(0)$. He

Figure 31 Earnings profiles for individuals differing only with respect to disposable time



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1) With the problem

$$\text{Max } J = \int_0^T [1-v(t)]\ a\ n\ H(t)\ e^{-rt}\ dt$$

subject to $\dot{(nH)} = \beta_0\ (vnH)^{\beta_1}$

the solution would be exactly the same for (nH) as for H in the Basic Model.

also invests more. This is due to the fact that an increase in time disposable for labor market participation raises marginal return (MR) more than marginal costs (MC). MC refers only to one date, while MR is the sum of a return at all remaining dates - each of which is increased if more time is disposable.[1] Potential earnings therefore grow faster for the individual disposing of more time. This holds true also for actual earnings as investment decline continually.

The situation is another for an individual who envisages expending more and more "time" on work and human capital production. This might be an individual who plans to increase his labor market participation successively, from,e.g., part-time to full-time. In this case, the effect of declining marginal returns due to the shorter period remaining for realization of the benefits of the investment, will be counteracted. If the effect of increasinglabor market participation is large enough, the individual might temporarily increase his human capital investments, and potential earnings would rise exponentially. For an individual disposing of less and less "time" the decline in investment incentives would be strengthened. We shall return to this question in Section 5 where the amount of "time" for work and human capital production will be determined within the model.

---

[1] During phase 2 we have:

$$(1) \qquad MC = \frac{a}{\beta_0\beta_1(vnH)^{\beta_1-1}} = \frac{a}{\beta_0\beta_1} \ (vnH)^{1-\beta_1}$$

and

$$(2) \qquad MR = \frac{an}{r} \ (1-e^{r(t-T)})$$

From (1) and (2) follows that the solution v^x is:

$$(v^x)^{1-\beta_1} = \frac{n}{r} \ (1-e^{r(t-T)})\beta_0\beta_1(nH)^{\beta_1-1}$$

$$v^x = [\frac{n}{r} \ (1-e^{r(t-T)})\beta_0\beta_1(nH)^{\beta_1-1}]^{\frac{1}{1-\beta_1}}$$

$$v^x = [\frac{1}{r} \ (1-e^{r(t-T)})\beta_0\beta_1]^{\frac{1}{1-\beta_1}} H^{\frac{\beta_1-1}{1-\beta_1}} \ n^{\frac{\beta_1}{1-\beta_1}}$$

$$\frac{\partial v^x}{\partial n} = [\frac{1}{r} \ (1-e^{r(t-T)})\beta_0\beta_1]^{\frac{1}{1-\beta_1}}(H)^{\frac{\beta_1-1}{1-\beta_1}} \ \frac{\beta_1}{1-\beta_1}(n)^{\frac{\beta_1}{1-\beta_1}-1} > 0 .$$

The situation with interruptions in labor market participation may also be analyzed with models of this type.[1] We may then write the maximization problem of the individual as

$$\text{Max} \int_0^T e^{-rt} \, P \, a \, H(1-v) \, dt$$

where $P = 1$ if the individual fully uses his capacity for human capital creation or work purposes;

$P = 0$ if the individual does not use his capacity at all for any of these purposes.

3.2.4 Growth and Vintage Effects

Growth and vintage effects are discussed by, e.g., S. Rosen (1973, 1975, 1976), C. Brown, and Y. Weiss and L.A. Lillard.[2] Here we try to make as simple as possible a statement of the issues raised by introducing these aspects in relation to the Basic Model.

Let us just begin by noting how growth and vintage effects may be related to a competitive equilibrium with a constant rental price for human capital. In a steady state growth economy, where the growth in human capital is achieved by an increase in population, we could have a constant equilibrium rental price. The main requirement is that the total stock of human capital should expand at the same rate as the rest of the economy.[3] This could otherwise be achieved by more efficient use of human capital or larger initial endowments of successive generations. In the former case the changed efficiency could also be expressed as a growth in the relative price of human capital.

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[1] Cf. Y. Weiss, R. Gronau, "Expected interruptions in labor force participation and sex-related differences in earnings growth." Working Paper No. 5-80, February 1980, Foerder Institute for Economic Research, Tel-Aviv.

[2] S. Rosen, op.cit., 1973.
S. Rosen, op.cit., 1975.
S. Rosen, op.cit., 1976.
C. Brown, op.cit.
Y. Weiss, L.A. Lillard, op.cit.

[3] C.J. Bliss, Capital Theory and the Distribution of Income. Amsterdam, 1975, Chapter 4.

In a situation where the rental price of human capital increases,1) investment incentives at a given date will be larger than where the rental price does not grow. With a rising rental price both costs and returns grow. However, the effects on returns will be greater than on costs. A one-time increase in the rental price has the same effect on costs and returns, but a continually growing rental price has a greater effect on returns. An exponential growth rate, e.g., g in the rental price a, could be said to have the same effect as appreciation or a lowering of the rate of discount from r to $(r-g)$.2)

If individuals who differ only in that they belong to different generations are compared in a situation where the rental price of human capital is rising exponentially over time, the younger ones would have larger lifetime earnings because of the higher rental price.

An early generation may begin its activities when the rental price is a_0. If the price should grow at a constant rate (g), it will have risen to a_0e^{gT} by the time these individuals leave the market. A later generation may start with rental price a_0e^{gS} ($S>0$) and at the end of its working life earn $a_0e^{g(S+T)}$ on each unit of human capital that is rented. So, if an individual

---

1) The analytic difference between growth and appreciation that we treated in the preceding section is their causes. The former is due to general efficiency improvements (innovations) in the economy and is the same for all human capital. The former is caused by changes in the individual and might differ among individuals.

2) Let us assume that the rental price of human capital grows at a constant rate g: $a(\tau) = a_0e^{g\tau}$.

Thus, $Y(\tau) = a_0e^{g\tau}H(\tau)\,(1-v(\tau))$.

The individual can then be assumed to maximize:

$$J = \int_0^T a_0H(t)\,[1-v(t)]e^{-(r-g)t}\,dt.$$

With the same production function as in the Basic Model we would e.g. during phase 2 have

$$MC(t) = \frac{a(t)}{\beta_0\beta_1(vH)^{\beta_1-1}} = \frac{a_0e^{g\tau}}{\beta_0\beta_1(vH)^{\beta_1-1}}$$

and

$$MR(t) = \int_0^{T-t} a(t)e^{g\tau}e^{-r\tau}d\tau = \frac{a(t)}{r-g}\,[1-e^{(r-g)(t-T)}] = \frac{a_0e^{gt}}{r-g}\,[1-e^{(r-g)(t-T)}].$$

At a given date both MC and MR would be higher with than without growth. However, this effect is greater for MR than MC.

during his life has supplied as much human capital to the labor market, he would have earned more if he had belonged to the later generation than to the earlier generation. However, as long as the rate of growth remains constant, investment incentives remain the same for individuals "identical" but for generation.

Similar effects appear if successive generations with different amounts of initial human capital stock enter the labor market. With the production function for human capital of the Basic Model, an exogenously given growth rate in initial endowments of successive generations entering phase 2 directly would only change the level of earnings, not investment behavior. These effects are exactly the same as for individual differences in initial stock of human capital ($\bar{H}_0$). They do not change investment incentives.

Especially if one works empirically with cross-sectional materials, it is interesting to distinguish vintage and growth effects. In the Basic Model the earnings profile for "identical" individuals would be the same, whether it was observed for one individual over time or for several individuals of different ages at a given date. However, when growth and vintage effects enter the picture, this is not true for individuals "identical" except as to the generation to which they belong.

If one makes more specific assumptions about the production function of human capital, more precise implications could be arrived at concerning the relation between cross section and time series observations of, e.g., earnings for "identical" individuals. In Figure 32 we illustrate the differences that appear with a linear MC-curve (type MC = k Q, where k is a constant). This example is worked out fully below. It is an adaptation of a similar example used by S. Rosen (1975) [1].

Let us assume that we have the production function of the Basic Model and $\beta_1 = \frac{1}{2}$.

We would then have

$$\frac{\partial C}{\partial Q} = 2 \frac{a}{\beta_0^2} Q$$

i.e., linear marginal costs of the type MC = k Q where k is a constant.

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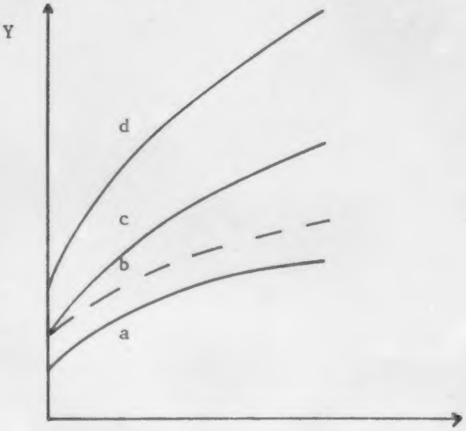
[1] S. Rosen, op.cit., 1975.

Figure 32 Earnings of individuals "identical" except as to the generation to which they belong, with linear marginal costs for human capital production



a) the development of earnings over time, based on the Basic Model without growth and vintage effects;

b) shows compared with a) the vintage effect comparing earlier and later generations;

c) shows compared with a) the effect of growth comparing earlier and later generations;

d) shows compared with a) the combined effect of growth and vintage comparing earlier and later generations.

Total investment costs are then

$$C = k/2\,\dot{H}^2 = \frac{a}{\beta_0{}^2}\,Q^2.$$

During phase 2 we should have MC = MR (t); i.e.,

$$2\,\frac{a}{\beta_0^2}\,Q(t) = \frac{a}{r}\,(1-e^{r(t-T)}).$$

As $H(t) = \bar{H}_0 + \int_0^t Q(\tau)\,d\tau$ for individuals who enter phase 2 directly, we have for them

$$H(t) = \bar{H}_0 + (\frac{1}{ur})t - \frac{1}{ur^2}\,e^{r(t-T)} + \frac{1}{ur^2}\,e^{-rT}.$$

where $u = 2/\beta_0{}^2$. Furthermore,

$$Y(t) = aH(t) - \frac{a}{\beta_0^2} Q(t)^2 = aH(t) - \frac{a}{\beta_0^2} \left[\frac{1}{ur} (1-e^{r(t-T)} \right]^2.$$

By substituting and rearranging we have:

$$Y(t) = a\bar{H}_0 + \left(\frac{a}{ur}\right) t - \frac{a}{2ur^2} \left[1+e^{2r(t-T)}\right] + \frac{a}{ur^2} e^{-rT}$$

or

$$Y(t) = a\bar{H}_0 - \frac{a}{2ur^2} + \frac{a}{ur^2} e^{-rT} - \frac{a}{2ur^2} e^{-2rT} + \frac{a}{ur} t - \frac{a}{2ur^2} e^{2rt}.$$

We could use this expression empirically, e.g., given T to estimate r, a, $\bar{H}_0$ and β_0.

Now, let us study growth effects within such a model. We assume that the rental price grows at a constant rate g and that

$$Y(t) = a_0 e^{gt} H(t) \left[1-v(t)\right]$$

$$Y(t)\, e^{-gt} = a_0 H(t) \left[1-v(t)\right].$$

With a linear MC curve we would then have during phase 2:

$$Q(t) = \frac{1}{u} \frac{1}{(r-g)} \left(1-e^{(r-g)(t-T)}\right)$$

where $u = 2/\beta_0^2$, and

$$H(t) = \bar{H}_0 + \frac{1}{u(r-g)} t - \frac{1}{u(r-g)^2} e^{(r-g)(t-T)} + \frac{1}{u(r-g)^2} e^{-(r-g)T}.$$

Since $a_0 H(t)\, v(t) = \frac{a_0}{\beta_0^2} Q^2$

we have

$$Y(t) e^{-gt} = a_0 \bar{H}_0 + \frac{a_0}{u(r-g)} t - \frac{a_0}{2u(r-g)^2} \left(1+e^{2(r-g)(t-T)}\right) + \frac{a_0}{u(r-g)^2} e^{-(r-g)T}.$$

We can use this model to investigate the relation between time series and cross-section data for individuals "identical" except as to the generation to which they belong.

At a given date we observe individuals of different ages (t), born at different dates or belonging to different vintages (S). Age is assumed to range from 0 to T. Individuals at age T are shown to belong to vintage $S=0$, and those at age 0 to vintage $S=T$. At the date of observation they are all paid the same rental price. This will be set at $a_0 e^{gT}$ so that a_0 was the rental price that generation $S=0$ was paid when entering the model.

We would then have

$$Y(T,S=0) = e^{gT}\left[a_0\bar{H}_0 + \frac{a_0}{u(r-g)}T - \frac{a_0}{2u(r-g)^2}(1+e^{2(r-g)(t-T)}) + \right.$$
$$\left. + \frac{a_0}{u(r-g)^2}e^{-(r-g)T}\right]$$

and generally

$$Y(t,S) = e^{gt}e^{gS}\left[a_0\bar{H}_0 + \frac{a_0}{u(r-g)}t - \frac{a_0}{2u(r-g)^2}(1+e^{2(r-g)(t-T)}) + \right.$$
$$\left. + \frac{a_0}{u(r-g)^2}e^{-(r-g)T}\right].$$

As $S = T - t$ in the cross section, for that we have

$$Y(t,S) = e^{gT}\left[a_o\bar{H}_0 + \frac{a_0}{u(r-g)}t - \frac{a_0}{2u(r-g)^2}(1+e^{2(r-g)(t-T)}) + \frac{a_0}{u(r-g)^2}e^{-(r-g)T}\right].$$

Let us also study vintage effects in the model above. We assume that the initial endowments of successive generations or vintages grow at a constant rate λ and use the following notation

$$\bar{H}_0(S) = \bar{H}_0(0)\,e^{\lambda S}.$$

During phase 2 initial endowments have no effects upon the investment behavior and we would then have:

$$Y(t,S) = a\,\bar{H}_0(0)\,e^{\lambda S} + \frac{a}{ur}t - \frac{a}{2ur^2}(1+e^{2r(t-T)}) + \frac{a}{ur^2}e^{-rT}$$

This would also be what we would observe in a cross-section if we set $\bar{H}_0(0) = \bar{H}_0(S=0)$, i.e.,

$$Y(t,S) = a\,\bar{H}_0(S=0)e^{\lambda S} + \frac{a}{ur}t - \frac{a}{2ur^2}(1+e^{2r(t-T)}) + \frac{a}{ur^2}e^{-rT}.$$

Applying the same notation as above, combined growth and vintage effects would mean that the initial earning capacity of successive generations would be

$$a_0\bar{H}_0(0)\,e^{(g+\lambda)S}.$$

With this model we would then for the cross section have

$$Y(t,S) = e^{gT}[a_0\bar{H}_0(S=0)e^{\lambda S} + \frac{a_0}{u(r-g)}t - \frac{a_0}{2u(r-g)^2}(1+e^{2(r-g)(t-T)}) + \frac{a_0}{u(r-g)^2}].$$

If we added depreciation (d) we would have (r+d-g) instead of (r-g) in the expression above.

3.2.5 Rents, Supply and Demand for Human Capital

In these Life Cycle Models - as in the Schooling Models where individual variations explained the choice of level of schooling - there will be one optimal level of human capital supply for each individual at each date. In these models it can vary over time, which was not the case in the Schooling Models.

If the supply is to change, something in the conditions for the supply of human capital would also have to change. In the Basic Model we saw that changes in the rental price of human capital did not have any effects in this direction, but that changes in the rate of interest did. In models where not only individual capacity was used for human capital production, but also other resources, changes in the rental price of human capital and the price of these other resources were also seen to affect the production of human capital.

As in the Schooling Models, rents will be earned on the initial stock of human capital. Moreover, all units except the last one created at each date will earn rents, because for them the marginal return exceeds marginal costs.

As in the previous models demand for human capital is determined by the firms given their production technology, other prices, etc.

3.3 Special Features of the Life Cycle Models

Compared with the Schooling Models, new exogenous factors of the Life Cycle Models which explain the investment behavior and earnings of the individual are:

$\bar{H}$, n - absolute or relative amount of capacity set aside for activities other than work and human capital production;

β_0, β_1 - the parameters of the more precisely but alternatively
$\beta_0, \beta_1, \beta_2$ formulated production functions of human capital;
$\beta_0, \gamma_1, \gamma_2$

S - the generation to which the individual belongs in the case of growing initial endowments or rental price of human capital;

λ - growth of initial endowments;

g - growth rate of the rental price.

The new price variable in these models is P_d, the price of "other resources" used in the production process for human capital.

The models could, of course, be elaborated further within this framework. The parameters β_0, β_1, β_2, δ, α, g, λ e.g. do not have to remain constant for each individual. If we specify how they develop over time we can arrive at certain implications concerning their effects. However, at this point we will make no further extensions but look at the implications obtained with the models thus far and discuss empirical testing strategies.

Let us note that in many fundamental respects these models are similar to the Schooling Models. The individual is still assumed to maximize the present value of earnings and the firms are assumed to operate as before. In relation to the Schooling Models two types of firms could be distinguished: schools and firms producing other goods and services. Here there could be three: schools, firms combining training with the production of other goods and services, and firms specializing in the latter type of production. With regard to goods, the production processes have to be flexible to ensure that no problems of divisibility will arise when combining work and training. The same would have to be true of the training processes.

The market for human capital is also assumed to operate in the same manner as before. As with the alternative formulations of the Schooling Models, the individual is not indifferent to the amount of human capital he supplies. At each date he supplies a given amount to the labor market. This amount is part of a program of optimal investments that extends over the individual's entire lifetime. Only if some parameters change would the supply change. As previously, the demand for human capital is determined by the firms, given their production technology, other prices, etc.

The implications of the models will be summarized in terms of the endogenous variables that seem most likely to have observable counterparts, i.e.:

- Y - actual earnings;
- P V Y - present value of actual earnings;
- t^x - end of human capital specialization period;
- v and (1-v) - amount of capacity or "time" devoted to human capital production and work.

The most readily observable variable would certainly be Y, i.e. "actual" earnings. Such earnings are received in phase 2 and 3. The end of the period of specialization (t^x) could also be observed as this is the date when "actual" earnings are first received. Whether or not this date coincides with the formal schooling period is an empirical question. Generally speaking, the models implied that the period of specialization comes first in the life cycle, if at all. During that period a constant amount of "time" is devoted to human capital production.

After the period of specialization the same constant amount of "time" will be devoted to work and part-time or on-the-job training. As for the amount of "time" devoted to human capital production and work respectively, it could only be observed directly if these activities were to be carried out at different places. In that event, we could also identify the beginning of phase 3, the pure work period.

In Table II the effects of the main explanatory factors on certain dependent variables are summarized, given a stable, stock-independent MC-curve. The relations are for phase 2 and for individuals entering that phase directly.

One first set of empirical testing possibilities thus consists of investigating the relations between observable variables such as Y_t, $\dot{Y}_t$ and PVY and these explanatory variables.

Another set of testing possibilities has to do with the pattern of phases and earnings. As regards phases, we have discussed three types: phase 1 with $v = 1$ and specializations in human capital production; phase 2 with $0 < v < 1$ and capacity divided between work and human capital production; and phase 3 with $v = 0$, which means full-time work. Different combinations of these phases proved possible. With the Basic Model they were 1 + 2 + 3 or only 2 + 3. We also noted that in this case phase 3 only occurred at the last date (T). Changing the production function for human capital other combinations might be optimal. A prolonged period of full time work could, e.g., either be preceded by phase 2, phases 1 + 2, or begin directly. Another possible combination was phase 1 + 3. In addition to a general testing of the different Life Cycle Models, empirical studies with respect to the pattern of phases provide information about production functions of human capital, and the individual differences in these respects that are consistent with optimizing behavior and the pattern of phases and earnings. This type of indirect information should also be very valuable for better model building.

Table II Some effects of explanatory variables on different dependent variables in the Life Cycle Models

Effect of higher \ on	PVY	Y_t	vH	v	(1-v)	$\dot{Y}_t$
$\bar{H}_0$	+	+	0	-	+	0
T	+	?	+	?	?	+
β_0	+	?	+	?	?	+
d	-	?	-	?	?	-
$\bar{H}$	-	-	0	+	-	0
n	+	?	+	?	?	+
S(λ)	+	+	0	-	+	0
S(g)	+	+	0	0	0	+
g	+	+	+	?	?	+
a 1)	+	+	0	0	0	+
a 2)	+	?	+	?	?	+
P_d	-	?	-	?	?	-
r	-	?	-	?	?	-

Definitions of symbols:

+ positive effect

- negative effect

0 no effect

? effect unknown until further specifications have been made

$\bar{H}_0$ = initial stock of human capital

T = end of planning horizon

β_0 = "ability" - i.e., efficiency parameter in the production function for human capital

d = net rate of depreciation

$\bar{H}$ = absolute amount of human capital set aside for activities other than work and investment

n = relative amount of human capital set aside for activities other than work and investment

S(λ)= the generation to which the individual belongs in case of vintage effects

1) in a model excluding D and P_d

2) in a model including D and P_d

S(g) = the generation to which the individual belongs in case of growth of rental price

g = growth of rental price

P_d = price of "other resources"

D = "other resources"

r = market rate of interest

PVY = present value of earnings

Y_t = actual earnings at date t

vH = units of human capital used for human capital production

v = share of capacity or "time" used for human capital production

1-v = share of capacity or "time" used for pure work

$\dot{Y}_t$ = growth in earnings at date t (comparing concave earnings profiles)

The individual's investment incentives determine whether he will go through all three phases or only two or one of them. With the Basic Model, the smaller the initial stock of human capital the greater the chance of passing through an initial phase of specialization and the later the arrival of t^x, the date when that phase ends. Also, given the stock of initial human capital and the costs of producing more, the larger marginal returns (MR) the greater the chance of this type of specialization. As we have seen, MR is higher when, e.g., the end of planning horizon (T) comes later, the rate of growth (g) is larger, or when depreciation (d), rate of labor force participation (n), and rate of interest (r) are smaller. A lower MC-curve, due to higher ability (β_0), reduces the risk of specialization.

If individuals that differ in any of these respects could be identified, numerous implications could be tested. Do they specialize or not as predicted by the models? If they specialize, do they do so for shorter or longer periods as predicted by the models? The Basic Model contains different implications regarding variations in initial endowments and ability in these respects.

As for the actual earnings profiles - whether earnings are constant, growing or decreasing, or growing or declining at a certain rate - the various Life Cycle Models give different implications. Empirical data could indicate what approach to the production function for human capital and depreciation - appreciation is most appropriate here.

Another way of using the models is to assign numerical values to the parameters and calculate the consequences. [1] By giving numerical values to a, r, $\bar{H}_0$, β_0, β_1 and T in the Basic Model, e.g., t^x and Y could be calculated. Variations in the numerical values result in variations in t^x and Y. The results in terms of t^x and Y could be compared with actual observations. Actual observations could also be used to see what is required in terms of combinations of the parameters to reproduce the actual results.

In each model, actual earnings can be expressed as a function of the parameters of the model. As a rule, these functions will be rather complicated. However, simplifications can be introduced and relations, empirically more tractable, are then implied. [1] One example is the relation we obtain with a linear MC-curve:

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[1] Cf. Y. Ben-Porath, op.cit., 1970.
T.D. Wallace, L.A. Ihnen, op.cit.

$$Y_t = a\bar{H}_0 - \frac{a}{2ur^2} + \frac{a}{ur^2} e^{-rT} - \frac{a}{2ur^2} e^{-2rT} + \frac{a}{ur} t - \frac{a}{2ur^2} e^{2\ rt}$$

where the notation is the same as that given in connection with Table II above and $u = 2/\beta_0{}^2$.

Once earnings functions have been estimated, they could be used for income distribution analysis.

Each specification of, e.g., the production function would lead to a different relation to be estimated. Each such relation then only holds for that specific production function. The testing of earnings functions thus provides another way of gaining information about consistent earning outcomes and assumptions as regards production functions, etc. However, as individuals may have different production functions and depreciation-appreciation characteristics, it would be valuable to combine the testing of earnings functions with information gained from other studies that may indicate suitable "homogenous" groupings. Aside from the type of analyses mentioned above, studies aiming directly at estimating the production functions of human capital ought to be undertaken.

As we have seen, the assumptions concerning the production function of human capital are crucial for the choices to be made by the individual. Independent information relative to that aspect would thus be extremely valuable. Of course, there are many problems involved in the estimation of production functions for human capital - with one being how to measure human capital. However, this does not prevent empirical work from being done in which different alternative proxy measures for human capital are employed. [2]

As for empirical work with Schooling Models, the Life Cycle Models provide the following type of information:

- If individuals have significant amounts of earnings during their formal schooling period, it might be better to redefine schooling as the period up to t^x.

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1) Cf. e.g. C. Brown , op.cit.

2) Cf. J.M. Ritzen, D.R. Winkler, "The Production of Human Capital over Time." The Review of Economics and Statistics, 1977, No.4. E.A. Hanushek, op.cit., and Appendix A.

- When post-school (post-t^x) investments are made, actual earnings do not very exactly reflect the effects of schooling on earnings. When working with Schooling Models, one might use as an earning concept either aH_{tx} as based on earnings functions or actual earnings a certain number of years after the entry into the labor market. In the latter respect, estimated earnings functions could also provide indications as to suitable period. Another possibility would be to use Schooling Models for groups with identical post-school investment incentives as identifiable from the variables influencing MR and MC during phase 2 or more simply, in the case of separately registered post-school investments, to individuals with the same adult training paths.

- The actual earnings concept in Schooling Models should take into consideration the amount of labor market participation (cf. also Section 5).

By thus combining work with Life Cycle Models and Schooling Models, the Schooling Models could also provide information about the production function for human capital.

Another approach to the relation between Schooling and Life Cycle Models is to highlight the effects of schooling on post-school activities.[1] Given the fact that the period of specialization comes first, the Life Cycle Models could be used to solve for the optimal earnings path, given an arbitrary schooling period. This path will be a function of s and dependent on the ways in which s influences H_{tx} (or alternatively the efficiency parameters in the production function for human capital - β_0, β_1, etc.). Studies of different schooling and ability groups could give information regarding the effects of schooling compared with the effects of ability on the post-school experience.

The important question in all these different types of studies is, of course, whether the information they yield can be reconciled and lead to a coherent interpretation of data with due respect to the qualifications that the more extended models can bring.

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1) Cf. S. Rosen, op.cit., 1977,/b/.

4. Simplified Life Cycle Models

In this section we are going to investigate how a type of Life Cycle Models that is widely used empirically - what we call Simplified Life Cycle Models - can be integrated into the general framework developed so far. For this purpose we first present a basic formulation. Similar formulations can be found in the works of other authors.[1] The classical one is found in Mincer (1974).[2] Mincer works with different assumptions. He uses, e.g., both a "linear" and an "exponential-decline-in-v-assumption" during a working period of given length that includes or excludes depreciation. These models are described in Section 4.1. However, to bring the models closer to the ones described in Section 3, we reformulate them in terms of a given plannning horizon (T), with v either restricted to equal one at the transition from the period of specialization or not so restricted. The details concerning these models are given in Section 4.2.

After having discussed these models in Section 4.3 , we analyze in Section 4.4 the possibilities of adding other explanatory variables.

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[1] See, e.g.,
S. Batlett, "Education, Experience, and Wage Inequality: 1939-1969." Journal of Human Resources, 1978, No.3.
G.S. Becker, op.cit., 1975.
B.R. Chiswick, "An Interregional Analysis of Schooling and the Skewness of Income." In B.F. Kiker (ed.), Investment in Human Capital. Columbia S.C., 1971.
B.R. Chiswick, op.cit., 1974.
B.R. Chiswick, J. Mincer, "Time-series Changes in Personal Income Inequality in the United States from 1939 with Projections to 1985." Journal of Political Economy, 1972, No.3 II.
J. Heckman, S. Polacheck, "Empirical Evidence on the Functional Form of the Earnings-Schooling Relationship." Journal of the American Statistical Association, June,1974.
I. Garfinkel, R. Haveman, D. Betson, "Earnings Capacity and its Utilization." Quarterly Journal of Economics, 1978, No.3.
C.B. Knapp, W.L. Hansen, "Earnings and Individual Variations in Postschool Human Investment." Journal of Political Economy, 1976, No.2.
J. Mincer, op.cit., 1970.
J. Mincer, op.cit., 1974.
G. Psacharopoulos, R. Layard, "Human Capital and Earnings: British Evidence and a Critique." Review of Economic Studies, July, 1979.
J.D. Welland, "Cognitive Ability, Schooling and Earnings:the Question of Functional Form." Review of Economics and Statistics, 1978, No.4.

[2] J. Mincer, op.cit., 1974.

4.1 Basic Models

A formulation of these "Simplified Life Cycle Models" that is widely used expresses the relation between earnings, schooling and labor market experience as [1]

$$(4:1)\quad \ln Y_F = b_0 + b_1 s + b_2 F + b_3 F^2$$

where Y_F = actual earnings after F years on the labor market
s = years of schooling
F = years of labor market experience.

In order to relate this model to our previous ones, it can be rewritten in terms of our previous notation. Then it would read

$$\ln Y_t = b_0 + b_1 t^x + b_2(t-t^x) + b_3(t-t^x)^2.$$

The logarithmic value of actual earnings thus depends on how long the period of schooling (≈period of specialization) has been and how much time has elapsed since it ended.

The Simplified Life Cycle Models combine two elements, one of which can be said to be borrowed from the Life Cycle Models in the preceding section and the other from Simple Schooling Models. According to the former, based on the Life Cycle Models, investments in human capital decline after the period of specialization. [2] The latter states that there is a "constant rate of return" on human capital investments, with average and marginal return equalling each other at each date and over time. [3] If the latter assumption is extended to all types of human capital investments, we could write

$$(4:2)\quad E_t = E_0 + \bar{r} \sum_{j=0}^{t-1} C_j$$

---

[1] Cf. J. Mincer, op.cit., 1974 and the authors mentioned above.

[2] This feature was part of the optimal solution to the Basic Model of Section 3.1 where both investment costs (avH) and share of human capital devoted to "learning" (v) was seen to decline over time after specialization. With the alternative formulations of that model this was not always necessarily so.

[3] This was a feature of the Simple Schooling Models but not of the Schooling Models with Individual Differences.

where E_t = potential earnings at date t which are equal to the actual human capital stock (H_t) times the rental price of human capital, i.e., aH_t;

$E_0 = a\bar{H}_0$ potential earnings from initial stock of human capital;

C_j = investment costs during period j;

$\bar{r}$ = rate of return on investments in human capital given to the individual.

If we assume that there were no direct investment outlays for human capital investments, and express investment costs as a fraction (v) of potential earnings [1] $C_j = v_j E_j$

we have

$$E_t = E_0 + \bar{r} \sum_{j=0}^{t-1} v_j E_j = E_0 \prod_{j=0}^{t-1} (1+\bar{r}\, v_j)$$

and

$$Y_t = E_t \, (1-v_t).$$

Moreover,[2]

$$\ln E_t \approx \ln E_0 + \bar{r} \sum_{j=0}^{t-1} v_j$$

and

$$\ln Y_t \approx \ln E_0 + \bar{r} \sum_{j=0}^{t-1} v_j + \ln(1-v_t).$$

Let us assume that the investment activity of the individual starts with a period of specialization in human capital (v=1) and that this period equals the schooling period of duration s. With these assumptions, we can then write

$$(4:3) \qquad \ln Y_t \approx \ln E_0 + \bar{r}\, s + \bar{r} \sum_{j=s}^{t-1} v_j + \ln\,(1-v_t)$$

Given more precise assumptions about the development of v after the period of schooling, the expressions above can be reformulated and simplified. Let us begin by assuming that v falls linearly over time [3] so that

$$v_F = \begin{cases} v_0 - \dfrac{v_0}{N} F & 0 \leq F \leq N \\ 0 & F > N \end{cases}$$

---

[1] In the event of no direct investment outlays this v corresponds to that of Section 3, varying from zero to one.

[2] Using the same type of approximation as for the Simple Schooling Models.

[3] Cf. J. Mincer, op.cit., 1974.

where F = numbers of years on the labor market ($F \approx t - t^x$ in our previous notation;

N = the period of post-school investments ($N \approx T - t^x$ in our previous notation;

v_0 = share of potential earnings devoted to human capital investments when entering the labor market ($v_0 \approx v_{t^x}$).

Cumulative investment shares up to date F would then be

$$(4:4) \qquad \sum_{j=0}^{F-1} \left(v_0 - \frac{v_0}{N} j\right) = Fv_0 - \frac{v_0}{N} \cdot \frac{F(F-1)}{2} \approx Fv_0 - \frac{v_0}{2N} F^2 .$$

Substituting (4:4) into (4:3) we have

$$(4:5) \qquad \ln Y_F \approx \ln E_0 + \bar{r}s + \bar{r}v_0 F - \frac{\bar{r}v_0}{2N} F^2 + \ln\left(1 - v_0 + \frac{v_0}{N} F\right).$$

We can approximate $\ln(1 - v_0 + \frac{v_0}{N} F)$ through a quadratic Taylor expansion around $F = N$ by

$$(4:6) \qquad -v_0 \left(1 + \frac{v_0}{2}\right) + \frac{v_0}{N} (1 + v_0) F - \frac{v_0^2}{2N^2} F^2 .$$

Substituting (4:6) into (4:5) we obtain an expression of the type mentioned above

$$(4:1') \qquad \ln Y_F \approx b_0 + b_1 s + b_2 F + b_3 F^2$$

where $b_0 = \ln E_0 - v_0 \left(1 + \frac{v_0}{2}\right)$

$b_1 = \bar{r}$

$b_2 = \bar{r} v_0 + \frac{v_0}{N} (1 + v_0)$

$b_3 = -\left(\frac{\bar{r}v_0}{2N} + \frac{v_0^2}{2N^2}\right)$.

The coefficients b_i are thus functions of the human capital stock at the beginning of the schooling period ($\bar{H}_0$), the fraction of potential earnings devoted to human capital production at the beginning of the post-schooling period (v_0), the period of positive post-school investments (N) and the rate of return ($\bar{r}$). Having estimated empirically the coefficients b_i, the structural parameter $\bar{r}$ is identified by b_1. From b_0, b_2 and b_3 consistent combinations of E_0, v and N could be identified.

A similar expression is obtained if depreciation is added to the model above.[1] With a constant rate of depreciation δ, we would have

$$E_t = E_{t-1} + \bar{r}\, C^x_{t-1} - \delta E_{t-1}$$

where C^x = gross investments.

If we define the gross investment fraction as

$$v^x_t = C^x_t / E_t$$

instead of (4:5) we then have

$$(4:7) \quad \ln Y_F \approx \ln E_0 + (\bar{r}-\delta)\, s + (\bar{r}v^x_0 - \delta)F - \frac{\bar{r}v^x_0}{2N^x} F^2 + \ln(1-v^x_F).$$

If we make the same type of Taylor expansion as above we get the expression

$$(4:8) \quad \ln Y_F \approx c_0 + c_1 s + c_2 F + c_3 F^2$$

where
$$c_0 = c_0(E_0,\, v^x_0)$$
$$c_1 = c_1(\bar{r}, \delta)$$
$$c_2 = c_2(\bar{r}, \delta, v^x_0,\, N^x)$$
$$c_3 = c_3(\bar{r}, \delta, v^x_0, N^x).$$

An empirical estimation of the coefficients c_i could be used to identify consistent combinations of the structural parameters.

If all individuals were alike or if we had subsamples of identical individuals with respect to $\bar{H}_0$, $v_0(v^x_0)$, $\bar{r}$, δ, and $N(N^x)$, the models would suggest that except for random variations the above expressions for $\ln Y_F$ describe earnings profiles.

If other assumptions are made regarding the decline in v during the N years, other relations would result. At the end of this section we will present a few variations on this theme, but before doing so we will continue to study the present models, and primarily (4:1'). First, we can note that if there are no post-school investments the above (4:1') reduces to

$$(4:9) \quad \ln Y_F \approx \ln Y_0 + \bar{r}\, s$$

This looks like another expression for the results of the Simple Schooling Models, i.e., the logarithmic value of earnings after schooling is approximately equal to the logarithmic value of earnings on initial human capital plus the "rate of return" ($\bar{r}$) times the number of school years. Whether they are identical or not depends on $\bar{r}$. Can it be identified with the market rate of interest (r), as in the Simple Schooling Models, and thus permit r to replace $\bar{r}$ in the expression above? We shall discuss this and a few related questions in Section 4.3.

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[1] Cf. J. Mincer, op.cit., 1974.

Figure 33 illustrates the development of earnings due to school and post-school investments for individuals with equal initial endowments, $\bar{r}$, v_0 and development of v over time according to the initial model. With no investments the age-earning profiles would be flat. This would also be true if only school investments were made, though the level of earnings would be higher with higher schooling levels, as in the Schooling Models. If individuals continue to invest after their formal schooling ends, earnings will continue to increase. However, a smaller and smaller share of potential earnings are devoted to investments. With the share of earnings invested developing in the same way for individuals with different schooling levels, the increase in the log of actual earnings will be the same for all individuals [1] but the increase in actual earnings will be larger for individuals at higher schooling levels as illustrated at the bottom of Figure 33.

The starting salaries of people with longer periods of schooling will be higher than those of individuals with less schooling if the development of their respective v is equal. The differences in absolute earnings over years on the labor market will then always increase. However, the earnings profile may or may not cross.

If we relied on the Life Cycle Models in Section 3 we would expect $v = 1$ at the transition from specialization in human capital production (that we assume here to be equal to the schooling period) to a period with some labor force participation. The presumption would then be that the earnings profiles cross. In that case, the absolute differences in annual earnings over age once the individuals are on the labor market, could first be expected to decrease and later to increase. The absolute variance comparing the annual earnings of individuals with the same number of years on the labor market would still increase continually with longer labor market experience.

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[1] $\frac{d \ln Y_F}{dF} = b_2 - 2\, b_3 F$

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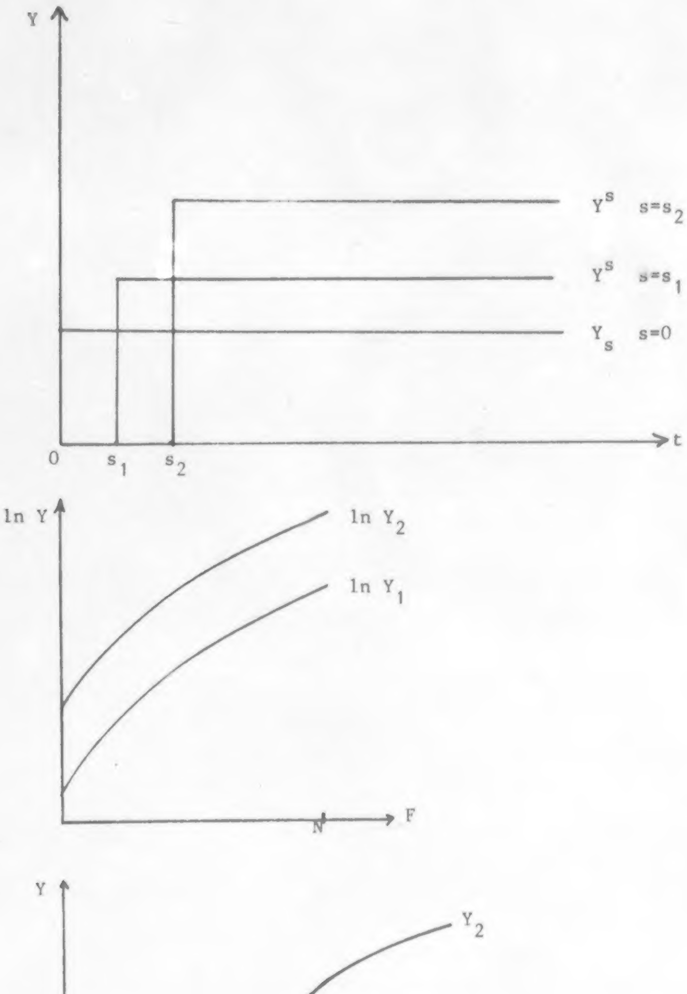
Figure 33 Age earnings profiles with and without post-school investments



Within schooling groups there would be no difference in earnings if we compare individuals with equal initial endowments and $\bar{r}$ and for whom v develops in the same way over time.

But what are the implications if there are variations in $\bar{H}_0$, v_0, $\bar{r}$ and N? Or, in other words: if we cannot distinguish homogenous groups in these respects and we still treat them as such, what are the risks of obtaining biased results? To illustrate this problem, let us return to the general formulation (4:2) and introduce a new notation. [1)]

According to (4:2) we would for individual i have

$$E_{iF} = aH_{0i} + \bar{r}_i \sum_{j=0}^{s-1} C_{ij} + \bar{r}_i \sum_{j=0}^{F-1} C_{ij}$$

or approximately

$$\ln E_{iF} = \ln aH_{01} + \bar{r}_i \sum_{j=0}^{s-1} v_{ij} + \bar{r}_i \sum_{j=0}^{F-1} v_{ij} \, .$$

Let us introduce the following notation that could, e.g., refer to one cohort

$\bar{\bar{H}}_0$ = average initial endowment

$H_{0i} = \bar{\bar{H}}_0(1-\alpha_i)$

$\bar{v}_j$ = average value of v_{ij} during schooling

$v_{ij} = \bar{v}_j + v^x_{ij}$ during schooling

$\bar{\bar{v}}_j$ = average value of v_{ij} during the post-school period

$v_{ij} = \bar{\bar{v}}_j + v^{xx}_{ij}$ during the post-school period

$\bar{r}$ = average rate of return

$\bar{r}_i = \bar{r} + \bar{r}^x_i$ rate of return for individual i.

Potential earnings of individual i, F years after the entry into the labor market could then be written:

$$\ln E_{iF} = \ln [a\, \bar{\bar{H}}_0(1+\alpha_i)] + \sum_{j=0}^{s-1} (\bar{v}_j+v^x_{ij})(\bar{r}+r^x_i) + \sum_{j=0}^{F-1} (\bar{\bar{v}}_j+v^{xx}_{ij})(\bar{r}+r^x_i)$$

---

1) This treatment of individual differences is similar to but more complete than the one found in B.R. Chiswick, op.cit., 1974.

If we use

$$\ln E_{iF} = \ln a \bar{\bar{H}}_0 + \bar{r} \sum_{j=0}^{s-1} \bar{v}_j + \bar{r} \sum_{j=0}^{F-1} \bar{\bar{v}}_j + U_i$$

U_i would stand for

$$U_i = \ln (1+\alpha_i) + \sum_{j=0}^{s-1} v_{ij}^{x} r_i^{x} + \sum_{j=0}^{s-1} v_{ij}^{x} \bar{r} + \sum_{j=0}^{s-1} \bar{v}_j r_i^{x} + \sum_{j=0}^{F-1} v_{ij}^{xx} r_i^{x} +$$

$$+ \sum_{j=0}^{F-1} v_{ij}^{xx} \bar{r} + \sum_{j=0}^{F-1} \bar{\bar{v}}_j r_i^{x}$$

If we disregard the individual differences contained in U_i and, e.g., in a least square regression analysis apply the formula $\ln Y = b_0 + b_1 s + b_2 F + b_3 F^2$ to individuals who are different in these respects we will, of course, explain less of the differences in earnings than would be the case for homogenous groups. However, from the expression U_i we can also see what the risks are of obtaining biased results. The explanatory variables s, F and F^2 should be independent of U. This would not be the case if, e.g., the schooling investments would be correlated both as to length and intensity with the individual rate of return. The same situation arises if the post-school investments are correlated with the individual rate of return.

It is possible, however, to reformulate the model to handle certain of these problems and bring it closer to the Life Cycle Models of Section 3. Let us, however, first present the results obtained by assuming, along with J. Min-Mincer, 1) that v falls exponentially.

If we thus assume for v that

$$v_F \begin{cases} v_0 \cdot e^{-\ F} & 0 \leq F \leq N \\ 0 & F > N \end{cases}$$

we can approximate $\sum_{j=0}^{F-1} v_j$ by

$$\int_0^F v_0 e^{-\beta t} dt = \frac{v_0}{\beta} - \frac{v_0}{\beta} e^{-\beta F}$$

and we have

$$\ln Y_F \approx \ln E_0 + \bar{r} s + \frac{\bar{r} v_0}{\beta} - \frac{\bar{r} v_0}{\beta} e^{-\beta F} + \ln(1 - v_0 e^{-\beta F}).$$

---

1) J. Mincer, op.cit., 1974.

$\ln(1-v_0\, e^{-\beta F})$ can by a quadratic Taylor expansion around $x = e^{-\beta F}$ and 0 be approximated by

$$- v_0\, e^{-\beta F} - \frac{v_0^2}{2}\, e^{-2\beta F} .$$

We could then write

$$(4{:}10) \qquad \ln Y_F = d_0 + d_1\, s + d_2\, X_F + d_3\, X_F^2$$

where $X_F = e^{-\beta F}$

$$d_0 = \ln E_0 + \frac{\bar{r} v_0}{\beta}$$

$$d_1 = \bar{r}$$

$$d_2 = - \frac{\bar{r} v_0}{\beta} - v_0$$

$$d_3 = - \frac{v_0^2}{2} \quad .$$

Having estimated d_i consistent sets of the structural parameters E_0, $\bar{r}$, v_0 and β could be identified.

Including depreciation and assuming that the share of gross investments are

$$v_F^x = v_0^x\, e^{-\beta F}$$

we would have

$$\ln Y_F = \ln E_0 + (\bar{r} - \delta)\, s - \delta F + \frac{\bar{r} v_0^x}{\beta}\, (1-e^{-\beta F}) + \ln\, (1-v_0^x\, e^{-\beta F}).$$

Using the same approximation as above, we could then write:

$$(4{:}11) \qquad \ln Y_F = e_0 + e_1\, s + e_2\, F + e_3\, X_F + e_4\, X_F^2$$

where $X_F = e^{-\beta F}$

$$e_0 = \ln E_0 + \frac{\bar{r} v_0^x}{\beta}$$

$$e_1 = (\bar{r} - \delta)$$

$$e_2 = \delta$$

$$e_3 = - \frac{\bar{r} v_0^x}{\beta} - v_0^x$$

$$e_4 = - \frac{v_0^{x2}}{2} \quad .$$

Having estimated e_i, the parameters $\bar{r}$, δ, v_0^x, β, and E_0 could be identified.

4.2 Models with a Given Planning Horizon

To bring the Simplified Life Cycle Models closer to the ones in Section 3, let us assume that individuals have a given planning horizon (T) and that they devote a declining share of their capacity to human capital production after schooling, i.e., from t^x to T. This period will thus be shorter the longer the period of specialization or the formal schooling, assuming that they coincide.

Let us assume that v falls linearly and thus

$$v_t \begin{cases} v_0 - \dfrac{v_0}{(T-t^x)} \quad (t-t^x) & t^x \leq t \leq T \\ 0 & t > T. \end{cases}$$

Cumulated investment shares up to date t could be approximated by

$$\int_{t^x}^{t} v_0 - \frac{v_0}{T-t^x} (t-t^x)\, dt = (t-t^x) v_0 - \frac{v_0}{2(T-t^x)} (t-t^x)^2 .$$

We would then have

$$\ln Y_t \approx \ln E_0 + \bar{r}\, s + \bar{r}\, (t-t^x)\, v_0 - \bar{r} \frac{v_0}{2(T-t^x)} (t-t^x)^2 + \ln[1 - v_0 + \frac{v_0}{T-t^x}(t-t^x)].$$

With the same type of approximation as used in (4:6) we would have

$$\text{(4:12)} \qquad \ln Y_t = \ln E_0 - v_0(1 + \frac{v_0}{2}) + \bar{r}\, s + [\bar{r}\, v_0 + \frac{v_0}{(T-t^x)}(1+v_0)]\, (t-t^x) - \\ - (\frac{\bar{r} v_0}{2(T-t^x)} + \frac{v_0^2}{2(T-t^x)^2})\, (t-t^x)^2 .$$

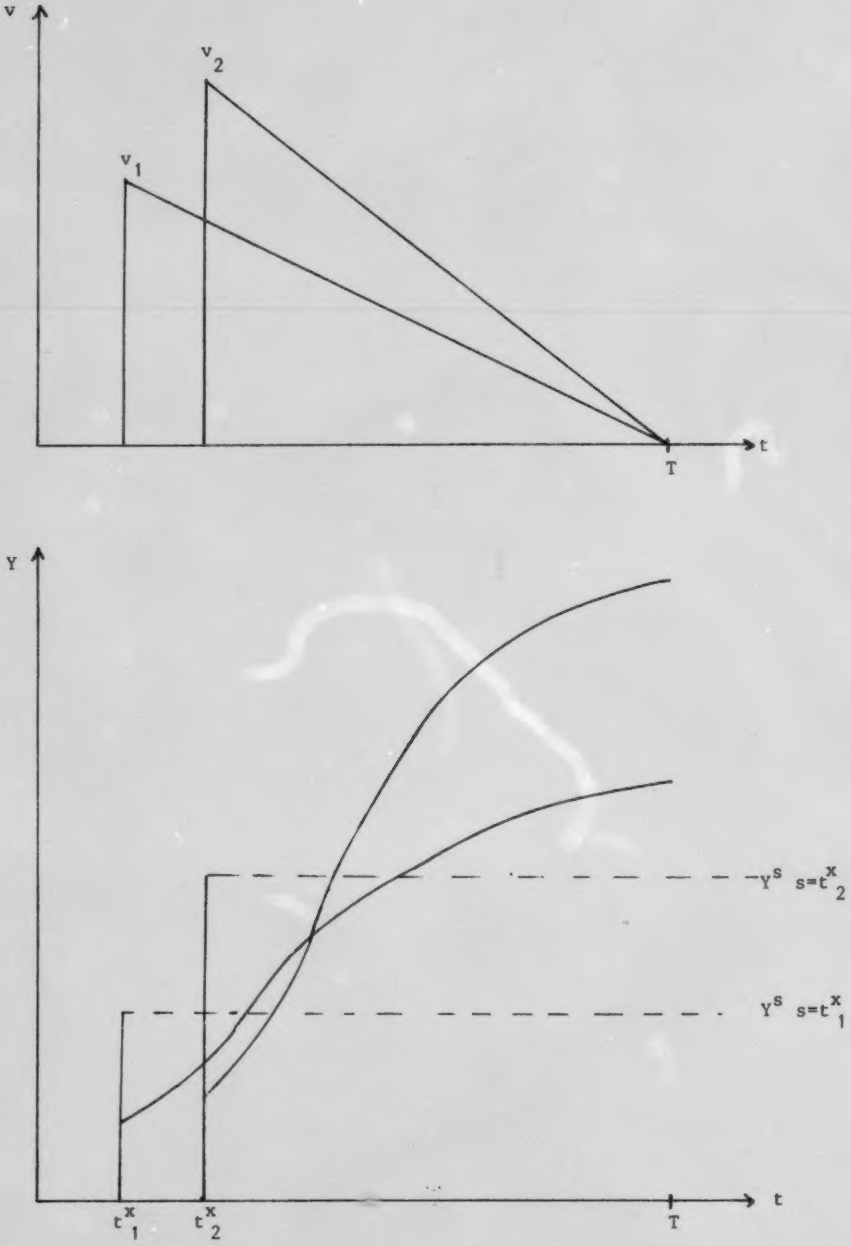
These relations would have to be estimated separately from empirical data for groups with different t^x, as v could not be the same for them.

We have

$$\frac{\partial v(t)}{\partial t^x} = - \frac{v_0}{(T-t^x)} (-1) - \frac{v_0 (t-t^x) \cdot (-1)}{(T-t^x)^2} \cdot (-1) > 0.$$

The earnings profiles resulting from (4:12) are illustrated in Figure 34. Compared with Figure 33 the increase in earnings for the individual with the longer period of schooling would now be more rapid relative to that for the individual with a shorter education. However, the period on the labor market

Figure 34 The development of the share of capacity devoted to human capital production (v), and actual earnings (Y) with a given planning horizon (T)



is shorter for the former. The actual earnings profile still may or may not cross, but we can no longer be sure that the starting salary of the individual with more education is higher. Therefore, the pattern of differences in earnings, either over age or years on the labor market, is more undetermined. In both cases it could either grow continually or first decrease and than increase.

With this approach we could not have $v = 1$ for all individuals at the time of transition from school to work. And empirically we would have to estimate a separate $\ln Y = b_0 + b_1 s + b_2 F + b_3 F^2$ relation for each schooling group.

However, a restriction may be imposed on v so that it equals one at the transition from schooling to work. Let us thus assume that $v = 1$ at t^x and then falls linearly and equals 0 at T. We would then have

$$v(t) = [1/(T - t^x)]\,T - [1/(T - t^x)]\,t.$$

Cumulated investment shares up to date t could be approximated by

$$\int_{t^x}^{t} [1/(T-t^x)]\,T - [1/(T-t^x)]\,t \; dt = \frac{1}{T-t^x}\,Tt - \frac{1}{2(T-t^x)}\,t^2 - 1.$$

We would then have

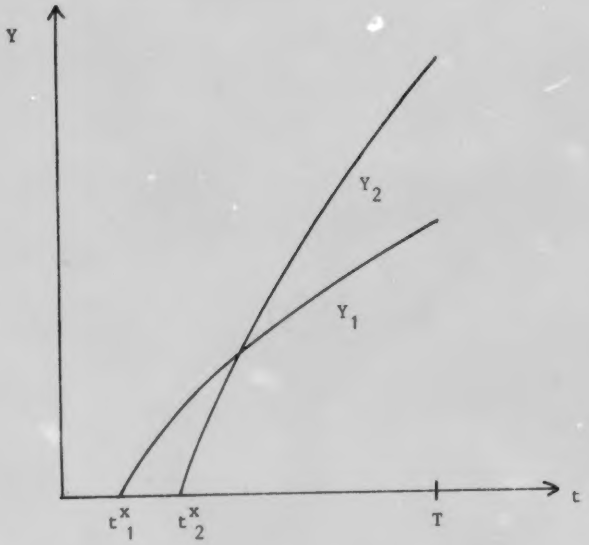
$$\ln Y_t \approx \ln E_0 + \bar{r}\,s + \bar{r}\,\frac{1}{(T-t^x)}\,Tt - \bar{r}\,\frac{1}{2(T-t^x)}\,t^2 - \bar{r} + \ln\left[1 - \frac{1}{(T-t^x)}\,T + \frac{1}{(T-t^x)}\,t\right]$$

or

$$(4{:}13)\quad \ln Y_t - \ln\left[1 - \frac{1}{(T-t^x)}\,T + \frac{1}{(T-t^x)}\,t\right] \approx \ln E_0 - \bar{r} + \bar{r}\,s + \bar{r}\,\frac{1}{(T-t^x)}\,Tt - \bar{r}\,\frac{1}{2(T-t^x)}\,t^2.$$

Compared with (4:12), we now obtain another relation between earnings, schooling and years on the labor market that could also be tested empirically. In this case, earnings profiles would cross as illustrated in Figure 35.

Figure 35 The development of actual earnings (Y) when the share of capacity devoted to human capital investments equals one at t^x



A final point before we turn to the discussion of the models in Sections 4.1 and 4.2. As seen from Figure 33, e.g., after a number of years the individual who makes post-school investments bypasses the earnings he would have had if only school investments were made. At this overtaking point, earnings (Y_{ov}) with and without post-school investments are equal, and

$$Y_{ov} = E_0 + F \sum_{j=0}^{s-1} C_j + r \sum_{j=s}^{ov-1} C_j - C_{ov} = E_0 + \bar{r} \sum_{j=0}^{s-1} C_j$$

and thus

$$\bar{r} \sum_{j=s}^{ov-1} C_j - C_{ov} = 0.$$

Returns on accumulated post school investments equal investment costs that year. If investment costs had been equally large each year, e.g.,

$\bar{C} = C_{ov}$, we would then have:

$$\bar{r} \cdot \bar{C} \cdot F_{ov} = \bar{C}$$

where F_{ov} = number of years on the labor market at overtaking

and we have $F_{ov} = \frac{1}{\bar{r}}$.

However, if absolute investment costs should decrease we would have $F_{ov} < \frac{1}{\bar{r}}$.

A comparison of the earnings of individuals with different lengths of schooling a few years after their entry into the labor market (F_{ov} years) could thus give a fairly clear picture of the isolated effect of schooling on earnings.[1]

4.3 Some Problems with the Simplified Life Cycle Models

The most problematic feature of the models in the previous sections is the assumption regarding a constant $\bar{r}$.

First of all, it cannot be interpreted as representing the rate of interest if there are differences in ability. Given the formulation of the model the only way in which individual differences in ability to turn monetary investments in human capital into earning capacity can be expressed is through individual variations in $\bar{r}$. However, in equilibrium the market rate of interest is equal to all individuals. If $\bar{r}$ is to vary among individuals it cannot thus equal the rate of interest. If we had not assumed the average rate of return to be constant, and thus equal to the marginal rate of return for each individual, we might have reconciled ability differences, varying returns and a given market rate of interest in the usual way: for marginal investments the rate of return equals the market rate of interest, and more able individuals can make more investments until the marginal rate of return is driven down to the market rate of interest.

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[1] This is not the only possibility. Estimation of the ln Y_F relation also permits a calculation to be made of earnings corresponding to different school levels. We have mentioned the above approach, based on J.Mincer (1974) as it was used in a study on Swedish data - S. Gustafsson, op.cit., 1976 - the results of which will be analyzed in a forthcoming report.

Secondly, it is problematic to motivate a constant average rate of return over time - higher for able individuals and lower for less able individuals.[1] If one dollar spent on human capital investment increases potential earnings by $\bar{r} \cdot 1\$$ at each date during the rest of the life of the individual the benefits of investing one dollar later in life would be smaller than one dollar invested earlier in life. If there are no capacity limitations there would be no reason for postponing investments. Assuming an infinite planning horizon and given an equilibrium situation, the return on marginal investments at each date would equal a constant market rate of interest. Moreover, if at each date there are constant returns in human capital production, average and marginal rates of return would coincide. With these assumptions, and further assuming no differences in ability we could expect a constant $\bar{r}$.

However, then we cannot explain investment decisions nor a forteriori motivate the assumption about the declining share devoted to post-school investments. In the models in Section 3, the main factor causing a decline in human capital investments was the approaching end of the planning period. The conclusion is that theoretically the Simplified Life Cycle Models can only as an approximation be reconciled with the previous models and an assumption of rational individual maximization behavior.[2]

A few theoretical and practical problems also arise in connection with the specification of v and its development over time. To keep in line with the Life Cycle Models of Section 3 the logical alternative - in the absence of direct investment outlays - would be to assume v to be equal to 1 until the time when the individual makes his first appearance on the labor market, which may be long before the end of formal schooling. From that time on, it may be assumed to fall from one to zero at T. Of course, the clear relation between earnings and schooling, that may have been one of the attractions of these and the Simple Schooling Models, is then lost. And this has not been the choice in the empirical studies. One reason for

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1) Cf. A.S. Blinder, "On Dogmatism in Human Capital Theory." Journal of Human Resources, 1976,No.1.

2) In the Schooling Models these problems could be avoided. Given the interest rate r, ability would determine the length of the schooling period. And it is also this more easily interpretable "r" that appears in the earnings-schooling relation of the Simple Schooling Models. Moreover, even in the Simple Schooling Models it was unnecessary to directly assume infinite life. The more acceptable assumption about a given length of the working period independent of the schooling period (or a finite planning horizon for approximate results) was enough.

retaining the length of the formal schooling period as an explanatory variable might be that the Simplified Life Cycle Models apparently permit the v to be treated in another way than in the Life Cycle Models in Section 3. In the latter case v represents the share of the individual's capacity (H), while in the former it represents the share of potential earnings. The share of earnings (but not the share of capacity) can exceed 1 in case of direct investment outlays. We can thus accomodate direct investment outlays within the Simplified Life Cycle Models - e.g., by letting the fall of v start from a level higher than 1.Direct investment outlays during the schooling period could also be included by raising the level of v above 1. In the same way, v could be reduced below 1 in case of labor market activities. Separate information about various factors that may raise or lower v compared with some general trend could thus be handled additionally. It may therefore seem appropriate to retain standard explanatory factors, such as the length of the schooling period, and the formulation (4:2) does, in fact, permit such an approach. However, this is not completely problem free, as can be inferred from our discussion of a reformulation of the Simple Schooling Models in terms of money costs or earnings shares.

If we had information about the amount of direct investment outlays during schooling e.g., how much above 1 should we set v? From what level above 1 should we let v start to fall? This we cannot know without an independent measure of potential earnings. Approximate estimations of these could, e.g., be obtained from information regarding earnings at the overtaking point.1) Still they will only be approximations but probably this is better than using earnings of individuals of the same age but out of school.2)

4.4 Introduction of Additional Explanatory Factors

When applying the basic models empirically, many different formulations have been used to adapt the models to the specific group investigated and to

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1) Applying the Simple Schooling Model to earnings observations a number of years (F_{0v} years) after school leaving implies an approximation, i.a. in the sense that only under special circumstances would the period F_{0v} be the same at different schooling levels.

2) P. Wachtel, "The Returns to Investment in Higher Education:Another View," in F.T. Juster (ed.), Education, Income and Human Behavior, New York, 1975, e.g., uses actual wages as a measure of potential earnings to measure the share of direct investment outlays.

increase the explanatory value of the models. A few examples of potential extensions and extensions that have been made are presented below.

Pre-school investments[1)]

Above, the Simplified Life Cycle Models were allowed to start at the beginning of the period of schooling. At that date the individual already possesses a stock of human capital $\bar{H}_{0i}$. This stock can hardly be assumed to be the one the individual was born with. At least in part it must result from investment activities. Considering the way the Simplified Life Cycle Models are constructed, there are at least three possible ways of adding this aspect.

One would be to return to the Basic Model in Section 3 and look for assumptions to use in this connection. It would then be natural to suggest that during the pre-school period, too, v=1. We would then, in line with the first Simplified Life Cycle Model, obtain the following relation to apply empirically

$$(4:14) \qquad \ln Y = n_0 + n_1 (s + AS) + n_2 F + n_3 F^2$$

where AS = age at the beginning of the schooling period.

A second possibility would be to stress the direct investment outlays and work from a formulation

$$Y_{it} = a\bar{H}_{0b} + \bar{r}_i \sum_{j=0}^{s_0-1} C_{ij} - \bar{r}_i \sum_{j=s_0}^{s_1-1} C_{ij} + \bar{r}_i \sum_{j=s_1-1}^{t-1} C_{ij} - C_{it}$$

where H_{0b} = human capital stock at birth
$j = 0,\ldots s_0$ pre-school period
$j = s_0,\ldots s_1$ schooling period
$j = s_1,\ldots t$ post-school period

[1)] Pre-school investments are treated, e.g., by A. Leibowitz, "Home Investments in Children," Journal of Political Economy, 1974, March/April Part II. She uses an approach of type 3b in our classification below. The other possible approaches - more directly integrated with these models - do not seem to have been discussed in the literature. Cf. also Sections 6 and 7.

This approach presents the same difficulties as the direct investment outlays discussed above, since C_{ij} always includes potential earnings.

A third approach, which is less highly integrated with the rest of the models, is to explain variations in the stock of human capital at the beginning of the schooling period using different indicators of pre-school investments. Aside from the capacity of the child, these could be: time devoted by mother, father or other persons to development activities together with the child; money devoted to the same type of activities, etc. As a measure of the stock of human capital at school start - the variable to be explained - one may use, e.g.

a) the estimated $\ln E_0$ i.e. the b_0 for different groups from $\ln Y = b_0 + b_1 s + b_2 F + b_3 F^2$

b) other measures of that stock of human capital, such as IQ, at the beginning of schooling.

Growth[1]

Autonomous growth in earnings could also be introduced into the Simplified Life Cycle Models. In Figure 36 can be seen how the youngest cohort in the cross section for 1965 moves to the right (---) in the successive cross sections (——). This movement - from A to C and from C to E - could be divided into two parts. From A to B (C to D) we have the increase in earnings resulting from normal investments made at that age, and from B to C (D to E) the growth effect.

Let us assume that there is an autonomous growth in earnings and write

$E(\Theta,F)$ = potential earnings for cohort Θ after F years on the labor market.

The cohort Θ could be defined by its age a given calendar year - e.g., 1965. A higher Θ index indicates an older generation. If there is positive growth in earnings, the earnings of the older generation would be lower after a given number of years on the labor market (F) than those of a younger

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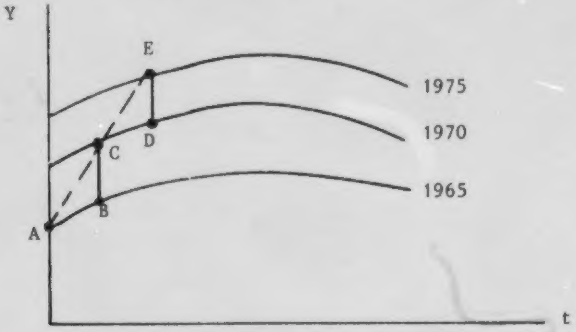
[1] Growth has been treated in the way described below by, among others, T. Johnson, F.J. Hebein, "Investments in Human Capital and Growth in Personal Income 1956-1966." American Economic Review, September,1974.

Figure 36 Cross-sections and longitudinal earnings



generation after the same number of years on the labor market. With a constant rate of growth (g), we thus have

$$E(\Theta,F) = E(0,F)e^{-g\Theta}.$$

Let us also assume that the rate of growth is constant for a given cohort but possibly varying between cohorts, e.g., so that

$$g(\Theta) = g_0 + 2g_1\Theta.$$

Integrating we then have

$$\ln E(\Theta,F) = \ln E(0,F) - g_0^{\Theta} - g_1\Theta^2.$$

With the linear assumption for the decline in v and including depreciation we have

$$\ln Y(\Theta,F) = \ln[E_0\ (1-v_0^x + \frac{v_0^x}{N^x} F)] + (\bar{r}-\delta)\, s + (\bar{r}v_0^x \delta)\, F - \frac{rv_0^x}{2N^x} F^2 - g_0\Theta - g_1\Theta^2$$

which could be written

$$(4{:}15)\quad \ln Y(\Theta,F) = a_0 + a_1\, s + a_2\, F + a_3\, F^2 - a_4\, \Theta - a_5\, \Theta^2$$

If access to several cross sections were available, this relation could be used to estimate the structural parameters.

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Variations in labor force participation[1]

Individuals do not always work full time or use all their time for work and training. They may even interruot their labor market career completely in order to pursue other activities during certain time periods. Could we still apply the Simplified Life Cycle Models in such situations? Let us discuss some solutions that have been proposed and used.

In the events of a complete interruption of labor market activity, the simplest assumption to make concerning the development of v would be that it develops as in the Basic Models during the periods of labor market participation and is otherwise zero.

The expression (4:2) $\ln Y_t \approx \ln E_0 + \bar{r}\, s + \bar{r} \sum_{j=s}^{t-1} v_j + \ln (1-v_t)$

can then be split up as follows if we assume that date t is one with labor force participation

$$\ln Y_t \approx \ln E_0 + \bar{r}\, s + \bar{r} \sum_{j=0}^{X_t-1} v_j + \bar{r} \sum_{j=0}^{NX_t} v_j + \ln (1-v_t)$$

where X = periods of labor market participation

NX = periods of non-participation on the labor market

$X_t + NX_t = t - s$.

Let us now adapt the linear assumption for v, so that v is assumed to equal zero except during the periods of labor market participation when it behaves according to

$$v_j = v_0 - \frac{v_0}{X}\, j \qquad j = 0,\ldots X$$

We then have

$$\ln Y_t \approx \ln E_0 + \bar{r}\, s + \bar{r}\, v_0\, j - \frac{\bar{r} v_0}{2X}\, j^2 + \ln(1-v_0 + \frac{v_0}{X}\, j).$$

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1) Labor market interruptions have been treated as describe below by:
J. Mincer, S. Polacheck, "Family Investments in Human Capital Earnings of Women." Journal of Political Economy, 1974, March/April Part II.
S. Gustafsson,"Rates of depreciation of human capital due to nonuse." Industrins utredningsinstitut. Working Paper No. 14, 1977.
Cf. also the discussion in Journal of Human Resources, vol XIII,No.1. "An Exchange: The Theory of Human Capital and the Earnings of Women,"
S. Sandell, D. Shapiro, "A Reexamination of the Evidence."
J. Mincer, S. Polachek,"Women's Earnings Reexamined."

As before, the last term can be approximated by

$$- v_0 [1 + \frac{v_0}{2}] + \frac{v_0}{X} (1 + v_0) j - \frac{v_0^2}{2X^2} j^2 .$$

We could then write

$$(4:16) \quad \ln Y_t \approx f_0 + f_1 s + f_2 X + f_3 X^2$$

where $f_0 = \ln E_0 - v_0 (1 + \frac{v_0}{2})$

$f_1 = \bar{r}$

$f_2 = \bar{r} v_0 + \frac{v_0}{X} (1 + v_0)$

$f_3 = -(\frac{\bar{r} v_0}{2X} + \frac{v_0^2}{2X^2})$

If a constant rate of depreciation for all dates including those for periods of absence from the labor market is applied we would have

$$\ln Y_t \approx \ln E_0 + \sum_{j=0}^{t-1} (\bar{r} v_j^x - \delta) + \ln(1 - v_j^x)$$

which can be split up in

$$\ln Y_t \approx \ln E_0 + (\bar{r} - \delta) s + \sum_{j=0}^{X-1} (\bar{r} v_j^x - \delta) + \sum_{j=0}^{NX} (\bar{r} v_j^x - \delta) + \ln(1 - v_j^x).$$

Assuming again that $v_j^x = v_0^x - \frac{v_0^x}{X} j$ during labor market participation and that is otherwise zero, we have

$$\ln Y_t \approx \ln E_0 + (\bar{r} - \delta) s + (\bar{r}\,\text{[illegible]}_0^x - \delta) j - \frac{\bar{r} v_0^x}{2X} j^2 + \delta NX + \ln(1 - v_t^x).$$

Using the same type of approximation as above we can write

$$(4:17) \quad \ln Y_t = g_0 + g_1 s + g_2 X + g_3 X^2 + g_4 NX$$

where $g_0 = \ln E_0 - v_0^x (1 + \frac{v_0}{2})$

$g_1 = \bar{r} - \delta$

$g_2 = \bar{r} v_0^x + \frac{v_0^x}{X} (1 - v_0^x)$

$g_3 = -(\frac{\bar{r} v_0^x}{2X} + \frac{v_0^{x2}}{2X^2})$

$g_4 = \delta.$

Thus, we can interpret g_4 as being the rate of depreciation. From estimates of g_1 and g_4, $\bar{r}$ and δ could be identified.

More specific assumptions concerning the development of the stock of human capital during the years away from the labor market could be introduced, of course. Human capital could, e.g., be made to depend on the type of activities of the individual undertakes during those years.

Regarding individuals whose pattern of their labor market interruptions is more regular - e.g., women staying at home with their first or second child - one may, as an alternative, assume that each segment of labor market activity has its own profile of investments. In the simplest case, v may be assumed to be constant during each segment but different from one segment to another.

If we assume that there are n segments of varying length (e_i years) of labor market participation, that post-school human capital investments are only undertaken during these segments, that there is no depreciation of human capital, and that during each segment v is constant, then we have approximately

$$\ln Y_t = \ln E_0 - \bar{r}\, s + \bar{r} \sum_{i=1}^{n} v_i\, e_i + \ln(1 - v_t)$$

which can be written

$$(4{:}18)\quad \ln Y_t = h_0 + h_1\, s + \Sigma h_i\, e_i$$

where $h_0 = \ln E_0 + \ln(1-v_t)$
$h_1 = \bar{r}$
$h_i = \bar{r}\, v_i \qquad i = 1.....n$.

Or, exemplifying with three periods

$$\ln Y_t \approx e_0 + e_1\, s + e_2\, M_1 + e_3\, M_2 + e_4\, M_3$$

where M_i = length of the respective labor market segment.

We are then to interpret e_2, e_3 and e_4 as $\bar{r} \cdot v$, i.e., the average return times the level of v for the respective segment.

One might also want to treat intermediate cases of variations in labor force participation. When the individual works less than full-time, the "part-time" potential earnings (E_t^x) must be related in some way to full-time potential earnings. Thus

$$E_t^x = f(E_t \cdot w_t)$$

where w_t = fraction of period worked.

Such considerations have led to the use of the following type of relation for empirical work

$$\ln Y_F = a_0 + a_1 s + a_2 F + a_3 F^2 + a_4 \ln W_F$$

where W stands for weeks worked during the year (Y_F being annual earnings) and a_4 is intepreted as the elasticity of earnings with respect to weeks worked.

If we go back to (4:2) and answer the following questions affirmatively - could C_j still be written $C_j = v_j E_j$ rather than $C_j = v_j E_j^x$; could v_F still be written $v_F = v_0 - (v_0/N)F$; could Y_t^x = actual earnings with variable amount of labor market participation be written $Y_t^x = E_t(1-v_t)h_t$ where h_t = fraction of capacity actually used - then we would arrive at

$$\ln Y_F = b_0 + b_1 s + b_2 F + b_3 F^2 + \ln h_F.$$

The only problem in relation to the expression above would then be to reconcile h_F with W_F and add that we would expect a_4 to equal one. But why make these ad hoc assumptions? If we should reformulate our theory to include such a variable as weeks worked, what would happen? It is not certain that we obtain implications that show weeks worked to be an explanatory variable or, if it is, that its effect is of the form of the expressions above. Moreover, it is not certain that the coefficients a_1-a_3 should be interpreted as before.1) If - as will be further discussed in the next section - hours worked or share of the human capital stock available for work (n) is variable, actual earnings are

$$Y = [a(1-v) H] n.$$

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1) A similar discussion of weeks worked is found in the following sources:
J. Mincer, *op.cit.*, 1974.
A. Schwartz, D.O. Parsons, "Time-series Changes in Personal Income Inequality in the United States from 1939 with Projections to 1985: A Comment." *Journal of Political Economy*, 1974, No. 5.
B. Chiswick, J. Mincer, "Time-series Changes in Personal Income Inequality in the United States from 1939 with Projections to 1985:Reply." *Journal of Political Economy*, 1974, No.5.
A.S. Blinder, *op.cit.*

Cf. also the rest of the discussion in *Journal of Human Resources*, 1976, No.1. "An Exchange: On the Appropriate Specifications of Human Capital Models,"
M.R. Rosenzweig, J.Morgan,"Wage Discrimination:A Comment."
M.R. Rosenzweig, "Nonlinear Earnings Functions, Age and Experience: A Non-dogmatic Reply and Some Additional Evidence."

And we could, e.g., have

$$(1-v)H = f_H\ (Z)$$

$$n = f_n\ (Z',vH)$$

where Z and Z' are sets of explanatory variables. We then have

$$Y = a\ f_H\ (Z) \cdot f_n\ (Z',vH).$$

Thus, n does not appear at all as an explanatory variable. Moreover, if one explanatory variable (e.g., Z_i) belongs to both Z and Z' its marginal effect on earnings is

$$\frac{\partial Y}{\partial Z_i} = a\ \frac{\partial f_H(Z)}{\partial Z_i}\ f_n\ (Z',vH) + af_H(Z)\ \frac{\partial f_n(Z',vH)}{\partial Z_i}$$

and its effect on earnings is no longer only via f_H.

Other Variables [1]

Many people who have used the formula $\ln Y_F = b_0 + b_1 s + b_2 F + b_3 F^2$ empirically have added further explanatory factors, such as socioeconomic background, number of siblings, direct investment outlays, ability, school quality, sex, civil status, nativity, religion, region, health, industry, and occupation.

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1) "Other variables" have been added by many authors, including
Z. Griliches, W.M. Mason, "Education, Income and Ability." Journal of Political Economy, 1972, No. 3 part II.
J.C. Hause, "Earnings Profile: Ability and Schooling." Journal of Political Economy, 1972, No.3 part II.
J.C. Hause, "Ability and Schooling as Determinants of Lifetime Earnings, or If You're So Smart, Why Aren't You Rich " in F.T. Juster (ed.) op.cit.
G.E. Johnson, F.P. Stafford, "Lifetime Earnings in a Professional Labor Market: Academic Economists." Journal of Political Economy, 1974, No.3.
G.E. Johnson, F.P. Stafford, "Social Returns to Quantity and Quality of Schooling." Journal of Human Resources, Spring 1973.
A. Klevmarken, J.M. Quigley,"Age, Experience, Earnings and Investment in Human Capital." Journal of Political Economy, 1976, No.1.
E. Lazear, "Age, Experience and Wage Growth." American Economic Review, 1976, No. 4.
A. Leibowitz, op.cit.
L.C. Solmon, "Schooling and Subsequent Success: Influence of Ability, Background, and Formal Education." ACT Research Report, No.57, 1973, IOWA.
P. Wachtel, op.cit.

Problems similar to those noted earlier also arise with the inclusion of these variables. What theoretical concepts should they correspond to, and how do they influence the behavior of the individual? How should the estimated effect of other variables be interpreted when the new ones are entered?

Different ways of including such variables have been tried empirically:

- entering the variables as continuous variables or dummy variables;
- permitting different types of interactions among these variables and the original ones;
- using different functional forms for entering the variables.

The more the need is felt to amend the model in this way the more natural it may seem to try any relation between earnings and the explanatory variables (e.g., between Y and not ln Y and such variables as mentioned above). The question may then become: what is the most efficient way of describing data?[1] Such work, of course, does not necessarily result in direct confirmation or rejection of hypotheses about the mechanisms that determine earnings, but it may do so indirectly.

4.5 Special Features of the Simplified Life Cycle Models

The Simplified Life Cycle Models explain individual actual earnings by the accumulated human capital investments as indicated primarily by the length of the schooling period and the labor market experience. The exact relation between earnings and these indicators depends on what more precisely is assumed about how considerable an investment these indicators represent. In the text we explained the effect of using different sets of assumptions for the investments related to the period before, during and after schooling, with and without an uninterrupted working period, etc.

In a certain sense these models can be viewed as elaborating an identity: realized capacity = the sum of capacities realized. The problem arises when an attempt is made to see them as deriving from a common theoretical framework with equilibrium and rational individual investment behavior. Why, in such a theoretical setting, should we try to obtain, as theoretical implications, relations between variables such as earnings, schooling and post-school investments , all endogenous variables? However, as we saw with the Simple Schooling Models, while this was not impossible, it might only be possible if less satisfactory assumptions were used. With the Simplified Life Cycle Models this situation is aggravated. This has especially to do with ability variations and how a constant rate of return should be combined with a rational investment behavior resulting in declining individual investments over a finite time period. We have not found any theoretically acceptable solution to this problem. Therefore, these models can at most be regarded as "approximately" belonging to the same theoretical family as the previous ones.

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[1] Cf. J. Heckman, S. Polachek, op.cit.
J.D. Welland, op.cit.

In the alternative formulations discussed in Section 4.4, the addition of further "endogenous" variables (e.g., weeks worked) as well as variables of a more exogenous nature (e.g., ability) was discussed. As regards theory, such additions create a rather confused situation.

Empirical work on the relations between earnings and explanatory factors (as discussed in Sections 2 and 3, and to be discussed in Sections 5, 6, and 7) as well as on the relations between earnings capacity and different indicators of the human capital stock (e.g., length of schooling) is very important. However, the latter type of work had better be motivated not as the testing of implications from models with optimizing individual behavior but, e.g., as an investigation of the possibilities constructing indices of the stock of human capital than can explain the earning capacity.1) However, in this perspective, there is no reason to limit the range of possibilities to the kind of relationships that emerge from the Simplified Life Cycle Models.

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1) Cf. Appendix A.

5. Life Cycle Models Including Leisure Activities

After the digression in Section 4, we will now extend the models of Section 3 by introducing and discussing the effects of more complicated preference functions. In Section 3 it was assumed that leisure activities had no value of their own and to find the optimal position it was sufficient for the individual to decide upon his labor market participation and human capital creation so as to maximize earnings. However, it is generally accepted that leisure activities are valuable to the individual and that, moreover, knowledge, skills and "human capital" can play an important role not only for productivity at work, but also for what the individual gets out of his leisure time. Therefore, it is no doubt important to investigate the possibilities to include these aspects in human capital models. With more elaborate preference functions the models will permit consideration to be given to consumption as well as to investment aspects of human capital and education.

Furthermore, as pointed out in Section 4, it would be more satisfactory to treat the choice between work and leisure inside the model and relate it to more basic explanatory variables. And we are interested in what happens to the conclusions presented in Section 3 regarding investment behavior - e.g., the declining share of capacity devoted to human capital production.

A certain amount of theoretical work has also recently been undertaken in this field. However, as always with generalizations, one risks losing precision in results. In this section, as in Section 3, where we experimented with different production functions, divergent implications will emerge when we discuss different preference functions. These are, of course, interesting to document. We shall concentrate on the more general aspects concerning the effects of introducing more complex preference functions. Throughout this section we use the same production function for human capital, we do not, e.g., investigate all possible combinations of production and preference functions and individual variations in the now rapidly increasing number of parameters. Probably the most fruitful research strategy would be a stepwise one, where empirical and theoretical work concerning preference and production function could lead successively to more precisely defined models.

In previous models the capacity of the individual (his human capital stock) could only be devoted to two types of activities, work and human capital production.1) Now we introduce a third type of activity: a consumption or leisure activity. We shall use the following notation:

v = share of individual capacity (H) devoted to human capital production,
l = share of individual capacity (H) devoted to labor activities,
c = share of individual capacity (H) devoted to consumption or leisure activities of different kinds,

with $v + l + c = 1$.

What difference does it make when human capital can be used in different activities? What will happen to our predictions about individual investment behavior? The answer to these questions depends on how the leisure activities are introduced and in what kind of a model they are analyzed. As will be shown, there are certain circumstances under which the predictions of the Basic Model of Section 3 will still hold.

The general approach used in all the models of this section is the same as that used by G.S. Becker (1975) and G.R. Ghez, G.S. Becker (1975),2) i.e. we use discrete time models with a composite preference function and investigate an interior solution.3) Specialization is discussed separately in Section 5.2.

5.1 Different Preference Functions

Before going into details, a short summary of the preference functions we are going to use is called for. Individuals are assumed to make plans for T periods denoted period $0, \ldots, T-1$. As regards the preference function, an initial choice has to be made as to which arguments should be included. Different possibilities are open in this respect and we have chosen to work with the following ones:

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1) At least these were the only two types of activities where the individual could make a choice.

2) G.S. Becker, op.cit., 1975.
G.R. Ghez, G.S. Becker, op.cit.

3) In the first model we, however, indicate the relation of this solution to a complete solution.

1. The individual maximizes a preference funtion U with one "commodity" (C) for each period as arguments. We write commodity within brackets, as this is not an ordinary (composite) good - but rather a composite result of different activities. It could be result of "having meals" and"the pleasure of driving cars"and of "visiting museums", for example. The activities combine a market good (x) with the leisure time of the individual (c).

2. As an alternative in the composite utility function we may assume that the effficiency of the individual in the various consumption activities is influenced by his stock of human capial (H).

3. However, direct satisfaction of individual wants is not necessarily restricted to leisure activities. It might also be assumed that the capacity spent on labor activities is considered valuable not only as providing the individual with earnings with which to buy the market good (x), but also as providing indivudal "satisfaction from work' (W). The level of the human capital stock could also be assumed to influence the production of this "commodity".

4. Finally, we add the possibility that the capacity spent in human capital production activities also provides direct satisfaction - "satisfaction from education" (E). This would correspond to the consumtion effects of education often reckoned with in the economics of education.

We shall describe below how the behavior of the individual would look with these different approaches to the preference function. First we discuss a non-specialization phase when neither c, v nor l equals one. Specialization is analyzed later. In order to concentrate on the alternatives opened up by the addition of consumption activities, one and the same production function for human capital will be used throughout. It is of the same type as that of the Basic Model of Section 3.(1)

5.1.1 Type 1 Preference Funtion – Leisure Welfare Produced Independently of the Stock of Human Capital

As indicated above, the first preference function can be written

$$U[\{C_i\}]$$

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1) $Q = \Psi(v \cdot H)$

where $\{C_i\}$ denotes the finite sequence $(C_0,\ldots,C_{T-1})$ and each C_i is produced according to [1]

$$C_i = f_i(x_i, c_i).$$

Our first model with preference function type 1 will have the same preference function but another human capital production function compared with the corresponding models in G.S. Becker (1975) and G.R. Ghez, G.S. Becker (1975)[2]. They concentrate on a production function without the stock of human capital as an argument, i.e.,of the type $[\psi(v)]$ which was discussed in Section 3. The same type of preference and human capital production function as used here is found in F.P. Stafford and P.E. Stephan (1973) and H.E. Ryder, F.P. Stafford and P.E. Stephan (1976)[3]. However, they formulate their analysis as a control-theoretic problem and relate their results to a more specifically designed preference function.

Let us add that leisure models have also been used in combination with schooling models. D.O. Parson (1974) and J. Schaafsma (1976)[4] use a preference function of type 1, while E. Lazear (1977)[5] uses a preference function where education is a direct argument. The latter is in line with

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[1] A more general formulation of this and the other preference functions would be

$$G(\{x_i\},\{v_i\},\{c_i\},\{l_i\},\{H_i\})$$

where { } represent the finitie sequence of the respective variables.

It does not impose the separability condition on the welfare "commodities" C_i, W_i and E_i. Still, we think it appropriate to start with the other kind of analysis that emphasizes the special relation between capacity and goods used at the same time, i.a. to keep in touch with empirical work on activity analysis. Further generalizations could be developed later.

[2] G.S. Becker, op.cit., 1975.
G.R. Ghez, G.S. Becker, op.cit.

[3] F.P. Stafford, P.E. Stephan, "Labor, Leisure and Training over the Life Cycle." Working Paper no. 7374-14, University of Michigan, 1973.
H.E. Ryder, F.P. Stafford, P.E. Stephan, "Labor, Leisure and Training over the Life Cycle." International Economic Review, 1976, No. 3.

[4] D.O. Parsons,"The Cost of School Time, Foregone Earnings and Human Capital Formation." Journal of Political Economy, 1974, No. 2.
J. Schaafsma, "The Consumption and Investment Aspects of the Demand for Education." Journal of Human Resources, 1976, No.2.
Cf. also W.S. Reece, "The Consumption and Investment Aspects of the Demand for Education: Comment." Journal of Human Resources, 1978, No. 3.

[5] E. Lazear, "Education: Consumption or Production?" Journal of Political Economy, 1977, No.3.

with preference function type 4, to be discussed below. V. Stoikov (1977)[1] lets schooling influence the (dis)utility of working.

There are additional assumptions that are common to all models including leisure. Before continuing we shall therefore enumerate them here:

- each composite preference function U will be assumed to be concave and continuous with positive derivatives for all argument of the respective U;

- a perfect capital market with a constant rate of interest (r) is assumed;

- the consumption commodity is produced by the individual according to a production function [f(.....)] that remains identical in each period. We assume it to be homogeneous of degree one and marginal products declining;

- the initial stock of human capital equals $\bar{H}_0$;

- the production function for human capital is assumed to be of the same type as in the Basic Model of Section 3, i.e., production of human capital in period i equals $Q_i = \psi(v_i \cdot H_i)$. More generally it can therefore be written $Q_i = h_i(v_0,\dots,v_i)$. It is assumed to be homogeneous of degree less than one and marginal products declining;

- human capital is not assumed to depreciate. We therefore have $H_{i+1} = H_i + Q_i$ and more generally $H_i = H_i(v_0,\dots,v_{i-1})$;

- the rental price of human capital - a - is the same in each period and is independent of the individual's stock of human capital;

- the price of the market good is assumed to be the same in each period and is set equal to 1;

- no other assets than human capital are considered.

Our first model can then be written:

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[1] V. Stoikov, "On some Models of the Educational Decision." Kyklos,1977,No.1.

(5:1) $\text{Max } U[\{C_i\}] \qquad \{C_i\} = (C_0,\dots,C_{T-1})$

subject to

(5:2) $C_i = f(x_i,\ c_i) \qquad i = 0,\dots,T-1$

(5:3) $v_i + l_i + c_i = 1 \qquad i = 0,\dots,T-1$

(5:4) $H_{i+1} = H_i + Q_i \qquad i = 0,\dots,T-1$

(5:5) $H_0 = \bar{H}_0 > 0$

(5:6) $Q_i = \Psi(v_i \cdot H_i) \qquad i = 0,\dots,T-1$

(5:7) $$\sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} = \sum_{i=0}^{T-1} \frac{a\,H_i l_i}{(1+r)^i}$$

The last restriction (5:7) assures that total outlays on the market good equal total earnings.

The necessary first order conditions for an interior solution to this problem will now be investigated. Let us note than this solution will be consistent with the solution for a planning period including specialization.[1)]

[1)] The relation of this and the other models investigated for a non-specialization phase to a model for the total planning period could be described in the following way:

Let us assume that the non-specialization phase consists of the periods n to n+m. Then

- a) call the period n period 0 and H_n^x (in the solution to the bigger problem) H_0
- b) call period $n+m+1$ for T and H_{n+m+1}^x for $\bar{H}_T$

The problem for the non-specialization period would then be

$$\text{Max } U[\{C_i\}] \qquad i = 0,\dots,T-1$$

subject to $C_i = f(x_i, c_i)$

$$v_i + l_i + c_i = 1$$

$$H_{i+1} = H_i + \Psi(v_i, H_i)$$

$$H_0 = \bar{H}_0 > 0$$

$$H_T = \bar{H}_T$$

$$\sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} = \sum_{i=0}^{T-1} \frac{a\,H_i l_i}{(1+r)^i} + Y$$

where Y = a lump sum corresponding to the surplus (or deficit) resulting from the rest of the periods in the solution to the bigger problem. These conditions will produce the same results as those of our models.

If we substitute and use $l_i = 1 - c_i - v_i$ and $H_i = H_i(v_0,\dots,v_{i-1})$ the Lagrangian function could be written

$$(5{:}8)\quad L = U[\{f(x_i,c_i)\}] - \lambda\{\sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} - \sum_{i=0}^{T-1} \frac{a[H_i(v_0,\dots,v_{i-1})](1-c_i-v_i)}{(1+r)^i}\}$$

and we have

$$(5{:}9)\quad \frac{\partial L}{\partial x_i} = \frac{\partial U}{\partial C_i}\,\frac{\partial f}{\partial x_i} - \lambda \frac{1}{(1+r)^i} = 0$$

$$(5{:}10)\quad \frac{\partial L}{\partial c_i} = \frac{\partial U}{\partial C_i}\,\frac{\partial f}{\partial c_i} - \lambda a\, H_i \frac{1}{(1+r)^i} = 0$$

$$(5{:}11)\quad \frac{\partial L}{\partial v_i} = -\lambda \frac{a\ H_i}{(1+r)^i} + \lambda \sum_{j=i+1}^{T-1} \frac{(1-c_j-v_j)a}{(1+r)^j}\,\frac{\partial H_j}{\partial v_i} = 0$$

From (5:9) we have

$$(5{:}12)\quad \frac{\dfrac{\partial U}{\partial C_i}}{\dfrac{\partial U}{\partial C_j}} = \frac{\dfrac{\partial f}{\partial x_j}}{\dfrac{\partial f}{\partial x_i}}(1+r)^{j-i}\ .$$

Dividing (5:9) by (5:10) and rearringing (5:11), the first order optimality conditions can the be written

$$(5{:}13)\quad \frac{\partial f}{\partial c_i}\Big/\frac{\partial f}{\partial x_i} = aH_i$$

$$(5{:}14)\quad \frac{aH_i}{(1+r)^i} = \sum_{j=i+1}^{T-1} \frac{l_j}{(1+r)^j}\ a\frac{\partial H_j}{\partial v_i}$$

or

$$(5{:}14')\quad a\ H_i = \sum_{j=i+1}^{T-1} \frac{l_j}{(1+r)^j}\ a\frac{\partial H_j}{\partial v_i}$$

with $i=0$

Let us now study how the leisure activities develop over time and condition (5:13) that regulates the efficient utilization of c_i and x_i in a given period consistent with the existent stock of human capital (H_i). The system is such that leisure activities can be studied conditional on the development of potential earnings, and human capital acitivites conditional on the leisure [c(t)] trajectory. In this sense the system is separable.

(5:13) implies that the marginal product of c_i should be relatively low when potential earnings (aH_i) are low, and relatively high when they are high. The production of the "commodity" should be capacity-intensive when earnings are low and goods-intensive when earnings are high. This is so because it is more profitable to devote human capacity to labor market activities when potential earnings are high.

Remembering that f( ) is homogeneous of degree one with the ratio of marginal products only depending on x/c, and also that marginal products are declining, (5:13) implies, if $a\,H_j > a\,H_i$, that

$$\frac{x_j}{c_j} > \frac{x_i}{c_i} \text{ and } \frac{\partial f}{\partial x_j} < \frac{\partial f}{\partial x_i} .$$

Also, remembering the assumptions on U and (5:12), we have

$$(5{:}15)\ \frac{\frac{\partial U}{\partial C_i}}{\frac{\partial U}{\partial C_j}} \overset{<}{\underset{>}{=}} (1+r)^{j-i} \qquad a\,H_j \overset{>}{\underset{<}{=}} a\,H_i .$$

Let us now assume, that U is homothetic in the C_i's. Then, if $\frac{\partial U}{\partial C_i} = \frac{\partial U}{\partial C_j}$ whenever $C_i = C_j$ (an assumption of neutral time preference) and if $r = 0$, according to (5:15) C_i would fall if earnings increased over time.[1] The higher the elasticity of substitution between different C_i's the more accentuated the fall in C. With $r>0$ and small, the decline would be smaller. For larger r, C would be expected to first fall and later to increase, and for sufficiently large r, to rise throughout.[2]

The absolute amount of "commodity" production is related to the development of potential earnings. When prospects are good for earning money, one does not produce so many commodities for oneself but transfers such activities to other periods. However, with a positive rate of interest on capital markets the incentive to shift "commodity" production from peak earning periods would be smaller.

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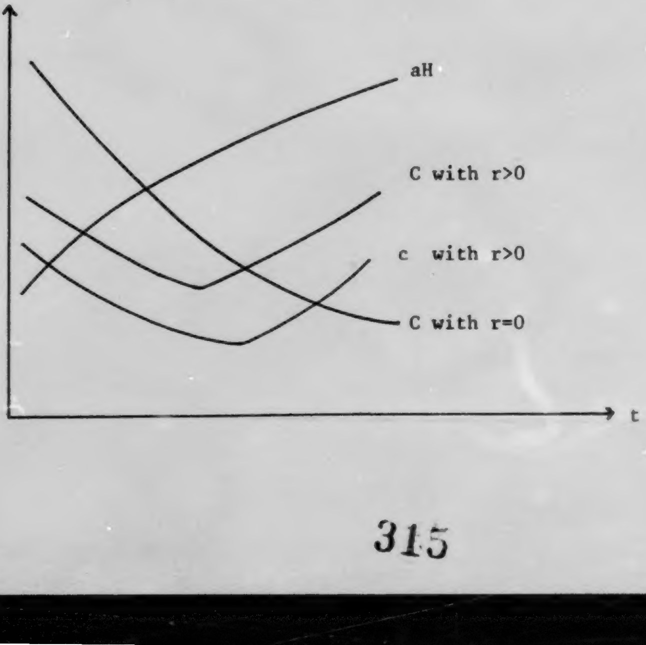
1) Earnings (aH_i) cannot fall in this model without depreciation. Note, however, that with falling earnings C_i would rise.

2) Time preference for the future has the same type of effect as a positive interest rate, and time preference for the present the opposite effect: cf. G.R. Ghez, G.S. Becker, op.cit., 1975, p.16.

The ratio c_i/x_i would fall as earnings increased. The higher the elasticity of substitution between c_i and x_i in commodity production the more rapid would this fall be. When C_i declines, c_i would also fall, to eventually rise as C_i rises. However, the turning point for c_i would in these cases come later than for C_i. If C_i is unchanged and aH increases, then c/x falls and so does c. These developments are illustrated in Figure 37. The corresponding implication for the share of capacity devoted to work and human capital production $(1+v)$ is that it would first rise and later fall. This variable $(1+v)$ could be identified operationally as time spent on the labor market (work and on-the-job training) plus time devoted to educational activities. Time devoted to these activities could be expected to fall later in life and might be inversely U-shaped.

Now, to the condition (5:14') that regulates the optimal production of human capital - the efficiency of capacity use over time. (5:14') is another version of the general result that in each period the human capital production should be pursued up to the fraction of human capital at which marginal returns equal marginal expense. In this particular case the expenses consist only of the earnings the individual foregoes as the amount of human capital used in production is increased. These earnings could otherwise be used to buy x, which could be used in producing satisfaction - i.e., to produce more "leisure."

Figure 37 The development of earnings and leisure



As for the returns, with this model the value of human capital is ultimately that it provides for earnings to buy x. Human capital has no value in terms of consumption. If the individual invests more he will have a larger stock of human capital in the future. For given l_j's he can then earn more money and buy more of x. This means that the future benefits depend on l in the respective period. But human capital also has an effect in human capital production that later - again depending on l - turns out in earnings.

To clarify what happens to marginal costs and benefits in the present model, let us compare it with the Basic Life Cycle Model in Section 3 where condition (5:14') corresponds to the expression

$$a\,H = \frac{1}{r}\left(1 - e^{r(t-T)}\right) a\,\frac{\partial \dot{H}(t)}{\partial v(t)} \quad \text{or}$$

$$a/\frac{\partial \dot{H}}{\partial (vH)} = \frac{a}{r}\left(1-e^{r(t-T)}\right).$$

In the Basic Life Cycle Model it was not necessary to take the future disposition of the human capital stock into consideration when deciding on present optimal investments. Future marginal costs were not influenced by the present human capital accumulation. In fact, they were independent of the level of the stock of human capital. Marginal returns fell over time, but independently of the human capital activities.

With the present leisure model the situation is not that simple. According to (5:14') the future disposition of the human capital stock has to be taken into consideration.[1] This is so for two reasons. First of all, the RHS of (5:14') is summed over the future l_j's. Secondly, the effect of an increase in v_i on the H_j's is not independent of the values of $v_{i+1},\ldots,v_{j-1}$ as we have assumed $\Delta H_j = Q_j = \Psi(v_j \cdot H_j)$.[2]

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[1] A similar situation arose in the models of Section 3 where the MC curve was not independent of the level of H.

[2] Illustrating by $Q_j = v_j H_j$ we have

$$H_j = H_0 \prod_{\tau=0}^{j-1} (1 + v_\tau) \quad \text{and} \quad \frac{\partial H_j}{\partial v_i} = H_i \prod_{\substack{\tau=0\\ \tau\neq i}}^{j-1} (1 + v_\tau) \; .$$

If instead of assuming $H_j = H_j(v_0, \ldots., v_{j-1})$, we had assumed that

$$H_j = A \cdot (v_i + \ldots.. + v_{j-1})$$

where A is a constant,
we would have had

$$\frac{\partial H_j}{\partial v_i} = A$$

and thus independent of future v's. [1] In this case we can say unequivocably that an increasing amount of human capital stock reduces investment incentives since it increases costs (aH) and these costs are not offset by increased efficiency in human capital production. Optimality condition (5:14') turns into

$$aH_i = \sum_{j=i+1}^{T-1} \frac{l_j}{(1+r)^i} aA$$

where H remains on the left-hand (cost) side but disappears on the right-hand (benefit) side. However, we would still have to consider the development of the l_j's for marginal returns.

For our model (5:14') means that the partial effect of increases in H on investment incentives is not unambiguous. A higher level of H_i means greater costs, but also greater returns, the latter depending, i.a., on the level of the future v's. The future v's are only decided simultaneously with the c's and l's.

As we are only considering interior solutions, we have $v_i > 0$. Human capital production will thus be positive.[2] And with no depreciation aH will therefore always increase with the effects on c, as discussed above. With r

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[1] This is a special case of the production function $Q_i = \Psi(v_i)$, used by G.S. Becker, op.cit., 1975 and G.R. Ghez, G.S. Becker, op.cit.

[2] A sufficient condition for this is

$$\lim_{v_i \to 0} \sum_{\substack{j=i+1 \\ i=0}}^{T-1} \frac{1-c_j-v_j}{(1+r)^i} \; a \frac{\partial H_j}{\partial v_i} > aH_i$$

which is the case with the production function used here as

$$\lim_{v_i \to 0} \frac{\partial \Psi(H_i v_i)}{\partial v_i} = \infty$$

and aH_i is always positive and finite.

positive and not too big c would first fall and later rise. The opposite would then be true for (1+v).

What happens to v and 1? As T approaches there will be a stronger and stronger tendency for v to fall, This would be due, e.g., to the shortening of the recoupment period and increased marginal costs for using v. If this occurs while c is rising, 1 may decline. A reduction in 1 further reduces the investment incentives. However, while c is falling, 1 must increase, and a rising 1 means increasing investment incentives. Therefore, v does not necessarily have to fall all the time.

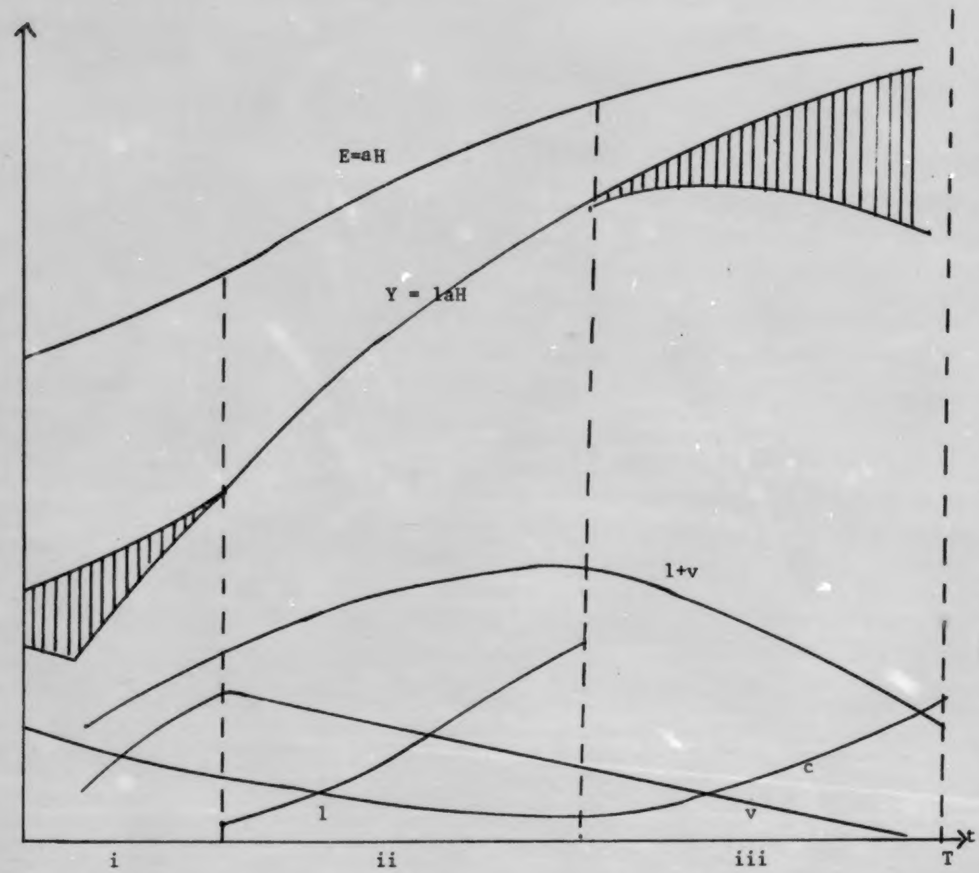
Some of these possibilities are illustrated in Figure 38. The period considered here corresponds to phase 2 of the Basic Model in Section 3. Potential earnings (E=aH) increase. As long as the amount of newly produced human capital falls, potential earnings increase at a decreasing rate. However, as we have just noted with this model, v might increase at least at an early stage. During such a period potential earnings would grow at an increasing rate. Contrary to the Basic Model of Section 3, E could thus grow at an increasing rate. The corresponding actual earnings may also rise exponentially, though even a fall in actual earnings is not excluded. With a constant or increasing 1, actual earnings would grow exponentially.

Subphase i and ii are both characterized by a declining share of capacity being devoted to consumption activities. During subphase ii this coincides with a falling v and thus increasing 1. At least for such a period the development of actual earnings (Y = 1aH) is clear. They increase for two reasons: the earning capacity grows and more of this capacity is devoted to income earning. With respect to the development of E, Y, 1 and v, subphase ii resembles the Basic Model of Section 3. However, as 1 increases not only due to the reduction in v, but also in c, the rise in Y would be more rapid - given the development of v.

During subphase iii, v is shown to continue to decline, and c to rise. Depending on the relative force in the decline in v and in the rise in c, 1 may grow or decline. Actual earnings may continue to grow or decline. A decline in actual earnings can thus occur without depreciation.

Let us note a few differences between these observations and the result of the Basic Model in Section 3:

Figure 38 Possible developments for earnings and capacity utilization in a model including leisure according to preference function 1



- leisure activities (c) could be expected first to fall and then to rise, and therefore labor market plus human capital activities (l+v) would be expected to have an inverse U-shaped form;

- human capital activities (v) do not necessarily decline during the entire period of non-specialization, but such a tendency will make itself felt later in life;

- work activities (l) do not necessarily rise all the time during non-specialization, they may, e.g., also be inversely U-shaped;

- potential earnings do not necessarily grow at a declining rate;
- actual earnings may fall without assuming depreciation.

The differences in v and l compared with the Basic Model of Section 3 - i.e. that they do not unambiguously fall and rise respectively - is due to the fact that it is not optimal to keep leisure activities constant during the life cycle.

5.1.2 Type 2 Preference Funtion – Leisure Welfare the Production of which Depends on the Stock of Human Capital

Here human capital has a value both in providing earnings with which to buy the market good and in making the production of the "commodity" C more efficient. Thus, the human capital stock has a value in all three possible types of activities (c, l and v). Formally we would now write:

$$U[\{C_i\}]$$

$$C_i = f_i(x_i, c_i, H_i).$$

With regard to these preference and commodity production functions, we have adopted an approach similar to the ones also found in G.S. Becker (1975), G.R. Ghez, G.S. Becker (1975) and J.J. Heckman (1976)[1]. The first two writers use $C_i = f(x_i, c_i; H_i)$, and the third $C_i = f(x_i, c_i \cdot H_i)$, both of which are special cases of the one used here. Their respective production functions for human capital are: $Q = g_1(v)$, $Q = g_2(v,H)$ and $Q = g_3(v \cdot H)$.

For a complete model we now have
Max (5:1)
subject to
(5:16) $C_i = f(x_i, c_i, H_i)$
(5:3)
(5:4)
(5:5)
(5:6)
(5:7)
which gives

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[1] G.S. Becker, op.cit., 1975.
G.R. Ghez, G.S. Becker, op.cit., 1975, pp 29-31.
J.J. Heckman, "A Life-cycle Model of Earnings, Learning and Consumption." Journal of Political Economy, 1976, No.4 II.

$$(5{:}17) \quad L = U[\{f(x_i, c_i, H_i)] - \lambda\{ \sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} - \sum_{i=0}^{T-1} \frac{a[H_i(v_0,\ldots\ldots v_{i-1}](1-c_i-v_i)}{(1+r)^i} \}$$

with the necessary conditions for a maximum:

$$(5{:}18) \quad \frac{\partial L}{\partial x_i} = \frac{\partial U}{\partial C_i} \frac{\partial f}{\partial x_i} - \lambda \frac{1}{(1+r)^i} = 0$$

$$(5{:}19) \quad \frac{\partial L}{\partial c_i} = \frac{\partial U}{\partial C_i} \frac{\partial f}{\partial c_i} - \lambda \frac{aH_i}{(1+r)^i} = 0$$

$$(5{:}20) \quad \frac{\partial L}{\partial v_i} = \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial C_j} \frac{\partial f}{\partial H_j} \frac{\partial H_j}{\partial v_i} - \lambda \frac{a\,H_i}{(1+r)^i} + \lambda \sum_{j=i+1}^{T-1} \frac{(1-c_j-v_j)}{(1+r)^j} \frac{\partial H_j}{\partial v_i} = 0.$$

The optimality conditions (5:18) - (5:20) can be written as (5:13) and

$$(5{:}21) \quad \frac{aH_i}{(1+r)^i} = \frac{1}{\lambda} \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial C_j} \frac{\partial f}{\partial H_j} \frac{\partial H_j}{\partial v_i} + \sum_{j=i+1}^{T-1} \frac{l_j . a}{(1+r)^j} \frac{\partial H_j}{\partial v_i} .$$

Optimality condition (5:13) is the same as for the first model and has the same implications.

However, (5:21) implies that the present value of marginal costs should equal the present value of earnings corresponding to labor market use of the human capital stock plus the present value of marginal satisfaction corresponding to use of human capital in consumption. This means that the incentives for human capital investment would be greater than in a situation when such effects are non-existent. Marginal investments will be registered as "financially" unprofitable, i.e., unprofitable if only the return in form of increased earnings is considered.

However, in qualitative respects - e.g., as pictured in Figure 38, the development would be the same as in the first model. It is possible, for example, that v first rises and then falls.

For special versions of how the stock of human capital affects the leisure activities there may, of course, be special implications. If we assume - as a special case - that the stock of human capital only influences the production of C_i through c_i, e.g., multiplicatively 1)

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1) Cf. the effect of H in the production of human capital in the Basic Model of Section 3 ($Q=\beta_0(v \cdot H)^{\beta_1}$).

$$C_i = f(x_i, c_i \cdot H_i)$$

optimality conditions (5:18) - (5:20) can be written

$$\frac{\partial f}{\partial(c_iH_i)} \Big/ \frac{\partial f}{\partial x_i} = a \tag{5:22}$$

$$\frac{aH_i}{(1+r)^i} = \frac{1}{\lambda} \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial C_j} \frac{\partial f}{\partial(c_jH_j)} c_j \frac{\partial H_j}{\partial v_i} + \sum_{j=i+1}^{T-1} \frac{l_j a}{(1+r)^j} \frac{\partial H_j}{\partial v_i} . \tag{5:23}$$

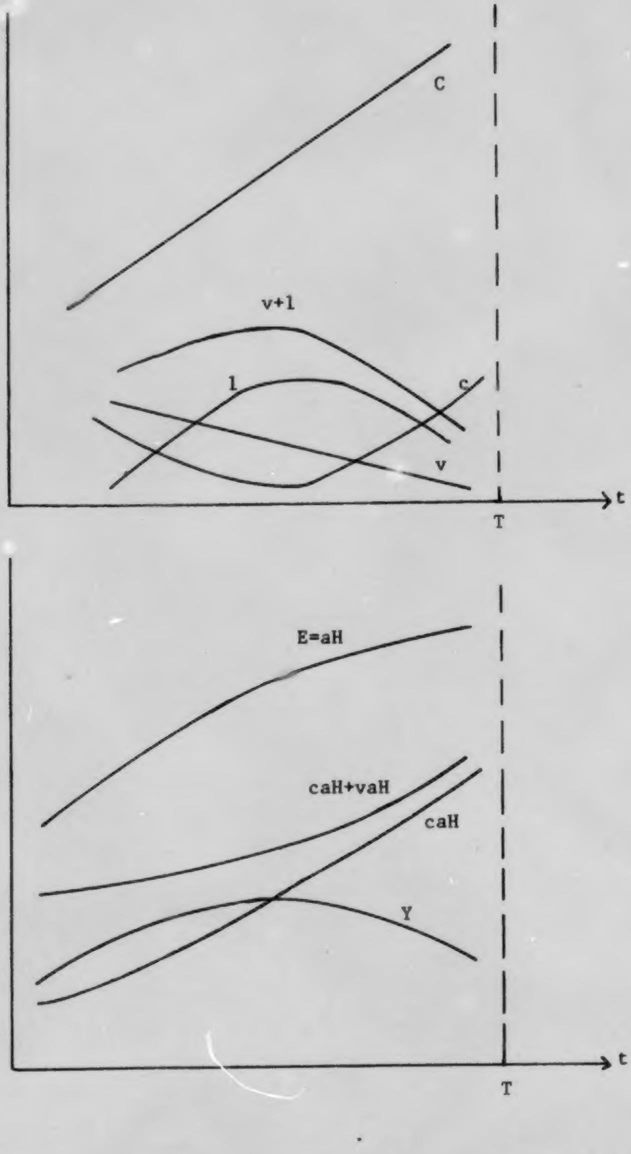
With this specification, if the individual devotes more of his capacity to consumption (c_i increases) he loses earnings equal to aH_i and gains "commodity" production $[\partial f/\partial(c_iH_i)] \cdot H_i$. As the stock of human capital enters the losses and gains in the same way, its level does not matter. So the ratio between the marginal productivity of capacity (c_iH_i) and of market goods should be constant and equal to a, as seen from (5:22).

Provided that $\frac{\partial^2 f}{\partial x_i \partial(c_i, H_i)} = 0$ and $\frac{\partial^2 f}{\partial(c_iH_i)\partial x_i} = 0$ it follows that when x increases cH must also increase according to (5:22). Or, the other way round: as H and eventually also c increases, x must also increase. As both x and cH increase, C also increases.

With this version of the second model where the effects of H are similar in consumption and on the market, there would be still fewer incentives than in the first model to shift periods when C is high to periods when potential earnings are low. C, x and cH would therefore be more likely to rise even though potential earnings are increasing. A pattern of development corresponding to subphases ii and iii of Figure 38 could come to dominate. However, the share of capacity devoted to consumption activities (c) does not necessarily rise while C, x and cH are increasing. The marginal productivity of cH could fall in the same proportion as the marginal productivity of x, with H growing and c falling, provided that H is growing fast enough. Actual earnings Y which equal laH = aH - avH - acH may peak. Potential earnings (aH) increase, while avH could be expected to fall and acH to rise.

Possible outcomes according to this model are depicted in Figure 39.

Figure 39 The development of earnings and different types of activities



The similar effect of H on "commodity" production and earnings also means that in this model a condition may be dervied that makes the investment decision of the individual independent of the division of capacity in the future between leisure and non-leisure.

If the value of the human capital stock in consumption and production were the same[1)], (5:23) could be written:

$$(5{:}24) \qquad aH_i = \sum_{\substack{j=i+1 \\ i=0}}^{T-1} \frac{(c_j+l_j)a}{(1+r)^j} \frac{\partial H_j}{\partial v_i} .$$

(5:24) implies that the marginal costs in period i should equal the present value to that period of an increased capital stock used for leisure and in the labor market and valued at market prices.

Here we see the beginning of what a "neutrality" assumption would be.

If we had, e.g.

$$(5{:}25) \qquad aH_i = \sum_{\substack{j=i+1 \\ i=0}}^{T-1} \frac{(c_j+l_j+v_j)a}{(1+r)^j} H_i \frac{\partial \Psi}{\partial (v_i H_i)}$$

i.e.

$$a = \sum_{\substack{j=i+1 \\ i=0}}^{T-1} \frac{a}{(1+r)^j} \frac{\partial \Psi}{\partial (v_i H_i)}$$

then the future uses of the stock of human capital would not matter.

With (5:25) it is evident that human capital investments (Q), vH and v would fall throughout the period of non-specialization. With the more general formulation of model 2 this is not necessarily so.

Thus, if human capital enters leisure and labor market activities in the same way and if it is also valued in the same way in both activities - i.e., at a - and if this is also the final value of human capital in human capital production, then the individual would be able to make investment decisions

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1) i.e. $\frac{1}{\lambda} \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial C_j} \frac{\partial f}{\partial (c_j H_j)} = \sum_{j=i+1}^{T-1} \frac{a}{(1+r)^j}$

without having to consider for what purposes the human capital stock will be used in the future. This is the same situation as in the Basic Model of Section 3 and the prediction is also the same: v must fall all the time during non-specialization. In this sense the Basic Model of Section 3 could be said to be a special case of a more general model which includes leisure. However, with the latter models we still do not have the implication that c is constant, which we used as an assumption in Section 3. To motivate a constant level of c to fit in with the Basic Model of that section would require more restrictive assumptions.

5.1.3 Type 3 Preference Function – Leisure and Work Welfare

With this preference function, not only "commodities" produced by the individual but also "satisfaction from work" (W) enter as direct arguments in the preference function. People like to work in addition to the fact that they can use the money they earn by working to buy market goods that also provide satisfaction. If we also assume that the stock of human capital influences how much "satisfaction from work" an individual obtains for a given work input - l - then we can formulate these assumptions as follows:

$$U[\{C_i, W_i\}] \qquad i = 0 \;.....\; T - 1$$

$$C_i = f_i(x_i, c_i, H_i) \qquad i = 0 \;.....\; T - 1$$

$$W_i = g_i(l_i, H_i) \qquad i = 0 \;.....\; T - 1$$

This can be thought of as a tentative representation of the fact that people with better paid jobs often have not only a more stimulating working life but also leisure time.(1) Though this may appear rather obvious, we have not found this type of human capital model in the literature.(2)

In accordance with what was said above, we could alternatively assume the effect of H_i on W_i to be multiplicative with l_i and write

$$W_i = g_i(l_i \cdot H_i) \qquad i = 0 \;.....\; T - 1.$$

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1) See, e.g., U. Bergsten, Adult Education in Relation to Work and Leisure. Stockholm, 1977.

2) Cf. Y. Weiss, "The Wealth Effect in Occupational Choice." International Economic Review, 1976, No.2, for a similar model with exogenous productive capacity.

This means that the effect of the human capital stock on the production of "satisfaction from work" will depend on how great a share of the stock is used on the labor market.

The general model would now look as follows:

(5:26) $\text{Max } U[\{C_i, W_i\}] \qquad \{C_i, W_i\} = (C_0, W_0, \ldots . C_{T-1}, W_{T-1})$

subject to

(5:27) $C_i = f(x_i, c_i, H_i)$

(5:28) $W_i = g(l_i, H_i) = g\,[(1-c_i-v_i), H_i\,]$

(5:3)

(5:4)

(5:5)

(5:6)

(5:7).

And we have

$$(5{:}29)\quad L = U[\{f(x_i, c_i, H_i), g[(1-c_i-v_i), H_i]\}] - \lambda\{\sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} - \\ - \sum_{i=0}^{T-1} \frac{a[h(v_0, \ldots, v_{i-1})](1-c_i-v_i)}{(1+r)^i}\}$$

The necessary conditions for a maximum are

$$(5{:}30)\qquad \frac{\partial L}{\partial x_i} = \frac{\partial U}{\partial C_i}\frac{\partial f}{\partial x_i} - \lambda\frac{1}{(1+r)^i} = 0$$

$$(5{:}31)\qquad \frac{\partial L}{\partial c_i} = \frac{\partial U}{\partial C_i}\;\frac{\partial f}{\partial c_i} - \frac{\partial U}{\partial W_i}\frac{\partial g}{\partial l_i} - \lambda\frac{aH_i}{(1+r)^i} = 0$$

$$(5{:}32)\qquad \frac{\partial L}{\partial v_i} = \sum_{j=i+1}^{T-1}\left\{\frac{\partial U}{\partial C_j}\;\frac{\partial f}{\partial H_j}\;\frac{\partial H_j}{\partial v_i}\right] - \frac{\partial U}{\partial W_i}\;\frac{\partial g}{\partial l_i} + \sum_{j=i+1}^{T-1}\frac{\partial U}{\partial W_j}\;\frac{\partial g}{\partial H_j}\;\frac{\partial H_j}{\partial v_i} - \\ - \lambda\frac{aH_i}{(1+r)^i} + \lambda\sum_{j=i+1}^{T-1}\frac{l_j a}{(1+r)^j}\,\frac{\partial H_j}{\partial v_i} = 0\,.$$

The optimality conditions (5:30) - (5:32) can be expressed as

$$(5{:}33)\qquad \frac{\dfrac{\partial U}{\partial C_i}\;\dfrac{\partial f}{\partial c_i} - \dfrac{\partial U}{\partial W_i}\;\dfrac{\partial g}{\partial l_i}}{\dfrac{\partial U}{\partial C_i}\;\dfrac{\partial f}{\partial x_i}} = a\,H_i$$

$$(5{:}34)\quad \frac{aH_i}{(1+r)^i} + \frac{1}{\lambda}\frac{\partial U}{\partial W_i}\frac{\partial g}{\partial l_i} = \frac{1}{\lambda}\left[\sum_{j=i+1}^{T-1}\frac{\partial U}{\partial C_j}\frac{\partial f}{\partial H_j}\frac{\partial H_j}{\partial v_i} + \sum_{j=i+1}^{T-1}\frac{\partial U}{\partial W_j}\frac{\partial g}{\partial H_j}\frac{\partial H_j}{\partial v_i}\right] + \sum_{j=i+1}^{T-1} - \frac{l_j a_j}{(1+r)^j}\frac{\partial H_j}{\partial v_i}$$

(5:33) corresponds to (5:13) in the first and second models. It regulates the relative use of c and x. However, in this model consideration must be given not only to the value of c and x for commodity production (C) and their market prices but also to the loss in satisfaction from working (W) that an increase in c means.

If the marginal utility of C and W were equal (5:33) would say that the ratio of the marginal productivity of c in commodity production plus its marginal direct disutility in work and the marginal productivity of x should equal the ratio of their market prices.

Assuming the marginal productivity of c in work to be constant and small (in absolute value) we would have the same development of c_i as in the second model. However, consumption incentives would be smaller - smaller the greater the marginal productivity of c in work (in absolute value).

(5:34) implies that there are two types of costs involved in using capacity in production of human capital :

- the earnings foregone;
- the foregone satisfaction value of spending capacity in work activities;

and three types of benefits from having more human capital:

- the value of more satisfaction, corresponding to use of human capital in consumption;
- the value of more satisfaction, corresponding to use of human capital in production;
- more "earnings" when working.

As before, at each date, v (and thus the production of human capital) should be increased up to the point where marginal costs and benefits are equal. The additional cost term (compared with the previous models) only stems from one date, but the additional benefit term relates to a number of periods. It could therefore be expected that with this model investment incentives would be greater. Actual capital investment could then be expected to be still more "unprofitable" if judged in financial terms alone.

As for the development of c and v, they may be the same in qualitative terms as in the previous models if the direct welfare effect of capacity in production is constant and small in absolute terms.

In this connection we might also discuss more specific assumptions about the way in which the human capital stock enters the production of satisfaction from working. It may do so in a neutral way - e.g., with

$$W_i = g(l_i;H_i)$$

$$\frac{\partial^2 g}{\partial l_i \partial H_i} = 0 \qquad \frac{\partial^2 g}{\partial H_i \partial l_i} = 0$$

Another possible alternative is

$$W_i = g(l_i \cdot H_i) .$$

With this assumption and a similar one for consumption

$$C_i = f(x_i, c_i \cdot H_i)$$

the optimality conditions (5:33) and (5:34) can be written

$$(5{:}35) \qquad \frac{\dfrac{\partial U}{\partial C_i} \dfrac{\partial f}{\partial (c_i H_i)} - \dfrac{\partial U}{\partial W_i} \dfrac{\partial g}{\partial (l_i H_i)}}{\dfrac{\partial U}{\partial C_i} \dfrac{\partial f}{\partial x_i}} = a$$

$$(5{:}36) \qquad \frac{aH_i}{(1+r)^i} + \frac{1}{\lambda} \frac{\partial U}{\partial W_i} \frac{\partial g}{\partial (l_i H_i)} H_i = \frac{1}{\lambda} \left[\sum_{j=i+1}^{T-1} \frac{\partial U}{\partial C_j} \frac{\partial f}{\partial (c_j H_j)} c_j \frac{\partial H_j}{\partial v_i} + \right.$$

$$\left. + \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial W_j} \frac{\partial g}{\partial (l_j H_j)} l_j \frac{\partial H_j}{\partial v_i} \right] + \sum_{j=i+1}^{T-1} \frac{l_j a}{(1+r)^j} \frac{\partial H_j}{\partial v_i}$$

(5:35) shows that now the level of H does not enter the determination of c_i/x_i. The marginal utility [1] of human capital in commodity production minus marginal direct utility of human capital at work should stand in a constant relation to the marginal utility of x in commodity production.

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1) Alternatively, productivity if $\frac{\partial U}{\partial C_i} = \frac{\partial U}{\partial W_i}$

As before, (5:36) shows that in each period the production of human capital should be pursued up to the point where the two types of marginal costs and the three types of marginal benefits are equalized.

Should it be assumed, as we did experimentally in the second model, that the value of human capital remained the same when used to earn money and when used to produce other benefits [1] (5:36) reduces to:

$$(5{:}37)\quad \frac{aH_i}{(1+r)^i} + \frac{1}{\lambda}\frac{\partial U}{\partial W_i}\,\frac{\partial g}{\partial(l_i H_i)}H_i = \sum_{j=i+1}^{T-1}\frac{a(c_j+2l_j)}{(1+r)^j}\,\frac{\partial H_j}{\partial v_i}$$

which again shows the presumably greater incentives to invest due to more benefits. And once again, investment decisions cannot be made separately from decisions concerning the future use of capacity, not even with this type of "neutrality."

5.1.4 Type 4 Preference Function – Leisure, Work and Educational Welfare

Here we augment the direct arguments in the preference function to three: satisfaction from leisure activities (C), satisfaction from work activities (W) and satisfaction from "educational" activities (E). The latter corresponds to what is often called consumption effects of education, though in this model no distinction is made as to human capital creation - e.g., in school or on the job.[2] This rather natural extension is not found in the literature.

If we assume that the level of the human capital stock influences the degree of satisfaction obtained from education and combine this with model 3 we could write:

$$\begin{aligned} &U\,[\{C_i,\ W_i,\ E_i\}\,] && i = 0 \;....\; T-1 \\ &C_i = f_i(x_i,c_i,H_i) && i = 0 \;....\; T-1 \\ &W_i = g_i(l_i,H_i) && i = 0 \;....\; T-1 \\ &E_i = h_i(v_i,H_i) && i = 0 \;....\; T-1 \end{aligned}$$

---

[1] That is $\frac{1}{\lambda}\sum_{j=i+1}^{T-1}\frac{\partial U}{\partial C_j}\,\frac{\partial f}{\partial(c_j H_j)} = \frac{1}{\lambda}\sum_{j=i+1}^{T-1}\frac{\partial U}{\partial W_j}\,\frac{\partial g}{\partial(l_j H_j)} = \sum_{j=i+1}^{T-1}\frac{a}{(1+r)^j}$

[2] In this model and the preceding one we define W_i and E_i as "goods" they might, of course, also be "bads".

And the general model

(5:38) Max $U[\{C_i, W_i, E_i\}]$ $\{C_i, W_i, E_i\} = (C_0, W_0, E_0 \ldots . C_{T-1}, W_{T-1}, E_{T-1})$
subject to

(5:39) $$C_i = f(x_i, c_i, H_i)$$

(5:40) $$W_i = g(l_i, H_i) = g[(1-c_i-v_i), H_i]$$

(5:41) $$E_i = e(v_i, H_i)$$

(5:3)
(5:4)
(5:5)
(5:6)
(5:7).

And we have

(5:42) $$L = U\{f(x_i, c_i, H_i), g[(1-c_i-v_i)H_i , e(v_i,H_i)\}] -$$
$$- \lambda \left\{ \sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} - \sum_{i=0}^{T-1} \frac{a[h(v_0, \ldots . v_{i-1})(1-c_i-v_i)]}{(1+r)^i} \right\}$$

with necessary conditions for a maximum:

(5:43) $$\frac{\partial L}{\partial x_i} = \frac{\partial U}{\partial C_i} \frac{\partial f}{\partial x_i} - \lambda \frac{1}{(1+r)^i} = 0$$

(5:44) $$\frac{\partial L}{\partial c_i} = \frac{\partial U}{\partial C_i} \frac{\partial f}{\partial c_i} - \frac{\partial U}{\partial W_i} \frac{\partial g}{\partial l_i} - \lambda \frac{aH_i}{(1+r)^i} = 0$$

(5:45) $$\frac{\partial L}{\partial v_i} = \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial C_j} \frac{\partial f}{\partial H_j} \frac{\partial H_j}{\partial v_i} - \frac{\partial U}{\partial W_i} \frac{\partial g}{\partial l_i} + \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial W_j} \frac{\partial g}{\partial H_j} \frac{\partial H_j}{\partial v_i} +$$
$$+ \frac{\partial U}{\partial E_i} \frac{\partial e}{\partial v_i} + \sum_{j=i+1}^{T-1} \frac{\partial U}{\partial E_j} \frac{\partial e}{\partial H_j} \frac{\partial H_j}{\partial v_i} - \lambda \frac{aH_i}{(1+r)^i} + \lambda \sum_{j=i+1}^{T-1} \frac{l_j a}{(1+r)^j} \frac{\partial H_j}{\partial v_i} = 0.$$

The optimality conditions (5:43) - (5:45) can be written

(5:46) $$\frac{\frac{\partial U}{\partial C_i} \frac{\partial f}{\partial c_i} - \frac{\partial U}{\partial W_i} \frac{\partial g}{\partial l_i}}{\frac{\partial U}{\partial C_i} \frac{\partial f}{\partial x_i}} = a\,H_i$$

$$(5{:}47)\quad \frac{aH_i}{(1+r)^i} + \frac{1}{\lambda}\left(\frac{\partial U}{\partial W_i}\ \frac{\partial g}{\partial 1_i}\right) = \frac{1}{\lambda}\left(\frac{\partial U}{\partial E_i}\ \frac{\partial e}{\partial v_i}\right) + \frac{1}{\lambda}\left[\sum_{j=i+1}^{T-1}\frac{\partial U}{\partial C_j}\ \frac{\partial f}{\partial H_j}\ \frac{\partial H_j}{\partial v_i} + \sum_{j=i+1}^{T-1}\frac{\partial U}{\partial W_j}\ \frac{\partial g}{\partial H_j}\ \frac{\partial H_j}{\partial v_i} + \sum_{j=i+1}^{T-1}\frac{\partial U}{\partial E_j}\ \frac{\partial e}{\partial H_j}\ \frac{\partial H_j}{\partial v_i}\right] + \sum_{j=i+1}^{T-1}\frac{(1-c_j-v_j)a}{(1+r)^j}\ \frac{\partial H_j}{\partial v_i}.$$

Compared with the previous model the only difference here is that two more benefits are associated with human capital investment activities: the satisfaction derived from "learning" and the increased possibilities in the future to reap such satisfaction. Investment incentives will therefore be larger. However, as these benefits are not of a financial type, judgements of human capital investments made solely on the basis of financial benefits underestimate their value.

5.2 Specialization

In addition to the specification of the arguments to be included in the composite preference function, the conditions imposed on both the preference function and on the production function for human capital are of importance. They will determine, e.g., what kinds of specialization phases are possible and the length of phases. As shown in Section 3, depending on the specification of the production function for human capital we could either have a model with:

- two phases: one with $v = 1$
  one with $v = 0$ or
- three phases: one with $v = 1$
  one with $0 < v < 1$
  one with $v = 0$ (at $t = T$ or for a longer period).

Now we have $c + v + 1 = 1$. If we do not rule out any possible combinations we could have the following seven types of phases.[1]

---

[1] No general discussion of conditions for the existence of different phases is found in the literature. Authors who work with specific functions and e. g. , control-theoretic formulations, derive results in relation to the specific assumptions. The model of F. Stafford and P. Stephan, op.cit., 1973, essentially permits two types of periods: one with $1=0$ and one with $0<1<1$.

J.J. Heckman, op.cit.1976 only discusses a period when $1>0$, $c>0$, $v>0$ (except at T). A.S. Blinder and Y. Weiss, op.cit., 1976, use a model where four types of periods are possible: schooling, on-the-job training, work

Footnote continued on next page.

Table III **Possible combinations of values of the variables c, v and l in Life Cycle Models Including Leisure**

variable value	c	v	l
	0	1	0
	0	m	m
	0	0	1
	m	m	0
	m	m	m
	m	0	m
	1	0	0

m = a value greater than zero but less than one.

With the preference function discussed above, C is always an argument in the composite preference function. If we impose the condition that the marginal value of c is unbounded[1] as c goes to zero, we rule out the first three types of phases (c=0) and are left with models with four types of phases that could be labeled:

I schooling, a period out of work but with positive investments (l=0,v=c=m)
II on-the-job training and/or recurrent education (v=l=c=m)
III work (v=0, l=c=m)
IV retirement, a period without work and training (v=l=0, c=1).

Footnote continued from previous page.

and retirement. However, as indicated in the footnote on page 70 , their approach is somewhat different. The concepts of human capital, of a production function for human capital, of a rental price for human capital and of possibilities of diverting capacity from earning money in order to create more human capacity are not used. Instead there are different jobs and an occupational index. At some jobs you can learn a lot but get less pay, the extremes being no learning (and high pay) and full-time learning (and no pay). So, in their model the production of human capital and its rental price is amalgamated into a relation between actual earnings and increments to earnings capacity. The distinction between v and l is not used. There is only c and time on the labor market (v+l). This means that there will be no separate implications for v and l which may be unfortunate in situations with mainly "recurrent education" rather than on-the-job training.

1) $\lim\limits_{c_i \to 0} \frac{\partial U}{\partial c_i} = \infty$

Figure 40 illustrates a possible life-cycle including these four periods. Actual earnings only appear during the "on-the-job training" and "work" phases. The date at which actual earnings begin to be registered marks the end of the schooling phase. The beginning of the retirement phase is marked by the disappearance of earnings.

The figure is based on the assumption that human capital does not depreciate. Therefore potential earnings grow as long as v is positive - i.e. during the first two phases. Thereafter they are constant. During the pure work period actual earnings fall due to a decreasing individual labor supply.

In this setting a wage rate concept may be distinguished that could be operationally defined and used in empirical analysis (contrary to the rental price for human capital). The "net wage rate" (NW) multiplied by the amount of capacity not used for leisure $(l+v)$ would give actual earnings.[1] This means that

$$NW = \frac{1}{v + l} aH$$

and thus actual earnings

$$Y = NW \cdot (l + v) = laH.$$

The "net wage rate" would be constant during the pure work period. During the on-the-job training phase that we discussed in Section 5.1, at least during subphase ii, it would increase.

On certain conditions the pure work period could be ruled out, as in the Basic Model of Section 3 (except at T-1). Let us consider the disposition of capacity between v and l (when what is needed for leisure has been taken away). If both v and l are positive, their marginal value is equal. Such a situation is illustrated in Figure 41 A. If v is to be zero the marginal value of l (MV_l) must be higher than the marginal value of v (MVv) at $v = 0$, as illustrated in panel B. However, this will never occur if, e.g., the marginal value of v becomes infinite at zero with the marginal utility

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[1] In the Life Cycle Models without leisure - e.g., the Basic Model of Section 3, this concept is not meaningful as

$$\frac{1}{1 + v} aH \equiv laH \quad \text{as } l = 1 - v.$$

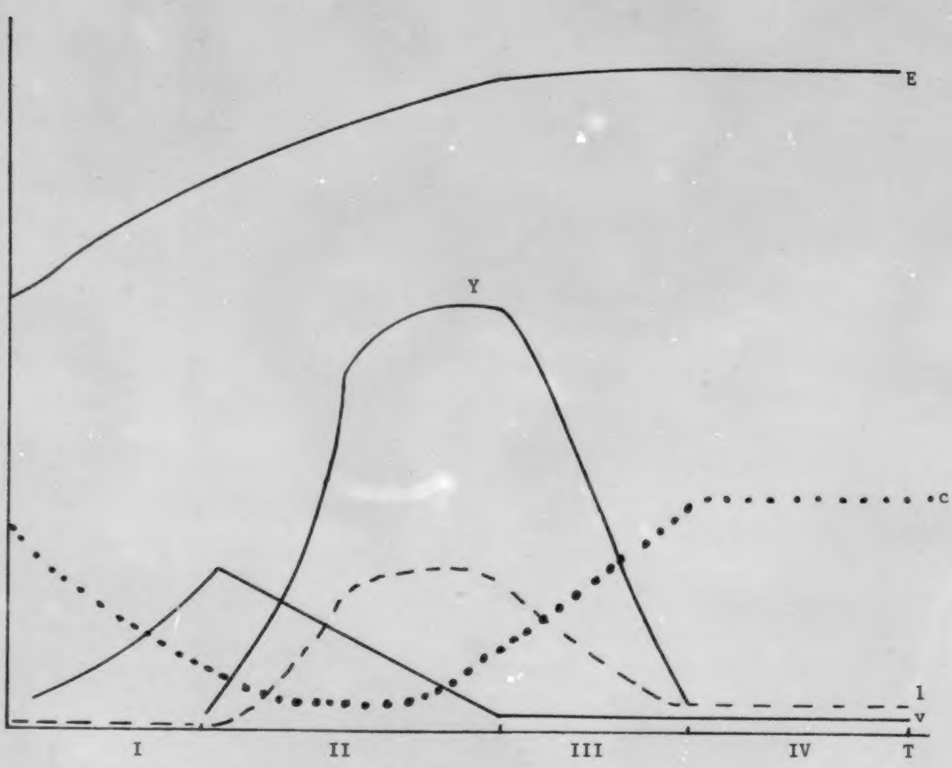
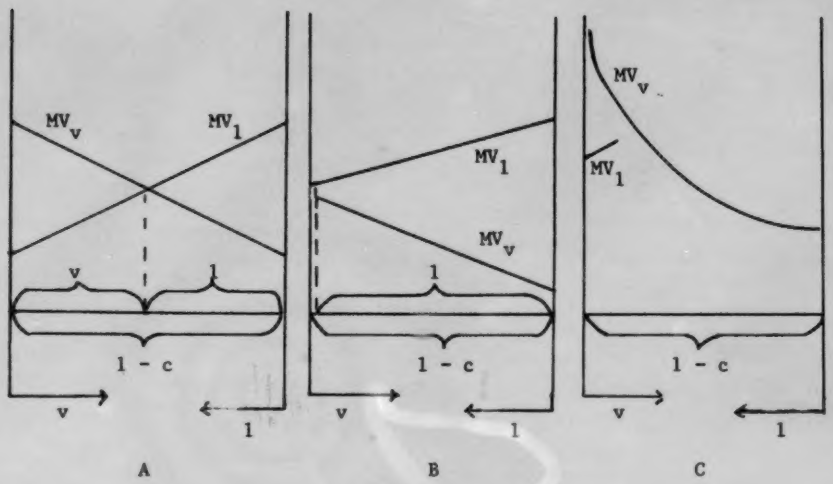


Figure 40 Life cycle with four phases: I, schooling; II, on-the-job training; III, work; IV, retirement

Figures 41 A, B and C The disposition of capacity between leisure and human capital production



of 1 being finite for all values of c between zero and one. 1)Panel C shows such a situation for a given value of c.

In such a case we could be left with a model with three types of phases:

- schooling,
- on-the-job training,
- retirement.

We could continue and ask what conditions would be sufficient to make the schooling period disappear. As long as for any given c the marginal effect of reducing v is greater than the marginal effect of increasing 1, a schooling period is attractive. A schooling period is excluded if the marginal value of 1 is infinite at 1 = 0, while the marginal value of v is finite.

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1) This would be the case, for example, with the human capital production function of the Basic Model of Section 3 where $\lim_{v_i \to 0} \partial Q_i / \partial v_i = \infty$ and with 1 only affecting the preferences of the individual through earnings and earnings changing with a finite amount when 1 is varied.

However, in all the models that we have worked with, the marginal value of l has been finite has far as its effects via earnings are concerned. But a schooling period could still be ruled out. Either the non-pecuniary effects of l may be infinite or, e.g., an individual may have such a large stock of human capital that the marginal value of increasing v at $l = 0$ would be very low (lower than the marginal value of working). If the schooling period disappears we are left with:

- on-the-job-training,
- retirement.

As for retirement, a necessary condition for this type of combination to be chosen is that the marginal utility of c at $c = 1$ is non-negative.[1)] At period T-1, v will equal zero, if v does not appear - e.g., via E - in the preference function. In period T-1 l will only be zero if the utility of raising l above zero to be able to buy more goods and/or have utility from work is less than the marginal utility of c when c equals one. Ceteris paribus, satisfaction from work reduces the likelihood of this retirement. For periods prior to T-1, the marginal utility of c at $c = 1$ must not only outweigh the marginal utility of l (earnings, satisfaction from work) but also the marginal utility of v (future earnings, consumption- and work satisfaction-effects, etc.) for retirement to be optimal.

The on-the-job-training period could also disappear. Certain individuals might only have a work and a retirement period. The condition for this to occur is, of course, that the marginal costs for such training exceed the marginal return. Among the preference functions considered here, the third one is the one with the most types of costs associated with human capital production - but this does not necessarily mean that investment incentives are smallest in that model.

Two further aspects concerning these periods should be mentioned - their order of appearance in the life cycle and the possibility of their reappearance - though we will not dwell on the subject at any greater length. These questions can only be fully analyzed in connection with more precisely defined models.[2)]

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1) $\underset{c\,=\,1}{\frac{\partial U}{\partial c}} \geq 0$ for any level of the other arguments in the preference function.

2) A.S. Blinder, Y. Weiss, op.cit., 1976, e.g., have treated at length these two aspects in relation to their explicit assumptions regarding the form of the different functions.

However, if they all are once (or several times) part of the optimal program it may be reasonable to expect them to appear in the following order: schooling, on-the-job-training, work and retirement. If human capital only affects the welfare of the individual through earnings to buy the market good (preference function type 1), there would only be incentives to concentrate on human capital investment (schooling period) in preparation for work - either pure work or in combination with on-the-job-training. In the same way, concentration on work would only be profitable after human capital has been accumulated. However, retirement - a period of neither work nor training - might optimally come first in the life cycle. Individuals who highly value early consumption might take this period first and work later, after preparing for work by making human capital investments. Given other preference functions, where human capital more directly influences the welfare of the individual, an early retirement period would be less likely. It would pay to prepare even for retirement by investing in human capital.

As for the recurrence of these phases, it cannot a priori be excluded that it is not optimal for the individual to have several work period, e.g. - perhaps shorter to begin with and longer later on, and prepared for by an on-the-job-training period or even a period of schooling. Such delayed schooling periods could be viewed as full-time "recurrent education." However, in these models they would not be motivated by the need for the individual to keep abreast with new knowledge, but as a take-off for more intensive work periods.

Subcycles including retirement-periods may be optimal. The likelihood for such a development could be increased if we introduced a new aspect of individual welfare, namely, that of having children (cf. Section 6). Figure 42 illustrates one such life cycle.

5.3 Special Features of the Life Cycle Models Including Leisure

In this section we widen the perspective of the human capital models by considering individual preference functions that no longer permit a simple maximization of lifetime earnings. We look briefly at models including leisure activities where human capital is an asset not only in human capital

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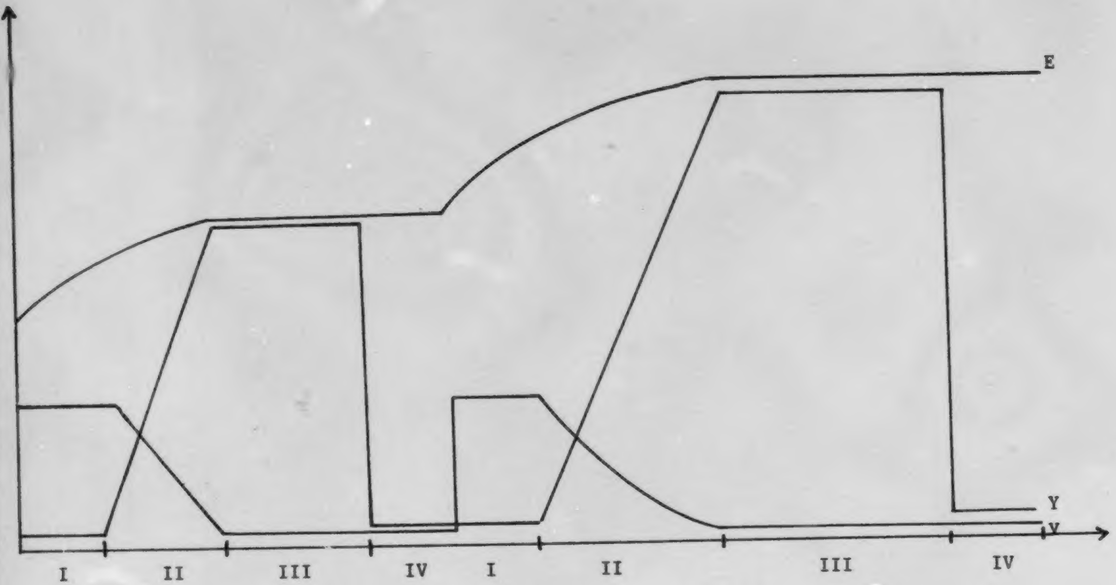
1) A.S. Blinder, Y. Weiss, op.cit., 1976, e.g., have treated at length these two aspects in relation to their explicit assumptions regarding the form of the different functions.

Figure 42 Alternations of four types of phases during a life cycle



production but successively also for leisure activities, satisfaction from work and education. Given all these theoretical possibilities, the purpose has not been to fully penetrate this field but rather to lay the preliminary basis for empirical research based on which more precise models could be developed.

The introduction of consumtion or leisure activities into the human capital models means that individual variations in earnings may stem from an increasing number of causes. And a given development in earnings could be explained by many alternative combinations of explanatory factors.

The different types of explanatory factors in the Basic Model of Section 3 are listed below. Explanatory factors in variations to this model are indicated within parentheses. The additional explanatory factors that appear in models that include leisure are presented in the right hand column.

Basic Model (with variations)	Models including leisure
prices:	
- rental price for human capital	
- rate of interest	
(- market price of inputs for human capital production)	- market price of inputs for commodity production
individual characteristics:	
- initial human capital stock	
- planning horizon	
- production function for human capital	
(- net rate of depreciation)	
(- amount or share of capacity set aside for other activities than work and human capital production)	
(- rate of growth of earnings)	
(- the generation to which the individual belongs and the rate of growth of earnings or of initial human capital from one generation to another)	- preference function
	- production functions for the arguments in the preference function.

One important aspect of the preference function is the arguments that for different individuals enter this function. Another significant factor is the substitution possibilities between arguments at a given date and between the same argument at different dates.[1] These are new areas where we could have individual variations. In addition to variations in the type and form of the preference function, individual variations are also possible in the production functions for the arguments in the preference function.

All individuals do not necessarily have the same arguments in their preference functions. Some find pleasure in learning; others do not. Some are more sensitive to the timing of consumption; other are less so. For given inputs, individuals may produce different amounts of the "commodities" that appear as arguments in the preference function. "Ability" may vary in these respects as well as in terms of production of human capital.

If the aim is to have models that explain as closely as possible the behavior of individuals, groups that are identical with respect to the preference function should be identified. A special model should then be applied to each group. Such groups may be identified directly through questions or indirectly through the behavior of the individuals - e.g., as judged from the pattern of phases or the pecuniary returns to educational investments. And for individuals in general, does the longer list of explanatory variables (including proxy variables for preference functions - their arguments, substitution and production possibilities) better explain such results as earnings than the shorter list from the Basic Model of Section 3? Are actual earnings better described according to the formulas discussed in connection with the Basic Model of Section 3, or for instance according to these formulas minus a term indicating the value of other activities? The latter would be implied assuming leisure model two with the special conditions making the investment decisions independent of the future use of capacity.[2]

As the various preference functions may be directly identified, through questioning, and do result in different sets of explanatory variables and

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1) In case of a simple relation between different dates, the latter equals the rate of time preferences.

2) J.J. Heckman, op.cit., uses this kind of procedure to test for the importance of leisure. The importance of non-pecuniary aspects of jobs might, e.g., be tested on the development of the labor market, cf. Y. Weiss, "The Wealth Effect in Occupational Choice." International Economic Review, 1976, No.2.

behavior, it would seem reasonable and possible in an empirical research strategy to first tackle the question of which preference function better explains the behavior of different groups at different times.

The more readily observable variables on which the implications of the models including leisure could be tested are:

1) actual earnings ($Y = laH$) as in the Basic Model of Section 3. However, with this type of models it can be meaningful to make a distinction between the "net wage rate" (NW) and earnings. Actual earnings equal the wage rate times labor supply. If an individual devotes part of his capacity to work and human capital production, and if the latter takes the form of on-the-job-training, then there exists an observable wage rate that is net of the investment costs.[1] This wage rate times the labor supply (then interpreted as both capacity for work and investment) gives actual earnings as

$$NW = \frac{1}{v+1}\, aH \qquad Y = NW \cdot (v+1)$$

and we could have implications for both.[2]

2) share of capacity devoted to leisure (c), work (l), and investment (v) activities. As discussed in connection with the Basic Model of Section 3, v is only observable when investment is undertaken separately, and not on the job. Then it could be approximated by share of time devoted to such activities. However, with this model we can use the implications for (v+1) or c, which could be more easily approximated empirically as share of time devoted to non-leisure activities and to leisure activities.

3) occurrence of phases, their date of beginning and ending, and their duration. The main effect of the additional factors in the models including leisure is that the role of human capital increases. Human capital production is no longer only motivated for investment or pecuniary reasons. It can also be valuable in leisure, work and educational activities. As such, its remuneration is non-pecuniary.

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[1] At least to the extent that all direct investment outlays are paid for by a wage reduction.

[2] This distinction could not be made in the models of Section 3, a factor that may also be used to separate the field of applicability for the different types of models.

Human capital had no direct use value in the models in Sections 2 - 4. The same is true of the first model of this section. In model two, human capital is of direct value in consumption, in model three also at work, and finally in model four even in education.

Investment incentives are higher the greater the benefits (the more uses human capital has, the more human capital means in these uses and the more valuable these activities are) and the lower the costs (foregone earnings and directly valuable uses).

Less time will be devoted to labor activities and more to investment activities with model one than with a model where there are no opportunities to influence earnings through investments. In the same way, with model two, where human capital also has a value in consumption, there will be greater investment incentives than in model one. In models three and four, human capital is useful in addition in providing for satisfaction from work and from education. However, the direct use value of human capital at work increases the costs of investments, which somewhat reduces the greater investment incentives that otherwise exist in model three compared with model two. In model four, investments incentives would be greater than in model three, as there is one more type of benefit - direct value of human capital in education - but no more costs.

Of course, the supply of human capital will be larger in an economy where the incentives to invest in human capital are greater. The rental price will tend to be lower. In all models including leisure variations in rental prices affect both marginal costs and returns to different extents except in very special cases. An individual who reckons with a lower rental price may have greater investment incentives than an individual who counts on a higher rental price if, e.g., human capital is important in leisure activities and for the satisfaction derived from working.

In models two, three and four, human capital investments do not have only "pecuniary" benefits. If marginal human capital investments were judged from their effects on earnings alone they might be expected to appear unprofitable.

In the Basic Model of Section 3 (and variations on that model) there is only one independent individual control variable (v or l). In the models with leisure there are two (e.g., v and c). When the amount of capacity set aside for human capital and leisure activities has been determined the labor supply is also given.

One general result concerning the models including leisure is that optimal investment decisions cannot be made without regard to future division of capacity among activities. Assumptions leading to "neutrality" in this respect might be found in certain cases, but appear even more restrictive in this extended setting than in Section 3.

We discussed such "neutrality" assumptions in connection with a restricted version of model two and then found that in such a case we arrived at the same prediction as in the Basic Model of Section 3, i.e. $-v$ must fall continuously during non-specialization. In this sense the Basic Model of Section 3 could be said to be a special case of models including leisure, though with the latter models a constant level of leisure activities would be hard to motivate.

If we exclude that type of specialization in which no time is devoted to leisure activities, the following four types of phases are possible:

- schooling,
- on-the-job-training,
- work,
- retirement.

Retirement is a new type of phase which did not appear in the models in Section 3. On certain conditions one or more of the above phases may disappear. A retirement period with no work and human capital production is less likely to occur, for example, with models three and four, where work has a direct value. With the production function for human capital of the Basic Model of Section 3, a pure work period with no human capital investments will, e.g., not occur with models one and two, but perhaps with models three and four.

We already noted in connection with the Basic Model of Section 3 and its variations that few precise qualitative statements could be made about the development of Y, e.g., once alternative production functions were allowed. In other words, almost any development in Y could be explained by reference to some special production function for human capital. That tendency is strengthened when we introduce a more complicated preference function and no longer assume that the individual only maximizes earnings. Preference functions could be specified in a number of ways. We have worked with four types here.[1) They could each be combined with differently specified production functions even though we have abstained from pushing our investigations further in that direction. There are hardly any a priori reasons to prefer one combination of preference and production function to another. And, of course, these models could also be extended to include depreciation, appreciation, growth effects [2), etc. Another rather natural extention would be to make the production functions - for human capital, leisure "commodities", etc. - individually age dependent.

However, it could be interesting to note that even without further elaboration a number of predictions concerning investment behavior and earnings that were not noted earlier become possible with these models:

- increasing share of capacity devoted to human capital production after specialization, even in case of the production function for human capital of the Basic Model of Section 3;
- actual earnings declining without depreciation taking place;
- the supply of capacity for work and training first rising and later falling.

The explanation for all these differences reduces to the fact that when the individual is free to choose his leisure time it will be optimal not to hold it constant, a fact that will simultaneously influence the optimal level

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1) Other examples would be models where the leisure activities are split up in different types with human capital of varying value in each of them. One example of such leisure activities could be that of bringing up children (B), another voluntary work in different organizations (Z). If these activities are added, the preference function of the individual may take the following form:

$$U[\{C_i, W_i, E_i, B_i, Z_i\}]$$

$$C_i = f_i(x_i, c_i, H_i) \qquad E_i = g_i(v_i, H_i)$$

$$W_i = g_i(l_i, H_i) \qquad B_i = b_i(t_i, H_i)$$

$$Z_i = z_i(h_i, H_i)$$

$$c_i + v_i + l_i + t_i + h_i = 1$$

t_i = share of capacity devoted to child-care activities

h_i = share of capacity devoted to voluntary work

2) Cf. for example G.R. Ghez, G.S. Becker, op.cit.

of work and human capital activities.

Certain results for the development of Y, e.g., still seem to be excluded, however, by these models. One example could be earnings that increase at a growing rate during the whole life cycle.

In these models a distinction could also be made between actual earnings and "net wage rate." For the latter the implication was that during a pure work period it would remain stable while it could be expected to rise for a certain time, at least, during the non-specialization period.

Explicit earnings-experience functions in terms of the explanatory variables are more difficult to obtain in the leisure models. A research strategy aiming at identifying production and preference functions would ultimately arrive at numerically specified functions and solutions in terms of earnings-experience relations that may be tested. Provisional numerical experiments provide another way of arriving at testable earnings functions. In the case of the first model and under certain circumstances for the other models, another alternative is also available. As was observed with the first model the condition for optimal human capital accumulation was separable from the rest of the system and would be the same as in a problem of maximizing of the present value of earnings for a given leisure time development. Hypothesing or estimating c-trajectories from actual data, the human capital accumulation and earnings paths may be solved for and used as approximate life cycle models for empirical testing purposes. 1)

These models which include other activities than work and training also point out certain problems of simultaneity that arise in simpler models and which should be taken into account when testing them:

- human capital investment and working activities are determined simultaneously. In a simpler model one cannot add an explanatory variable such as "weeks worked" without risking biased results.
- human capital investment, working and other activities such as child rearing are determined simultaneously which means that, e.g., number of children cannot be used as an exogenous variable in earnings functions without risking biased results.

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1) Cf. S. Rosen, op.cit., 1977 |b|.

As suggested above, Life Cycle Models may be applied for given leisure time patterns which would reduce problems regarding "weeks worked". Similar procedures could also be used to take into account child rearing activities.

As for Schooling Models, an alternative would be to apply them - extending the suggestions made in Section 3 - to groups of individuals with similar leisure time patterns or, e.g., the same number of "weeks worked" and children. The Life Cycle Models Including Leisure also point out a number of other factors that ought to be considered in these models. The use of the individual's own time in schooling must be weighed against its value not only at work but also in other activities. And with the fourth model, schooling might have important direct consumption effects.

Highlighting the effect of schooling on the returns to working and other post-school activities, it may be interesting to study - in line with what was suggested in Section 3 - different schooling and ability groups to investigate the effects of schooling and ability not only on working but also on other activities.

6. Intergenerational Models of Individualistic Type

The models of the previous sections could be said to be individualistic in the sense that the individuals planned their lives for themselves without directly considering the wishes or potentialities of others, as they might have been expected to do in, e.g., a household setting. Moreover, there was no direct link between generations. In this section we are going to make a preliminary investigation of the possibilities of turning these individualistic models into more explicit intergenerational models. In such models a mechanism would exist through which the actions of one generation influences the potentialities of the next.

Household models might be an alternative. However, as they are also interesting in other respects, they will be treated separately. Though household models have been treated fairly extensively in the literature - as indicated in the next section - very few intergenerational human capital models of the individualistic type exist.1)

There are, of course, many problems involved in constructing intergenerational models and we will discuss a few which are of immediate interest from a human capital point of view and relate them to the previous models. Some suggestions are made regarding possible model formulations that ought to be further developed in the same kind of setting as used here and also in more complex settings. The main purpose of this section is to investigate the new questions that are brought into the realm of analysis for the human capital models with this extension and how this expansion of the perspective affects the interpretation of the less comprehensive human capital models.

6.1 Individualistic Models

An intergenerational aspect could be said to have appeared already in the models discussed in Section 5. As indicated there, one aspect of the welfare of the individual could be to have children. With such a model we can discuss

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1) For schooling models of this type cf., T. Ishikawa,"Family Structures and Family Values in the Theory of Income Distribution." Journal of Political Economy, 1975, No. 5,
A. Drazen, "Government Debt, Human Capital, and Bequests in a Life Cycle Model." Journal of Political Economy, 1978,No.3,
and with a macro orientation, Sheng Cheng Hu, op.cit.
For a "non-leisure on-the-job training model" cf., A. Razin, "Optimum Investment in Human Capital." Review of Economic Studies, Vol.89,p.455.

what this means for the supply of labor and human capital investments for the individual who has children. However, this type of models could be further developed to analyze intergenerational effects - distribution of welfare and earnings between generations, distribution of welfare and earnings within generations as affected by the actions of earlier generations, economic growth over generations, etc.

The link between the generations would be the time and market resources the individual "parent" invests in his child, and the resources that are thus created and embodied in the child in addition to the initial endowments. They are at the disposal of the child for further human capital investments and other activities.

The connection between generations could be depicted as in Figure 43. The individual of the first generation lives to T_{11} and has one child. His behavior could be described by a model of the same type as in Section 5. However, we now use a preference function, where one argument would be the activity of having and raising children (B), e.g.:

$$U[\{C_i, B_i\}] \qquad i = 0, \dots, T_1 - 1$$

$$C_i = f(x_i, c_i)$$

$$B_i = b(x_i, t_i)$$

where t_i = share of capacity (H_i) devoted to children.[1]

Or, this activity could be specified to be influenced by the stock of human capital:

$$B_i = b(x_i, t_i, H_i).$$

---

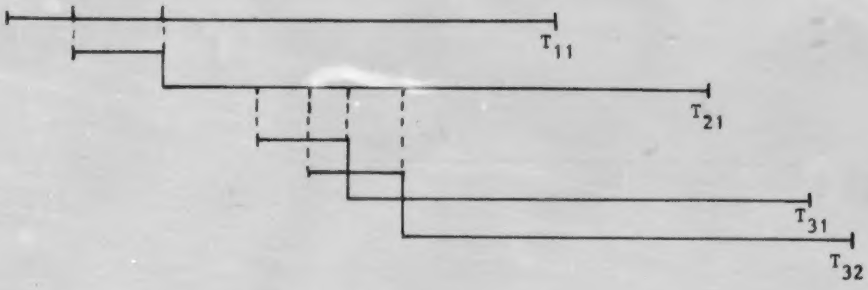
[1] Actually, this activity should be seen as the result of two activities: "producing" children (P), and taking care of children (K), e.g.:

$$\left.\begin{aligned} P_i &= p(x_i, s_i) \\ \Sigma K_{ij} &= k(x_i, k_{ij}) \end{aligned}\right\} \qquad B_i = b(x_i, t_i)$$

where s_i = share of capacity (H_i) devoted to "producing" children (P),
k_{im} = share of capacity (H_i) devoted to taking care of child m (K_m),

$$m = 1, \dots, n \quad \text{and} \quad n = \sum_{j=0}^{i-1} P_j$$

Figure 43 Three successive generations



The child lives with the individual for a certain period of time and the individual devotes himself to the child-care activities. The child is brought up according to the preferences of the individual of the first generation in line with a production function such as one of those indicated above. This upbringing plus the child's genetic endowment constitute his resource base when he leaves home. This can be assumed to be the moment when he begins to make decisions for himself. The latter decision-making could once again be described, e.g., by a model similar to the one for the individual of the first generation, etc. In Figure 43 the moment the individual of the second generation leaves home is indicated by a turn in his "lifeline". He is supposed to live until T_{21} and have two children who successively leave home.

There are two especially important aspects of this process: the exact meaning of the argument B in the individual's preference function and the link between generations. If what the individual does to his child makes a difference for the child's further development then the link between generations can be analyzed with the type of model sketched above.

If we use the human capital production function of the Basic Model of Section 3, $Q_t = \beta_0(v_t H_t)^{\beta_1}$, at least one of the following factors characterizing the child must depend on the actions of the person who rears him.

$$\beta_0 = f_1(I)$$
$$\beta_1 = f_2(I)$$
$$\bar{H}_0 = f_3(I)$$

where I = activities undertaken by the parent.

If genetic factors alone determined the resource base of the child, other types of models would be more interesting.

Let us assume that the intergenerational effects work themselves out via the stock of human capital and that the child rearing activity could only take place during a restricted period during the life of an individual. A formal model might then look as follows

(6:1) First generation Max $U[\{C_i, B_{ij}\}] \qquad i = 0 \ldots T_1-1$
subject to

(6:2) $C_i = f(x_i, c_i) \qquad i = 0 \ldots T_1-1$

(6:3) $B_{ij} = b(x_{ij}, t_{ij}) \qquad i = T_1-n \ldots T_1-m \quad n>m$

(6:4) $v_i + l_i + c_i + \sum_j t_{ij} = 1 \qquad i = 0 \ldots T_1-1$

(6:5) $H_{i+1} = H_i + Q_i$

(6:6) $H_{01} = \bar{H}_{01} > 0$

(6:7) $Q_i = \Psi(v_i H_i) \qquad i = 0 \ldots T_1-1$

(6:8) $$\sum_{i=0}^{T_1-1} \frac{x_i}{(1+r)^i} + \sum_j \sum_{i=T_1-n}^{T_1-m} \frac{x_{ij}}{(1+r)^i} = \sum_{i=0}^{T_1-1} \frac{a H_i l_i}{(1+r)^i}$$

(6:9) $t_{ij} = 0 \qquad T_1-m < i < T_1-n$

B_{ij} = welfare contribution in period i from child j
t_{ij} = share of capacity devoted to child j in period i.
x_{ij} = market resources devoted to child j in period i.

The effects of the actions of the individual of the first generation on individuals in the next generation could analyzed accordingly:

(6:10) Second generation Max $U[\{C_i, B_{ij}\}] \qquad i = T_j \ldots T_2-1$
subject to

(6:11) $C_i = f(x_i, c_i) \qquad i = T_j \ldots T_2-1$

(6:12) $B_{ij} = b(x_i, t_{ij}) \qquad i = T_2-n \ldots T_2-m, \ n>m$

(6:13) $v_i + l_i + c_i + \sum_j t_{ij} = 1$

(6:14) $H_{i+1} = H_i + Q_i$

(6:15) $H_{02j} = \bar{H}_{02j} = h(x_{ij} t_{ij}) \quad i = T_1-n_j \ldots T_1-m_j$

(6:16) $Q_i = \Psi(v_i H_i) \qquad i = T_j \ldots T_2-1$

$$(6{:}17) \qquad \sum_{i=T_j}^{T_2-1} \frac{x_i}{(1+r)^i} + \sum_j \sum_{i=T_2-n}^{i=T_2-m} \frac{x_{ij}}{(1+r)^i} = \sum_{i=T_j}^{T_2-1} \frac{a\,H_i l_i}{(1+r)^i}$$

(6:18) Third generation Max $U[\{C_i, B_{ij}\}]$
etc.

These linked models illustrate how the actions of the individual of the first generation to maximize his utility function (6:1) subject, i.a., to (6:3), will affect the initial possibilities of individuals of the second generation. As shown by (6:15) the initial human capital stock of an individual of the second generation depends on the actions of the individual of the first generation.

As for the exact meaning of the argument in the individual's preference function that says he is interested in having and bringing up children one extreme may be that this argument is the wishes of the child. The individual cares about the wishes of the child and gets pleasure from doing his best to see them fulfilled. Then the U of the child should enter as B in the U of the parent. If, e.g., the child wants his lifetime earnings to be maximized, his lifetime earnings would be the argument in the parent's preference function. The latter would then invest his time and market resources to provide the child with the best possible start in this respect. How much the individual would invest in the child would depend on:

- his preferences for B (exemplified above by the lifetime earnings of the child) compared with the other arguments in his preference function.
- his potentialities. Due to the "income effect" an individual who is "wealthier" in the sense that he could have more of all the welfare aspects in which he is interested [1] would - assuming that child welfare is a normal good - choose to have more of that good.
- relative prices. The higher the price of "child welfare", i.e., the more costly its production, the less the individual could be expected to choose of this good.

As regards relative prices a complication arises - as time may be an important factor in bringing up the child. For an individual with high potential earnings this good may then be relatively expensive and he would be induced to have less of it due to the "substitution effect." On the other hand, higher earnings could, as mentioned above, mean a higher demand for this good [1] - due to the "income effect." The net effect of a higher relative price and higher earnings could therefore still be an increase in demand if the "income effect" outweighs the "substitution effect."

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[1] Higher initial endowments and ability make an individual "wealthier."

We would also expect individuals with higher portential earnings to have a more goods- than time-intensive type of child welfare production. If child welfare is a more time-intensive type of production, individuals could also be expected to substitute other welfare commodities for it (i.e., C_i for B_{ij}).

The children of individuals whose child welfare preferences are stronger and whose parents are "wealthier" [2] could be expected to have higher earnings (welfare). The same effects result with greater efficiency in the production of B if there are efficiency differences among parents in this respect. As regards price differences, individuals who have to pay a higher price for the goods purchased for use in child-caring activities would tend to substitute human capacity for purchased goods. As they will also be "poorer," because of the higher price, they will tend to demand less of all normal goods including child welfare, assuming it is a normal good. Individuals who are discriminated against on the labor market (who receive a lower rental price for their human capital) would also be "poorer" and would demand less child welfare if the "income effect" is not offset by the "substitution effect."

A paternalistic and perhaps more realistic alternative for the preference function of the parent may be a preference function that has as an argument the aspects the individual thinks the child ought to be interested in (e.g., the same things as he himself is interested in). If this does not coincide with the wishes of the child, the individual's investments would be misdirected from the point of view of the child. The parent's demand for what he thinks is the correct child welfare is, of course, determined by the same factors as described above and will have the same result for child welfare, at least as measured by the definition of child welfare made by the parent.

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[1] Earnings from non-labour sources would only have an "income effect."

[2] Provided that child welfare is a normal good and that the "income effect" outweighs the "substitution effect."

As for possible intergenerational conflicts, we saw in Section 5 that investment incentives vary with the preference function. The parent and the child could have different opinions about how great an investment should be made in the child and thus disagree as to what preference function the parent should apply. However, as long as the stock of human capital is assumed to be homogeneous there could not be any conflicts regarding its composition. Such effects could arise if the parent has a choice between investing to affect the ability to produce human capital (e.g., β_0, β_1 in a production function for human capital) or to affect the stock of human capital. Moreover, if the decision-making period of the parent overlaps the beginning of the child's own decision-making period, they could have different opinions as to the time profile of investments.

The possibility exists, of course, that the individual may not be interested at all in what happens to the child later in life, but only in what he himself derives from his activities while rearing the child - e.g., the pleasure of participating in a developmental process. The demand for this "consumption" good would also be determined by the same factors as mentioned above concerning the demand for the child welfare: tastes, "wealth", production techniques, and relative prices. This good might be expected to be even more own-time-intensive. The net effect of individual difference in earning potentialities may therefore be smaller differences in the demand for such a consumtion good than for an investment good like future child welfare.

However, even though the individual is not interested in the child as an investment, his treatment of the child could still influence the child's further possibilities. This would require us to explicitly introduce this relation in order to have an intergenerational model. As in the formal model above, there would then be three elements:

- the consumption aspect of having children, which would appear in the parent's preference function (cf. 6:1);
- the production function for this consumtion good (cf. 6:3);
- a relation between the inputs used to produce this consumption good and the later investment possibilities of the child (cf. 6:15).

Our discussion above suggests, however, that only two elements might be necessary in case of an investment in the child:

- the argument in the parent's preference function that says what aspects of the child's investment possibilities the individual is interested in;

- the production function for this good.

In case of the formal model above this means that (6:3) and (6:15) would be identical.

When the individual is not interested in what happens to the child later in life there may, of course, be intergenerational conflicts over the investments in the child. This may result in a compromise whereby the preference function of the parent is replaced by a preference function representing the interests of both the parent and the child - i.e., a "household" preference function (cf. Section 7).

Another possible outcome of intergenerational conflicts is that the child leaves the individual earlier than the latter had planned. Whether the child has any incentives to do so would depend in these models on the relative benefits contributed by the resources invested by the parent compared with those that the child's own resources could produce. The child's resources were not considered explicitly in the model above.[1] The production function $B_{ij} = b(x_{ij}, t_{ij})$ could either be interpreted as defined for $c=1$, $v=l=0$ for the child, or $l=0$, $v=k$, and $c=1-k$, where k is a constant. With $v<1$ there might be incentives for the child to leave the individual.

The "individualistic" intergenerational models discussed so far describe intergenerational aspects rather abstractly. To be fully developed a number of specific questions that we have not dealt with must be considered, e.g.:

- The circumstances of the child's birth and the manner by which he comes under the jurisdiction of the individual ("parent");
- the relation between number of children and other aspects of rearing children;
- how and when the decision-making power is transferred from parent to child with regard to such questions as human capital invesments, for example.

However, the type of intergenerational models discussed here could be used as first approximations of mechanisms in the transfer of wealth in the form of human capital from one generation to another and for explaining general tendencies in the connection between generations.

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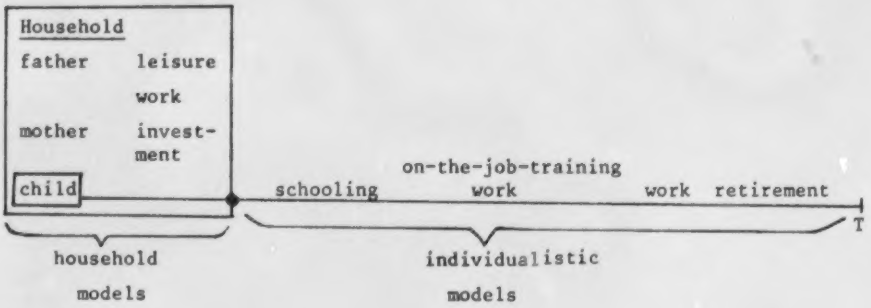
[1] This will be done in the household models.

As mentioned above, household models are an alternative to the intergenerational models of individualistic type. The individualistic models are unrealistic in the sense that children are not usually brought up by one individual, but rather in a family, and then grow up surrounded by a number of people, relatives, friends, etc. Instead of trying to incorporate these diverse aspects into the individualistic models, it may be preferable to include them in household models, reserving the individualistic models for use in the analysis of general long-run tendencies in intergenerational transfers of wealth.

A household approach may be suitable for use in analyzing a few more - though, of course, not all - complexities in child rearing compared with the intergenerational models of individualistic type. It will probably not be practicable in one model to analyze more in detail the household context, the upbringing of the child, what this upbringing means for the child with respect to further investment possibilities, etc., as well as how successive generations of individuals and generations of households are related to one another. In the latter case one would also have to treat the question of how households are formed and dissolved. A game theoretic approach might be more appropriate for the latter aspects, though it is by no means clear how this would fit in with an equilibrium setting. By using the household models for more static and detailed questions one could - for given households - analyze their behavior vis â vis children. These results could be used as inputs in models of the type discussed in Sections 2, 3 and 5, for example. The transition of children from given households to schooling and working life could then be studied as indicated in Figure 44.

However, the household models were not primarily developed for the analysis of such intergenerational questions. They have, e.g., been used to study how the capacity of the different individuals within a household could be expected to be used for different activities - work at home, on the labor market etc. As this gives a new dimension to these choices, compared with those already discussed in previous sections, we will in Section 7 first describe this approach in general terms and show how it complements our earlier models. After that, we will return to the intergenerational aspects.

Figure 44 Combining household and individual models in an intergenerational perspective



6.2 Special Features of the Individualistic Models

The human capital models in this section are sketched rather roughly. They could be used, however, for exploratory purposes to analyze investments made in children. Important aspects in the models were:

- the situation of the individual making these investments, as described by the preference function and the production functions for the arguments in the preference function;
- the "wealth" of the individual;
- the relative price of the child related argument in the individual's preference function.

The child related argument in the preference function could either be directly related to variables of importance for the child's future investment possibilities or only indirectly related to these variables. The latter is the case when the child-care activities are seen as a consumption "commodity." We also indicated the possibility of studying intergenerational conflicts regarding these investment decisions with the help of these models. They may also be interesting from a policy point of view, as they point out a number of factors that influence the incentives to spend time in child-care activities. Some are related to benefits

- what the individual gets out of the resources devoted to bringing up children: pleasure in the activity, or interest in the child's future welfare or earnings.

Others are related to the cost side:

- direct investment outlays for the purchase of the market goods that goes into the child-care activity;
- opportunity costs associated with the diversion of the individual's capacity from other valuable activities to that of child-care.

Disregarding legal enforcement of cartain behavior, the incentives to invest in children could be reinforced both via returns and costs. The former could be increased by making people more interested in children - e.g., through general or special educational programs for parents. Costs could be influenced by subsidizing the market good used in child investment, by subsidizing the use of human capacity in child investment, and by making people invest more efficiently in children.

For many reasons the marginal investment in the child may not be "financially" profitable. The individual may in his decision not only be motivated by future earning possibilities of the child and the child may not use his human capital stock exclusively for earning purposes.

The implications for the observable variables on which these models could be tested are:

- capacity (or time) use of the parent, for different purposes, i.a. child-care activities;
- use of market goods by the parent for different purposes, i.a. child-care activities;
- indicators of the human capital stock or ability of the child (IQ, schooling, etc.);
- length of the period of specialization in training of the child;
- earnings (welfare) of the child.

As for the implications of these models, let us summarize a number of qualitative statements that could be used as starting points for empirical investigations and regression analysis.

On the assumption that the "child" argument in the preference function of the individual is investment-related and a normal good, we would expect that individuals having the same preference and production functions would:

- with greater wealth make more substantial investments;
- with higher potential earnings, be induced to make more investments if the positive income effect outweighs the negative substitution effect;
- with higher potential earnings, produce child welfare, more goods-intensively than time-intensively; and that
- children of such individuals would have higher lifetime earnings or welfare.

Individuals with less pronounced preferences regarding child welfare and less efficiency in its production are also expected to invest less in their children. In disequilibrium situations, individual differences in the prices paid for the market good that goes into the child-caring activities, the rental price for human capital and the rate of interest would also influence the behavior of the individual with respect to investments in the child. In these respects less favourably treated individuals could be expected to invest less in their children, since they would be poorer. Their input combinations would also be influenced.

To test the models for maximum explanatory power, to be able to interpret the results in terms of these models, and also to avoid biases, one ought to standardize for:

- variations in the preference functions. Groups with different attitudes to children and education could, e.g., be distinguished.
- variations in the production functions. Groups with different aptitudes to produce "child welfare" could be identified through preliminary analysis of the relation between the amount of child-care activities and the results. It would, however, be important to include not only quantity but also quality indicators for the parent-child interactions.
- non-equilibrium. Groups paying different prices for market goods and with different borrowing possibilities could be distinguished. Due to regulations, individuals may be forced to invest more (in case of compulsory education) or less (in case of restricted entry to certain educational levels) in their children than they wish to, a factor that also has to be taken into account in order to avoid biases.

Once the investment process gets started, the stock of human capital will increase from one generation to another as long as each generation has the

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same preference functions, human capital production possibilities do not deteriorate, and child investment remains a normal good. This may be one mechanism through which the steady state economy remains on its growth path with a constant population. International cross-section data and national time series data could be investigated to find out to what extent they could be reconciled with models of the type discussed in this section. And more precisely what type of individualistic intergenerational models would perform best.

Returning to the Schooling Models, these intergenerational models indicate a certain number of explanatory factors that could be incorporated in a simultaneous or recursive way with the former models. As we have seen in this section the intergenerational link, once the child starts his own decision-making, is an exogeneous datum. If schooling decisions are taken by the child given the investments of the parent, the intergenerational models are of interest as they could be used to identify groups homogeneous with respect to schooling potentialities. If schooling decisions are better analyzed within the "home" setting, this may either be done in terms of of the "individualistic" intergenerational models of this section or the household models to be described in Section 7. The "individualistic" intergenerational models do in this perspective first of all indicate certain factors that explain how much has been invested in the child before schooling begins. Secondly, the analysis of the schooling decisions may be similar to that of Section 2 (though enlarged with the input of the resources of parents) - i.e., if parents are interested in the maximization of the child's lifetime earnings. They do, however, point to the fact that these investments have to be related to other uses of the parents' time (as did the models of Section 5 point to the fact that "own investments" in schooling has to be compared with alternative uses of the individual's time) as well as they do indicate what factors that may cause differences among parents in this respect. These models also point out the theoretical possibility that schooling even in an intergenerational perspective may be much of a consumption activity (as did the last model of Section 5 from the point of the individual).

7. Household Models

The household models to be discussed in this section are structurally similar to those in Sections 3 and 5. Thus, all members of the household are assumed to have a common preference function - which includes production functions for preferred "commodities". In these production functions that are common to the household, the capacity of more than one individual is the input factor. As a rule, the household has to decide such matters as how much goods to buy on the market, how much of the capacity of each individual in the family should be devoted to different activities, i.a. work and human capital formation. The investment activity could aim at increasing the human capital stock of the individual himself ("own investments") or that of other members in the family - e.g., that of children. The household perspective may be useful as a complement to the "individualistic" type of models discussed earlier, as they point out further explanatory variables for investment choices. As to "own investment" in human capital, we will only note the type of new explanatory variables that emerge with a household approach and discuss at somewhat greater length the intergenerational perspective in household models.

7.1 Household Models and ''Own Investments'' in Human Capital

Let us first discuss how the household situation may influence decision-making as regards the individual's "own investment" in human capital.

Let us first see what a household model corresponding to the first model in Section 5 would look like. For two individuals - labeled 1 and 2 - we would have:

(7:1) $\text{Max } U = U\,[\{C_i\}] \qquad i = 0,\ldots.\ T-1$

subject to

(7:2) $C_i = g(x_i, c_{i1}, c_{i2})$

(7:3) $c_{i1} + l_{i1} + v_{i1} = 1$

(7:4) $c_{i2} + l_{i2} + v_{i2} = 1$

(7:5) $H_{01} = \bar{H}_{01}$

(7:6) $H_{02} = \bar{H}_{02}$

(7:7) $H_{(i+1)1} = H_{i1} + Q_{i1}$

(7:8) $H_{(i+1)2} = H_{i2} + Q_{i2}$

(7:9) $Q_{i1} = \Psi_1 \, (v_{i1}H_{i1})$

(7:10) $Q_{i2} = \Psi_2 \, (v_{i2}H_{i2})$

(7:11) $$\sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} = \sum_{i=0}^{T-1} \frac{aH_{i1}l_{i1}}{(1+r)^i} + \sum_{i=0}^{T-1} \frac{aH_{i2}l_{i2}}{(1+r)^i}$$

While each individual in the household is subjected to his own capacity restriction (7:3, 7:4), the common resources of the pair are used - as seen from (7:2) and (7:11) to maximize the preference function (7:1).

Given the Lagrangian function

$$L = U[\{C_i\}] - \lambda\left[\sum_{i=0}^{T-1} \frac{x_i}{(1+r)^i} - \frac{aH_{i1}l_{i1}}{(1+r)^i} - \frac{aH_{i2}l_{i2}}{(1+r)^i}\right] - \sum_{i=0}^{T-1} \sum_{j=1}^{2} \mu_{ij}(c_{ij}+l_{ij}+v_{ij}-1)$$

the necessary conditions for an interior solution would be

(7:12) $$\frac{\partial L}{\partial x_i} = \frac{\partial U}{\partial C_i} \frac{\partial g}{\partial x_i} - \lambda \frac{1}{(1+r)^i} = 0$$

(7:13) $$\frac{\partial L}{\partial l_{ij}} = \lambda \frac{aH_{ij}}{(1+r)^i} - \mu_{ij} = 0 \qquad j = 1,2$$

(7:14) $$\frac{\partial L}{\partial c_{ij}} = \frac{\partial U}{\partial C_i} \frac{\partial g}{\partial c_{ij}} - \mu_{ij} = 0 \qquad j = 1,2$$

(7:15) $$\frac{\partial L}{\partial v_{ij}} = \lambda\left(\sum_{m=i+1}^{T-1} al_{mj} \frac{\partial H_{mj}}{\partial Q_{ij}} \frac{\partial Q_{ij}}{\partial v_{ij}} \frac{1}{(1+r)^m}\right) - \mu_{ij} = 0 \qquad j=1,2$$

From (7:12 - 14) we have:

(7:16) $$\frac{\partial U}{\partial C_i} = \lambda \frac{1}{(1+r)^i} \quad \pi_i$$

(7:17) $$\pi_i = \frac{1}{\partial g/\partial x_i} = \frac{aH_{i1}}{\partial g/\partial c_{i1}} = \frac{aH_{i2}}{\partial g/\partial c_{i2}}$$

and from (7:14 - 15):

(7:18) $$\frac{aH_{ij}}{(1+r)^i} = \sum_{m=i+1}^{T+1} a \, l_{mj} \frac{\partial H_{mj}}{\partial Q_{ij}} \frac{\partial Q_{ij}}{\partial v_{ij}} \frac{1}{(1+r)^m} \qquad j = 1,2$$

In our earlier models the idividual had to decide how his own capacity should be used. Many possibilities were open: specializing in training, work or leisure, a combination of work and training, etc. In a household the same type of decision has to be made. However, in the household context the result of the decision-making process may differ. There are two basic reasons why this may be so. One is that the preference function of the household (7:1) may be different from that of the individual. The other is that the production functions for the welfare "commodities" must change as they are to include the capacity of more individuals (7:2). It would then no longer be sufficient for the individual to compare his marginal efficiency in consumption activities with that of the market good. He would also have to compare his marginal efficiency with that of the other members of the household (7:17). In order to maximize the common preference function the allocation of time of each member will be determined so that the marginal efficiency of each member in each activity and over activities is equalized.

Both these factors - changes in the preference function as regards the arguments it includes and their evaluation, and also the relative efficiency of each member of the household in performing different activities - may influence the individual's investment incentives. If two individuals entering a household relationship are not as efficient on the margin at their former level of, e.g., leisure activities given the household evaluation there will be a redistribution of the utilization of the capacity of each.

An interesting factor - with certain implications for "own investments" in human capital - is whether two individuals in a household will specialize in their activities - e.g., one in the labor market and the other in the home. The person making the latter choice would have zero labor market activities ($l=0$). The former would not have zero leisure activities ($c=0$), but rather a minimum or restrained level of leisure activities ($c=\bar{c}$).

Now, why should individuals specialize in this way? Let us begin by assuming that prior to entering a household relationship, the two individuals have used their capacity for both work and consumption activities and that they possess an equal amount of human capital. From the point of view of the household they would be equally valuable on the labor market and in line with our models we may assume that there are no returns to scale in that activity. The only reason then for specialization would be indivisibilities and increasing returns to scale in activities at home.1) Either of the individuals could then be chosen to specialize in household activities. If there are differences as to the amount of human capital investments that would be optimal during one career or the other, these differences would influence "own investments" and bind the individuals to their respective activities. If an equal amount of human capital investments is optimal, there would be no binding effects, as we have assumed human capital to be homogeneous.

The possibility of specializing in the future in either work or homemaking activities in a household situation may also influence the behavior of individuals before the household period. An individual who expects to specialize in homemaking activities may make a smaller investment in human capital. However, this would only be true if human capital is of less value for home activities than for "work".

If, for whatever reason, either of the two individuals who enters a household situation has more human capital than the other, the analysis as regards specialization and further human capital investments would be the same as above. The marginal value of each individual's capacity for each activity should be equal. Whether this implies specialization in work or home activities and more or less human capital investments by different members of the household depends on the shape of the respective production functions.

If human capital does not influence efficiency in home activities as in (7:2), it would be optimal for the individual with less human capital to specialize in home activities, while the other specializes in market activities where the amount of human capital makes a difference in financial terms. In the same context, if an individual expects to specialize in home activities there would, of course,be smaller investment incentives. And the opposite would be true for one who expects to specialize in market activities where human capital influences the returns.

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1) One might, of course, also assume that the efficiency of the one stock is greater than the other, e.g., according to

$$C_i = a\, x_i^{\alpha}[w_1(t_{i1}\ H_1) + w_2(t_{i2}\ H_2)]^{1-\alpha} \qquad \begin{array}{c} 0 < \alpha < 1 \\ w_1 \neq w_2 \end{array}$$

where a, α, w_1 and w_2 are constants and 1 and 2 indicate the respective individual. However, this is contrary to the homogeneity approach that we have used throughout this paper.

It is not our intention here to explicitly introduce and experiment with various specifications of the preference function and the different production functions. This is a vast field of research in itself. It obviously has implications from a human capital point of view, as it introduces new explanatory factors that influence the investment behavior of the individual before, during - and after - a household period. So far, we have seen these factors to be the importance of human capital for different types of activities, expectations about specialization, "locking in" effects, the efficiency of the individual in different household activities relative to that of other members of the household, and the evaluation of different activities at the household level. Also in this context a stepwise research procedure would seem adequate - first investigating, e.g., household production functions and preferences, and later building more complex models explaining, i.a., investment behavior.

There are, of course, many other possible model formulations than the ones discussed so far. One could hypothesize, for example, that the level of one individual's human capital stock influences that of other grown-ups in the same household.1) However, we will not elaborate the analysis of "own investments" in the household context further but return back to the question about investment in children.

7.2 Hosehold Models and Intergenerational Investments

Household decision-making with respect to children has been analyzed , for instance, by R.J. Willis (1973) and G.S. Becker and H.G. Lewis (1973) 2). A. Leibowitz (1974) and C.R. Hill and F.P. Stafford (1974) have made empirical investigations of the importance of home investments in children, based on the household approach.3) Studies of this kind have generally been

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1) L. Benham, "Benefits of Women's Education within Marriage." Journal of Political Economy, Vol. 82, p.557, sketches a model for human capital accumulation within the household, where the level of the stock of human capital of the husband influences the level of the stock of human capital of the wife, and vice versa.

2) R.J. Willis, "A New Approach to the Economic Theory of Fertility." Journal of Political Economy, March/April, 1973.
G.S. Becker, H.G. Lewis, "On the Interaction between the Quantity and Quality of Children." Journal of Political Economy, March/April, 1973.

3) A. Leibowitz, op.cit.
C.R. Hill, F.P. Stafford, Family Background and Lifetime Earnings. Department of Economics, University of Michigan, Mimeo 1974.
C.R. Hill, F.P. Stafford, "Allocation of Time to Preschool Children and Educational Opportunity." Journal of Human Resources, 1974,No.3.

undertaken from the perspective of schooling (cf. C.R. Hill, F.P. Stafford (1974, 1978) D.O. Parsons (1975, 1978) [1]). J. Mincer and S. Polachek (1974) used the earnings function which they have developed for women (cf. Section 4.4) in order to estimate the investment costs for children.[2] They did not relate it to the further development of the children.

The "household" type of intergenerational models to be discussed here are inspired by these works, although we relate them to the investment models of Sections 3 and 5.

We shall not discuss the consumption aspect (including indirect investment effects) of bringing up and taking care of children.[3] We assume that the "development possibilities of children" represents an argument in household preference functions and that normal household decision-making theory can be applied to that commodity. A simple formal model for a household consisting of two grown-ups (individual 1 and 2) and one child (individual 3) could be constructed as follows.[4] The analysis starts when the child is born - age zero. The planning horizon for the household extends to date F. At a predetermined date, S, the child is assumed to leave the household and make decisions for himself, in accordance with a model of the same type as the Basic Model in Section 3. An argument in the preference function of the household is the human capital stock of the child at date S (H_{S3}). An exogenous maturing process is assumed, and the original human capital stock the child therefore also grows autonomously ($\bar{H}_{03}(i)$). The grown-ups in the household are not assumed to invest any further in themselves. The only investment decision thus concerns the investment in the child. Given these assumptions and extending the first model of Section 5, the household situation could be described by the following model:

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[1] C.R. Hill, F.P. Stafford, ibid.
C.R. Hill, F.P. Stafford, "Intergenerational Wealth Transfers and the Educational Decisions of Male Youth: an Alternative Interpretation." Quarterly Journal of Economics, 1978, No. 3.
D.O. Parsons, "Intergenerational Wealth Transfers and the Educational Decisions of Male Youth." Quarterly Journal of Economics, 1975,No.4.
D.O. Parsons, "Intergenerational Wealth Transfers and the Educational Decisions of Male Youth: the Mother's Hometime Hypothesis." Quarterly Journal of Economics,1978, No.3.

[2] J. Mincer, S. Polachek, op.cit.

[3] As shown in the preceding section, models including investment aspects of having children could easily be turned into models having only consumption aspects.

[4] As in the previous section, we do not discuss how children and how many children are brought into existence and under the jurisdiction of the household. Different authors have treated these questions, cf.,e.g.:
G.S. Becker, N. Tomes, "Child Endowments and the Quantity and Quality of Children."Journal of Political Economy, August, 1976, II.
S.H. Cochrane, "Children as By-products." Population Studies 1975,No.3.
M. Fulop, "A Survey of the Literature on the Economic Theory of Fertility." The American Economist, 1977, No.1.
B.A. Tuchi, "Microeconomic Theories of Fertility: a Critique." Social Forces, 1975, No.1.

(7:19) $\text{Max } U = U[\{C_i\}, H_{S3}]$ $\quad i = 0....F -1$

subject to

(7:20) $C_i = f(x_i, c_{i1}, c_{i2})$ $\quad i = \ ....F -1$

(7:21) $H_{S3} = \bar{H}_{03}(S) + \sum_{i=0}^{S-1} Q_{i3}$

(7:22) $Q_{i3} = q(x_i, t_{i1}, t_{i2})$ $\quad i = 1....S$

(7:23) $c_{i1} + t_{i1} + l_{i1} = 1$

(7:24) $c_{i2} + t_{i2} + l_{i2} = 1$

(7:25) $\sum_{i=0}^{F-1} \frac{x_i}{(1+r)^i} = \sum_{i=0}^{F-1} \frac{aH_1 l_{i1}}{((1+r)^i} \quad \sum_{i=0}^{F-1} \frac{aH_2 l_{i2}}{(1+r)^i}$.

According to (7:23) and (7:24), the capacity of each of the grown-ups has to be divided between the three activities: leisure (c), work (l), and education of the child (t). This has to be done in such a way that (7:19) is maximized subject to the budget constraint (7:25).

As for the education of the child an alternative would be

(7:26) $Q_{i3} = q(x_i, t_{i1}, t_{i2}, H_1, H_2)$, or more specifically

(7:27) $Q_{i3} = q(x_i, t_{i1} \cdot H_1, t_{i2} \cdot H_2)$.

With this formulation the human capital stock of those who interact with the child makes a difference for him. However, this reformulation would be more interesting if we had not assumed the human capital stock of the grown-ups to be given, as these alternatives would have important implications for the optimal investment pattern of the individuals preparing for child-care activities.

The child is assumed to maximize

(7:28) $\text{Max} \sum_{i=S}^{T-1} \frac{1}{(1+r)^i} (1 - v_{i3})\, a\, H_{i3}$

subject to

(7:29) $H_{S3} = \bar{H}_{S3}$

(7:30) $H_{(i+1)3} = H_{i3} + Q_{i3}$ $\qquad i = S.....T-1$

(7:31) $Q_{i3} = \beta_0(v_{i3}H_{i3})^{\beta_1}$ $\qquad i = S.....T-1$

(7:32) $v_{i3} + l_{i3} = 1$

As discussed in Section 6, other possible assumptions would be that the child-relevant argument in the preference function equals either β_{03} with $\beta_{03} = \beta_0(x_i, t_{i1}, t_{i2})$, or β_{13} with $\beta_{13} = \beta_1(x_i, t_{i1}, t_{i2})$.

The types of household models sketched here highlight the questions of how much of the grown-ups' capacity should be devoted to the education of children, as compared with other possible uses of that capacity.

The model makes it possible to analyze decisions about investments in the child in terms of income and substitution effects. If the commodity "development possibilities of the child" is seen as a normal "good", higher incomes would induce a household to obtain more of this good. The substitution effect resulting from a higher relative price for this good (i.e. a higher price for the market goods and the human capacity that goes into its production) would reduce the household's interest in this good. As noted above in connection with the "individualistic" type of models, these effects are mixed to a certain extent. Therefore, let us examine these phenomena a bit more closely.

If a household had non-labor earnings, the effect of such earnings would not be difficult to analyze. Households with higher non-labor earnings would demand more of all normal goods. Consequently, if "development possibilities of the child" is a normal good, households with greater non-labor earnings would invest more in their children.

The effect of changes (or differences) in earning capacity can be of the same type as for non-labor earnings. This is the case when the earning capacity of an individual who is not participating in the production of "development possibilities of the child" is raised or is higher than in other households.

When there is an increase in the earning capacity of an individual who is participating in this type of production, a reallocation of duties within the household could result. Besides that effect there would be two other effects. The cost of producing "development possibilities of the child" and also of producing other welfare "commodities" using this human capacity would rise. In all such production activities there would be a tendency to substitute market goods for human capacity. However, there may also be a substitution between activities. Certain activities may be more capacity-intensive than others. The capacity-intensive type will be relatively more expensive and, if possible, replaced by other activities. The production of "develoment possibilities" may be one example of a capacity-intensive activity. Households where the potential earnings of the individual who is in charge of this activity is increased would thus be expected to demand less of this good and more of other welfare commodities (C_i).

However, a great earning potential influences not only the relative price of goods but also earnings. Increasing earnings may have a positive income effect. The latter influence may even be strong enough to wipe out a negative substitution effect.

In brief, we could say that, assuming "development possibilities of the child" is a normal good

- households with higher non-labor earnings and higher earnings for individuals not participating in the education of the children would invest more in children;
- households with higher earnings for the individuals participating in the education of children would invest more if the income effect dominates over the substitution effect.

However, these factors are not the only ones in this model that create differences among households with respect to investments in children. As always, differences in the preference function also lead to different choices. To what extent are different households interested in children? As child welfare only materializes later in the child's life, less imaginative or more shortsighted households may demand less of this good.

The investment costs are the sum over the investment period of

- the market goods bought for this purpose;
- the value of the human capacity used for this purpose.

If the individual uses some of his capacity for work activities the capacity part of investment costs could be measured in terms of the loss of potential earnings. This potential loss would, of course, be greater than the loss of actual earnings if some investment in the individual himself is being made. In models in which depreciating human capital is a factor, there may also be a cost due to depreciation, to the extent that human capital depreciates differently depending on the activities of the individual.

On what conditions would there be a definite relation between these investment costs and the investment behavior or earnings of the child? To answer these questions let us first consider the following one:

What difference does it make for the child that more or less has been invested in him at "home"? To answer that question we could use the results from the Basic Model of Section 3:

- individuals who leave the "home" with a greater human capital stock or more ability (β_0) will have greater lifetime earnings;
- individuals who have a greater human capital stock will have a shorter period of specialization in training, but the increases in earnings after specialization will not differ from those of individuals with a smaller stock of human capital;
- individuals with more ability (β_0) would have a longer period of specialization in training and larger increases in earnings after specialization.

Models including leisure activities of the type discussed in Section 5 suggest that the welfare of children in whom more has been invested is greater than the welfare of children in whom less has been invested. Lifetime earnings are not necessarily increased.

To answer the question about investment costs (C) and earnings of the child the discounted value of the child's earnings would be raised by the same amount - C 1) - if

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1) $$C = \sum_{i=0}^{S-1} \frac{p_i x_i + \sum_j w_{ij} t_{ij}}{(1+r)^i} = \Delta Y = \sum_{i=0}^{T-1} \frac{a \Delta H_S}{(1+r)^i}$$

w_j = value of time of household member i
ΔH_S = increment in human capital stock due to household investments
r = market rate of interest.

- average and marginal rates of return on "home" investments are equal;
- the returns to "home" investments are equalized by the market;
- the investor and the child are only interested in financial returns to human capital and there is no specialization period for the child after leaving "home" or marginal returns are constant for investments made by the child.

We could have another variation on the theme of the household and children if we viewed the children as participating in the production processes of the household. One plausible version - which does not seem to have been envisaged in the literature - would result if children are assumed to be participating in the production activities of households, i.e., in the "production" of the arguments in the preference function. In the simple formal model below, increments to the children's stock of human capital are assumed to be produced only by inputs of market goods and the children's own capacity.

(7:33) $\text{Max } U = U[\{C_i, Z_i\}] \qquad i = 0,\ldots,F-1$
subject to

(7:34) $C_i = g(x_i, c_{i1}, c_{i2}, c_{i3} \cdot H_{i3})$

(7:35) $Z_i = z(H_{i3})$

(7:36) $c_{i1} + l_{i1} = 1$

(7:37) $c_{i2} + l_{i2} = 1$

(7:38) $c_{i3} + l_{i3} + v_{i3} = 1$

(7:39) $H_{i1} = \bar{H}_1$

(7:40) $H_{i2} = \bar{H}_2$

(7:41) $H_{03} = \bar{H}_{03}$

(7:42) $H_{(i+1)3} = H_{i3} + Q_{i3}$

(7:43) $Q_{i3} = q(x_i, v_{i3} \cdot H_{i3})$

(7:44) $$\sum_{i=0}^{F-1} \frac{x_i}{(1+r)^i} = \sum_{i=0}^{F-1} \frac{aH_i \cdot l_{i1}}{(1+r)^i} + \sum_{i=0}^{F-1} \frac{aH_2 \cdot l_{i2}}{(1+r)^i} + \sum_{i=0}^{F-1} \frac{aH_{i3} \cdot l_{i3}}{(1+r)^i}$$

This type of model highlights the problem of how best to use the capacity of the child from the point of view of the household. As seen from (7:34), the child is assumed to participate in the production of the welfare "commodity" C, and the level of the human capital stock of the child has significance for this activity. However, according to (7:33) and (7:35) the household is also interested in the level of the human capital stock of the child itself - an expression of its interest in the child's future well-being. The level of the child's human capital stock develops according to (7:43) and depends, i.a., on how much of the child's own capacity is put into human capital production. The budget constraint (7:44) ensures that the earnings of the three members of the household will cover the outlays for market goods.

According to (7:43), child capacity and market goods are the only inputs for the development of the human capital stock of the child. This means that variationns in the earnings of other members of the household would only have income effects. More productive children would be more expensive to invest in. But, since there will be an income and a substitution effect of higher child productivity, the net effect may still be greater investment incentives.

In this model we have not assumed that the child leaves the household at a fixed date. The possibility that the child leaves the household could be incorporated, e.g., by making the possibility of c_{i3}, l_{i3} and v_{i3} equalling zero an increasing function of H_{i3}. In that case a risk would be involved in child investments, which would have a deterrent effect on household investments in the child. However, as the household is interested in the level of H_3 itself, the moment when H_3 reaches the critical value that causes the child to leave "home" may still be approached.

Still another version would result if we assume that the household has some market oriented production activity in which "child" - or "youth" - labor is used. M.R. Rosenzweig (1977) e.g. added to the household approach such a "firm" activitiy for the family and made an empirical study of farm-families.[1] A formal model in line with the one presented above may look as follows:

(7:45) $\text{Max } U = U[\{C_i, Z_i\}] \qquad i = 1,\ldots,F-1$
subject to

(7:46) $C_i = g(x_i, c_{i1}, c_{i2})$

(7:47) $Z_i = Z(H_{i3})$

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[1] M.R. Rosenzweig, "Farm-family Schooling Decisions: Determinants of the Quantity and Quality of Education in Agricultural Populations." *Journal of Human Resources*, 1977, No.1.

(7:48) $c_{i1} + l_{i1} = 1$

(7:49) $c_{i2} + l_{i2} = 1$

(7:50) $l_{i3} + v_{i3} = 1$

(7:51) $H_{i1} = \bar{H}_1$

(7:52) $H_{i2} = \bar{H}_2$

(7:53) $H_{03} = \bar{H}_{03}$

(7:54) $H_{(i+1)3} = H_{i3} + Q_{i3}$

(7:55) $Q_{i3} = q(x_i, v_{i3} \cdot H_{i3})$

(7:56) $O = o(x_i, l_{i1} \cdot \bar{H}_1, l_{i2} \cdot \bar{H}_2, l_{i3} \cdot H_{i3})$

(7:57) $$\sum_{i=0}^{F-1} \frac{x_i}{(1+r)^i} = \sum_{i=0}^{F-1} \frac{p_o O}{(1+r)^i} .$$

where O = output of the "household-firm" market good

p_o = price of this good.

In this perspective the investment decision will also involve the question of how much of the capacity of youth that should be taken away from the "household-firm" (7:56) to develop the human capital of the youth (7:55). Moreover, what amount of market goods should be purchased for this purpose? The investment costs are thus measurable in terms of foregone sales revenues and direct investment outlays.

The gain to the "household-firm" from such human capital accumulation would take the form of increased future firm production possibilities and sales revenues. Another benefit of the development of the child may also result if it figures as an argument in the preference function of the household (cf. 7:45 and 7:47). As in the model above, a negative effect may be included if the risk of youth quitting the household increases with the level of human capital.

7.3 Special Features of the Household Models

In a human capital context the household models could be used as complements to the individualistic models. One area of applicability is the changes in incentives for "own investments" in human capital brought about by the household context. These changes relate both to the period before, during and after a household phase. The importance of human capital for different types of activities, expectations about specialization, actual specialization, and the preference and production functions in the household context would be new explanatory variables compared with the individualistic approach. What is important for the use of individual capacity in different activities is the relative efficiency of the different members of the household.

In a household context there may be more room for specialization in, e.g., work and home activities, a factor which may in turn influence the investment behavior of the individual. However, neither such specialization, nor differential investment behavior among grown-up members of a household, or by individuals preparing for a household situation is a necessary outcome of the approach. What is decisive in this context is the specification of the welfare function and the production functions for the arguments in the household preference function. If an individual expects to specialize in home activities, for example, and if human capital does not influence such activities there would, of course, be small investment incentives. The opposite would be true for an individual who expects to specialize in work where human capital does make a difference.

The various assumptions relative to preference functions and production functions have very different implications for individual behavior. Therefore, as suggested in the preceding section, a stepwise research strategy seems to be appropriate and possible. Preliminary investigation of production technology, for example, could be undertaken before more definite models are elaborated.

As for the "household" type of intergenerational models we first discussed models where the children do not participate in the production activities of the household. The same type of factors discussed in connection with the "individualistic" type of models will determine the investment decisions. A difference arises, however, if one member of the household does not participate in the child investment activity. Greater earnings for that person will only have an income effect.

When children do participate in the production activities of the household - whether these activities are market oriented or not - a comparison must also be made of alternative uses of child capacity: home production/market oriented production against human capital investment,e.g.

To the extent that the home production is market oriented, farm household-firms being one example, the production function may be more easily defined The economic literature about firm production functions contains much information which might be useful in such analyses.

If child capacity and market goods are the only inputs for the development of the child, all effects of variations in the earnings of other members of the household will be of the income type. More productive children will be more expensive in terms of investments. However, the productivity of the child has both a substitution and an income effect. The net effect may still be greater investment incentives. However, more goods-intensive methods could be expected to be used.

We also indicated that if the risk that the child will quit the household depends on how much has been invested in him, this may constitute a deterrent to household investment in the child.

In a way similar to the "individualistic" intergenerational models of Section 6, the models in this section point out explanatory factors that should be taken into consideration when applying less extended models such as the Schooling Models. Important factors, the variation of which these models provide explanatory factors for, are the amount of resources that have been invested in the child before schooling starts, the amount of resources that during schooling are devoted by other members of the household and the individual himself to this activity compared with other activities.

8. The Human Capital Models – Interpretation, Empirical Application and Further Research Possibilities

The models presented in Sections 2 - 7 - each in its special fashion - describe how individuals make educational choices, what factors influence these choices and what relations results between the explanaotry factors, human capital and earnings.

In the Schooling Models the educational choice was only related to the length of an initial schooling period. In the Life Cycle Models not only the length of such a period of specialization but also the level of training in the form of on-the-job training and recurrent education later in life had to be decided. In Section 5 the perspective was further extended, so that these educational choices had to be made not only with regard to their effects on earning capacity but also to their effects on "leisure" activities of different kinds. Finally, in Section 6 and 7, the activities of grown-up individuals were coupled to the provision of initial starting possibilities for individuals of the next generation.

In presenting these models we have tried to stick to a common frame of reference, i.a. to be able to analyze to what extent they are reconcilable, what implications the more extended models carry for the empirical application of the less extended ones, and what a reasonable empirical testing strategy might look like. The purpose has also been to relate as much as possible to human capital models existing in the literature. Thus, we have covered most types of human capital models. At the end of this section when we discuss further theoretical development possibilities for the human capital approach, some models that lie outside the realm of this survey will be noted.

Earning differences are explained in the models in terms of two principal factors: inputs to the human capital production process and type of individual. This relation: resources → stock of human capital → earnings, was not, of course, the only result obtained from the models (nor was it that simple in all models). Other types of implications also emerged that are important to use in empirical work.

We have summarized the special explanatory variables in each group of models and the relations implied. In this connection we also discussed the implications that the more complete models have for empirical work dealing with the more restricted Schooling Models. From the aspect of educational policy it would be a rationalization if empirical work could concentrate on properly delineated Schooling Models. However, we are not going to review all these details here. As for explanatory variables, let us merely list the ones used in Sections 3 and 5:

preference ordering

- the arguments that enter the individual preference function:
  - leisure activities lumped together or divided in different groups,
  - work satisfaction,
  - educational satisfaction, etc.;
- the substitutability among the different arguments in the preference function at a given date and over time;

transformation possibilities

- production function of human capital (ability);
- net rate of depreciation of human capital;
- production functions of the arguments in the preference function;
- planning horizon;
- technical progress in the form of growth of the efficiency of human capital over time;
- technical progress in the form of growth of the efficiency of initial human capital from one generation to another;

endowments

- initial stock of human capital and ability.

The prices affecting individual behavior in these models were:

- the rental price of human capital;
- the rate of interest;
- the market price of inputs for human capital production;
- the market price of inputs for leisure activities.

These variables will explain such observable conditions as

- occurrence and duration of phases (schooling, on-the-job training, work, retirement);
- earnings and the present value of earnings;
- the division of capacity or "time" between activities on and outside the labor market.

The same type of variables - tastes, production possibilities, endowments, prices - will also determine how much different individuals will invest in their children. Simply investigating the explanatory power of these variables (or proxies for them) in relation to the different dependent variables represents a rich field for empirical work as discussed more fully in the respective sections.

Moreover, certain (more or less precise) qualitative implications were obtained from the models. These included how investment incentives, phases, earnings profiles, etc., are influenced by variations in the explanatory variables. The more specific the assumptions, the more exact the relations would be, for example, between explanatory variables and the earning profiles implied by the models.

It also becomes important, both for the empirical application of these models and for a general understanding of them, to see how they relate to general economic theory. Their general interpretation will be discussed in Section 8.1. In Section 8.2 their characteristics as part of a distributional theory are examined, while in 8.3 and 8.4 questions related to their empirical application and to further research are treated.

8.1 The Interpretation of the Human Capital Models

The human capital models that we have presented are obviously partial models in the sense that they do not explicitly describe the whole economy. Moreover, in the sector described - the production and rental of human capital - the emphasis is on individual decision-making. Still the models are supposed to explain such factors as realized earning differences, which makes it possible to interpret them in different ways. We may talk about a strong and a weak version, 1) with the strong version of the human capital models implying an equilibrium setting. Their results then hold for competitive equilibrium. Referring to general economic theory it is then also easy to see what type of development for the economy at large these models would require.

With the weak version, the human capital models are not restricted to describing situations of equilibrium. For whatever reason, individuals may

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1) Cf. Y. Weiss' lectures on human capital theory at Handelshögskolan, Stockholm, Spring 1980.

reckon with a given rental price for human capital, a given rate of interest, etc. and the given rental price, interest rate,etc. is also realized. The models are then viewed as true individual partial models holding ceteris paribus. Changes in the human capital sector or in the rest of the economy that alter these assumptions prevent the implications of the models from being realized.

There is also another sense in which we can speak of a strong and a weak interpretation. This has to do with the human capital concept itself, whether it is seen as an essential, functional element of the theory or not. In the latter case it might be eliminated and replaced, for example, by a direct relation between foregone earnings and future earnings.1)

The interpretation that is given to the human capital models is of great importance for their empirical application and for the understanding of the results. For this reason we will try to clarify what these interpretational differences mean.

If we make a strong interpretation of the human capital models in the two senses mentioned above, we obtain a consistent theoretical explanation of educational choices and realized earnings differences among individuals. The explanation of earning differences is then made in terms of variations in human resources - i.e., human capital as a "physical" capital concept related to individual productive capacity. However, this consistent explanation of earnings is only achieved at a cost. It only holds for restrictive settings.

When interpreting the human capital concept used in our models as a fundamental and essential description of productive capacity and its manner of creation, we may note that "human capital" is homogeneous. Consequently, the use of this assumption imposes certain restrictions. In Appendix A we set forth in some detail the general conditions under which we can have a homogeneous human capital concept that captures the physical productivity aspects of a multidimensional reality, e.g., as regards different skills.2) These conditions can be summarized as follows:

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1) Cf. S. Rosen, op.cit., 1977 /b/.

2) In Appendix B we briefly discuss macro production functions with homogeneous human capital exemplified with the Simple Schooling Model presented in Section 2.

- the productivity of each individual is independent of the configuration of the rest of the economy and its development;
- the stocks of human capital of different individuals are perfect substitutes, which also means that original human capital and that which is later produced are perfect substitutes;
- the individual ability to produce individual human capital is unidimensional.

In a steady state economy that remains on its given equilibrium growth path, such a homogeneity assumption would be satisfied. All factors of production (both those produced and non-produced ones) would then grow at the same rate. They would always be used in the same proportions, and marginal rates of substitution would remain unchanged. The economy would behave as though the conditions for having an aggregate human capital stock (H) as a simple sum measured in units of H are satisfied (cf. Appendix A). Moreover, we can use individual planning models to describe actual outcome.

We can thus visualize our human capital models as enacted in an intertemporal general equilibrium world, given a competitive, steady state economy, perfect information and foresight.1) We then have an orderly development of the economy over time. What is planned will be realized. There is a single market for the rental of human capital, where the rental price is constant and the same for all firms and individuals.

Such a strong interpretation of the human capital models offers many advantages. A clear demarcation of the area of application for the models facilitates empirical testing. We can avoid an "overestimation of the range of applicability" 2) of the models and refrain from using them in situations for which they are not suited, thereby misjudging their potentialities. If we apply them to situations for which they are known to be suited and they do not "fit", they can be rejected. At the theoretical level it may also be interesting to analyze what the restricticity of a possible (more complete) set of assumptions looks like, as this may stimulate a discussion of alternatives to be used for different purposes - distributional analysis and educational policy analysis, for example.

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1) Cf. T. Ishikawa, op.cit., 1975. This does not hold for the intergenerational models including both household and individualistic time segments as it is not clear how such a combination should be done theoretically.

2) T.C. Koopmans, op.cit., p.147.

Let us just add - to avoid possible misunderstandings - that a human capital approach does not necessarily have to be integrated in general economic theory by using these strong assumptions. Intertemporal general equilibrium models do normally not require that physical capital, for instance, can be aggregated.[1] Every unit of physical capital, every machine with special productive characteristics could, if necessary, be distinguished. The same holds for similar models including human capital.[2] Every individual and his specific innate characteristics and development possibilities could be accounted for separately.

In relation to such general models the strong interpretation of the human capital models can be seen as resulting from the imposition of a certain number of restrictive assumptions. They must then be understood in relation to the purpose of the theory. Distribution theory is one of the focal points of capital theory in general and of human capital theory in particular. As such, human capital theory is not primarily a theory for educational policy but rather for distributional analysis.

With the weak version of human capital models we cannot have a consistent theoretical explanation of realized earnings differences among individuals. What we can achieve is a conditional distribution theory, or - incorporating the weak human capital concept - a conditional description of income distribution. Certain problems also arise as to the interpretation of empirical results. If we test the implications of the models in terms of earnings profiles and explanatory variables, for instance, what we are essentially

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1) See, e.g., K.J. Arrow, F.H. Hahn, General Competitive Analysis. San Fransisco, 1971.
E. Malinvaud, "Capital Accumulation and Efficient Allocation of Resources." Econometrica, 1953, vol.21.
L.E.O. Svensson, "On competitive markets and intertemporal resource allocation." Institute for International Economic Studies, University of Stockholm, 1976.

2) Within this research project different alternative ways of introducing human capital in this type of models were discussed at a preliminary stage. A formal presentation of one alternative has been made by B.C. Ysander. (B.C. Ysander, op.cit., pp. 236-259.) It is an intertemporal general equilibrium model where a sequence of temporary equilibria is considered. A complete set of spot markets and a market for one-period loans exists at each date. Perfect information and perfect foresight are assumed. The agents are consumers, firms and holding companies. The latter manage the human capital, which can be rented to firms and is created by development activities at firms.

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doing then is applying an individual planning model to ex post market observations. Why should we expect there to be any special relation between these two worlds? If there is, the models have helped us to discover a real world regularity than can be used for predictive purposes. However, it still remains to be explained why that regularity is realized. Or the other way around, if the implications are falsified this does not mean that individuals do not plan as described by the models. There can be many reasons why a lifetime plan is not realized.

As for the weaker interpretation of the human capital concept, it does not seem to lay ground specifically for human capital investment models. If a relation exists between foregone earnings and future earnings (due to, for example, schooling), this may very well induce an investment type of behavior by individuals. Models describing this behavior can be constructed. However, as long as we have not specified why this relation exists, these models might as well be "sorting-investment" models as "human capital-investment"models.

While it is not uninteresting to be able to describe how individuals behave under certain restrictions, we cannot stop at that point if we want to arrive at a theory of distribution or a normative theory for educational policy. It is necessary to know why the relation between foregone and future earnings exists.

Since an important function of the strong version of the human capital models is to provide a distributional theory, let us specify what it would look like.

8.2 Human Capital Models as Distributional Theory

The human capital models presented contain two fundamental concepts - homogeneous human capital and earnings. By giving the models a strong interpretation, an endogenous explanation of the distribution of earnings is obtained. In a condition of equilibrium,marginal investments earn the same rate of return.[1] Moreover, in a steady state economy the rate of return is equal to the constant market rate of interest. The rate of interest would also be determined endogenously, along with the rest of the prices.

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[1] In the models including leisure the benefits outside the labor market will then be included in the rate of return for human capital investment.

The fundamental determining factors are preferences, technological possibilities, initial endowments and the maximizing behavior of individuals and firms. [1)]

The personal distribution of labor earnings now depends on how much human capital each individual possesses, how much of that capital the individual rents, and the return paid on the market for human capital. Starting out with certain initial endowments of human capital, the individual determines the extent to which his stock should be increased. Given assumptions about a competitive equilibrium, perfect information, no external effects in the production and use of human capital, and decreasing returns to scale in the production of human capital, there wouldbe no reason to expect the individual decisions to be made but ina socially optimal way, given the preferences of individuals. With the homogeneous human capital concept used here, education or human capital could hardly be a public good. However, a public sector financed by "neutral" (non efficiency disturbing) taxes - e.g.,on the rents on innate human capital and ability - is "compatible" with this solution. It would not prevent the rest of the resource allocation from working out efficiently. The same is true for an income policy that only consists of a neutral redistribution of rents.

The introduction of other types of income policies or of an educational policy would create efficiency problems in this setting. Such distribution policies or any educational policy have to be weighed against efficiency losses. With educational policy efficiency losses arise if, e.g., the educational costs of certain groups were subsidized, if mandatory schooling was enforced, and a numerus clauses introduced. Also in another sense is there no room for an income policy. A normal characteristic of models using aggregate capital concepts has been the indeterminancy of the functional income distribution - that between labor and capital. Some factor exogenous to the economic model has been needed to settle this question. With human capital models of the type discussed here, this particular question disappears.

As regards distributional policy there are three possibilities with these models: [2)]

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1) C.J. Bliss, op.cit., pp. 61-93.

2) Cf. G. Persson, op.cit., who sees human capital theory as always imposing a choice between efficiency and equality.

1) to pursue distributional policy by means of rents without coming into conflict with an efficient allocation of resources;
2) other types of income policies and, for instance, educational policy to influence earnings, but with losses of efficiency;
3) "wealth" policy directed at initial endowments - human capital, ability, preferences, technology.

As for the last point, this type of policy is to a certain extent more difficult to pursue with respect to human capital than physical and financial capital. As initial human capital and ability are embodied in the individual, they cannot be redistributed once created, though the endowments of future generations may be affected by policy measures. However, according to the intergenerational models the intitial stock of human capital of an individual consists both of an innate stock and the investments made by the parents, and the latter - changeable part - can also be expected to be taken care of on decentralized markets in an efficient way - at least from the point of view of the parents. If the efficient resource allocation is not to be disturbed and innate human capital and ability are assumed not to be influenced by policy measures, the only categories in group 3 with some human capital connection that are subject to policy actions are educational preferences and technology.

In the strong version of the models the efficient working of the decentralized decision-making and the endogenous character of the formation of human capital and earnings would tend to indicate that at least educational and income policy are not advisable as distributional policy. The tendency with these models would be to shift distributional policy to some kind of more remote "wealth" policy. The exception to not interfering with earnings for efficiency reasons are taxes on rents. However, rents cannot be identified until the inheritance of human capital, ability and the production and supply of human capital has been clarified. This is an important field for further research that will be further discussed towards the end of this section.

8.3 Empirical Application of the Human Capital Models

We are going to discuss two aspects related to the question of empirical application of the implications of the human capital models and the interpretation of such results. The first pertains to the equilibrium or non-equilibrium assumptions of the models. The second has to do with the distinction between endogenous and exogenous variables.

In the human capital models, individuals are assumed not only to try to make optimal investments, but also to succeed in doing so. If the implications of the models are to be properly tested, and estimated parameters are to be interpretable in terms of the theory, it is necessary to investigate the situations furnishing data for the tests. Can we, e.g., expect the rental price for human capital to have been "given" or constant? With the strong version of the models we saw that this question could be reformulated: do the data come from a period that could reasonably be reconciled with the idea of an economy in a semi-stationary development or different such developments in various periods? Do markets function smoothly, so that the transition from one type of steady state development to another is done quickly? If not, some but not all decisions taken at present may reflect earlier types of development. For decisions already made this would irreversibly be the case (e.g., schooling decisions of older people). We could then either test the models only on recent decisions if the development situation had just changed or test them on a greater spectrum of decisions during stable periods. When concentrating on recent decisions we must require that markets function rather smoothly. Otherwise, there is no guarantee that the rental price of human capital and market rates of interest are equal, that different outlays on, e.g. schooling quality, represent specific amounts of human capital creation, etc.

If stable periods are rare both longitudinal and cross-sectional data concerning e.g. earnings and education become difficult to use and interpret. Cross-sectional data are more frequently available. They show the earnings of individuals of different ages with different types of education. The investment decisions reflected in these earnings refer at least to a time span of 50 years. Only if the same equilibrium semi-stationary growth development has prevailed during this period can the equilibrium version of the models be applied to all these decisions. To keep initial conditions constant it may be preferable to work with longitudinal, individual data. However, if one is to interpret all the investment decisions during the life cycle of an individual in terms of the equilibrium situation of the strong version of the models this still requires that the same steady state has prevailed for as long a period of time.

It is also of interest to identify situations for which the strong versions of the human capital models are or are not applicable from the point of view of a weak interpretation of the models. If the implications of the models hold equally well in both types of situations, we can conclude that there must be another explanation for this than the one provided by the strong version.

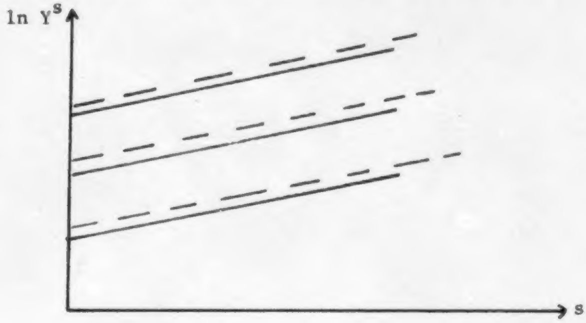
In the successive extensions of the human capital models, more and more of the variables that can be considered as open to individual choice are described within the models - i.e., they are made endogenous. The smaller models can be viewed as simplifications of the more comprehensive ones.

How then should these exogenous variables, that can always be regarded as only "provisonally" exogenous in the face of more extended models, be treated? The same question arises in partial analysis for prices when not all individuals confront the same prices. One possibility would be to identify groups of individuals identical in these respects. As we discussed in Section 4, this may be preferable to, e.g., standardizing for "provisionally" exogenous variables in a regression analysis. The advantages of a general approach becomes apparent when it comes to empirically identifying these identical groups. The more complete human capital models give important information that can be used in less extended models. They suggest variables that, e.g., explain differences in human capital investments made by parents in their children (income and relative prices) and in post-school investments made by the individual (length of time on the labor market). Such variables could be used to identify the necessary identical groups in less extended models. But, of course, for the exogenous variables appearing even in the most complete human capital models the identification of identical groups must be done on other premises.

Let us exemplify this and the previous point about equilibrium by using the Schooling Model.

In the Schooling Models the optimal level of schooling for an individual depends on such things as initial stock of human capital, ability, time horizon, and the rates of depreciation and appreciation. However, with a linear specification of the influence of schooling on earnings, individuals identical in these respects are indifferent towards different amounts of schooling. Figure 45 illustrates such a situation for three groups of

Figure 45 Earnings and schooling, linear model



individuals varying with respect to initial human capital and/or ability. The broken line represents the prevailing rate of interest with which individuals should compare the marginal effect of schooling (s) on the logarithmic value of earnings (ln Y^s) in order to maximize earnings.

With this model there is no correlation between s and initial endowments or ability, and an empirical estimation of b in

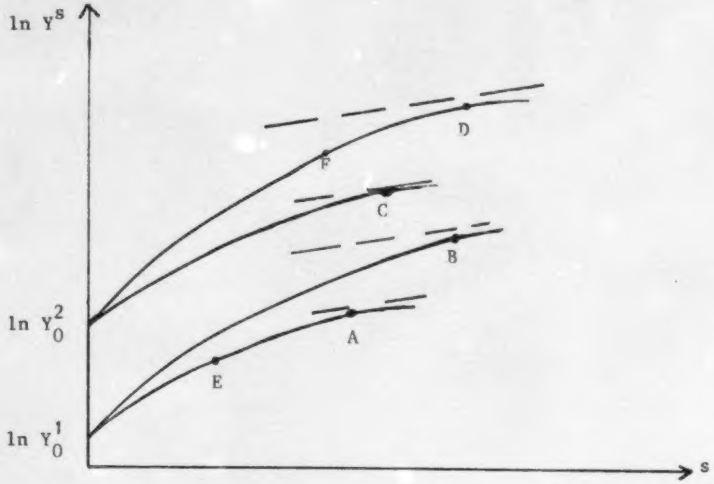
$$\ln Y^s = a + bs + u$$

would give a correct general picture of the effect of s on ln Y if, e.g, regression analysis is used. Of course, the share of earnings variations explained could be small as long as variations in initial endowments and ability are not considered and thus the explanatory power of the model is underestimated.

The interpretation of b is not entirely straightforward, however. Theoretically it includes the effects of depreciation and appreciation. As long as we do not know anything about the different initial endowment and ability groups in this respect, b cannot be precisely interpreted. That parameter does not necessarily have to be composed in the same way for these groups. If a separate analysis were made for each group, a further elaboration of this and similar questions would be possible.

Let us now turn to a situation where the linear model does not hold. In Figure 46 there are two levels of initial endowments (represented by ln Y_0^1 and ln Y_0^2) and two levels of ability (represented by the two different curves for school effects on earnings). People with finite time horizon (T) and $\delta = \alpha$ are supposed to choose points A, B, C, D.

Figure 46 Earnings and schooling, non-linear model



People with shorter time horizons and $\delta = \alpha$ would then choose shorter schooling periods - points E and F, for example. Different combinations of T, δ and α could make any point on the ability curves the optimal one.

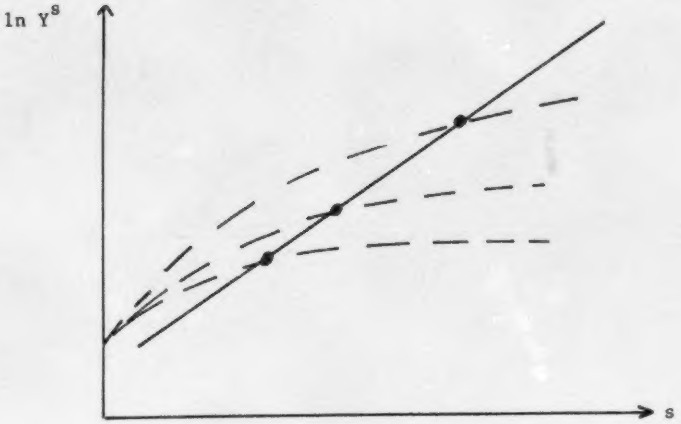
The estimation of

$$\ln Y^s = a + b\ s + u$$

in this situation would, understandably, not give much information about what is going on. A regression analysis may result in a good fit (a high R^2) as illustrated in Figure 47 but that does not help us to identify how schooling increases earnings. Working with different ability groups in this case turns out to be an imperative.

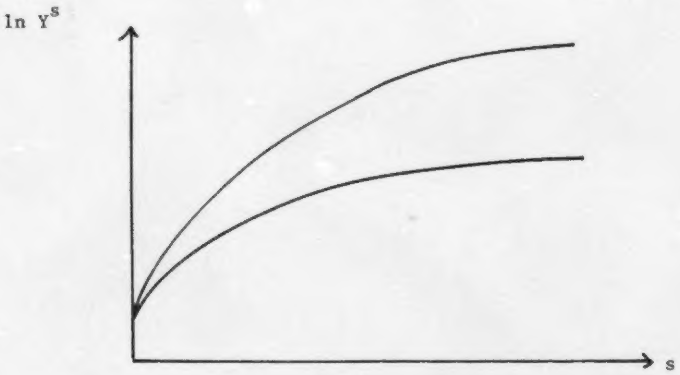
Let us now analyze what happens if the model is applied in a situation not characterized by equilibrium. In the absence of equilibrium there is no guarantee that a year of schooling, or a given amount of direct investment outlays would increase the earnings of identical incividuals by the same amount. The rental price for human capital could vary among individuals, as might the market rate of interest. If we do not have any specific information about the disequilibrium situation it is impossible to interpret the empirical results obtained when applying the Schooling Model. We might obtain results like those of Figures 46 and 47 but they would be uninterpretable.

Figure 47 Estimated earnings and schooling relation



However, suppose we know that only the capital market is in disequilibrium and that individuals have to pay different rates of interest for financial resources. Let us assume that we could distinguish different ability groups and identify within ability groups people with the same T, δ, α and Y_0. Figure 48 illustrates the relation between earnings and schooling for two ability groups of otherwise identical individuals. Within each ability group, people with lower rates of interest would choose a longer schooling period. This could permit identification of the curves. The deviation from a straight line could be estimated.

Figure 48 Earnings and schooling with different market rates of interest



These examples indicate the importance of investigating the situation as to equilibrium conditions and of identifying identical groups. In non-equilibrium situations the model could only be applied and interpreted if we have a thorough knowledge of the type of disequilibrium prevailing. Are there differences in the rental price of human capital, interest rates, prices of educational services, prices of other goods and services? Are these differences constant? If we want to test the models properly and be able to interpret the estimated parameters it is necessary also to identify groups identical with respect to these prices. This way of applying the models for disequilibrium situations can, of course, only result in a partial treatment of disequilibrium. And the question arises how far one can go in this direction and still remain within (at least the strong version of) the human capital models.

As stated earlier, when it comes to identifying these identical groups empirically, the more complete human capital models provide important information that could be used in less extended models. If models - like the Schooling Models - are not tested on homogeneous but heterogeneous groups of individuals the results would be difficult to interpret. This is true whether the implications of the model "correspond" to empirical data or not. If the individuals are not alike, the implications of the model should in general not hold. If they do we cannot explain why.

8.4 Further Research Possibilities

From our exploration of assumptions used in human capital models we think many questions are raised that ought to be further penetrated both empirically and theoretically within models of this type. The most important areas are in our opinion the following:

1. The preference functions of individuals as reflected in the types of effects they seek from education. More precise work with human capital models can only be achieved when better information about these aspects is available. Do individuals value education for its effects on job activities, political or cultural activities, or work at home? Do groups of individuals differ in these value judgments? What information do individuals have when they make their educational choices?

2. The educational production process and its relation to individual performance in different activities. To what extent is education skill-

increasing rather than efficiency-increasing (increasing the stock of human capital rather than the efficiency parameters in production functions of human capital, leisure activities,etc.)? To what extent is education an information-seeking, risk-decreasing, preference-changing process (aspects not included in the human capital models)?

3. Practical work to relate abstract concepts in the human capital approach, such as ability, educational inputs, net rate of depreciation and stock of human capital, to indicators for these concepts, and also to relate them to earnings (as outlined, e.g., in Appendix A).

4. Identification of rent elements in earnings (as outlined, e.g., in Appendix A).

In connection with the different models, we have also discussed suitable testing strategies and the possibilities of using Schooling Models without neglecting the important explanatory aspects brought out by the more complete models. If the focus of interest is on educational policy it would be practical to concentrate on such models and eventually to make them more detailed.

A different field for the application of human capital models is as models of expectations.Our models can either be interpreted as assuming equilibrium to prevail or as not assuming anything specific on that point. However, it may be possible to turn them into explicit instruments for analyzing disequilibrium situations. One might then want to use human capital models to describe the formation of expectations and behavior of adaptation. Suppose we have an intertemporal general equilibrium model describing the steady state development of an economy. This description includes the choice of human capital investments and supply of human capital from different individuals. Then some change occurs that may, for example, increase the rental price for human capital, or an educational reform is undertaken that increases the educational incentives for certain groups. If we then use the model to describe the planning of the individuals, we could calculate how much they would like to increase their stock of human capital and their supply to the labor market. Rather sophisticated predictions could be made about the reactions of individuals at different ages and with different preference functions, if the human capital models were used in this manner. However, all these plans would certainly not be realized as the individuals would change their plans as they saw the reactions of other individuals, and the models would only provide an indication of the first step in a dynamic theory of adaptation - unless they were complemented by hypotheses, e.g., about the speed at which these adaptations take place.

The primary purpose of these models would be to describe the adaptational process. It would be necessary to indicate how individual decision-making is done in disequilibrium situations, and how individuals calculate expected costs and benefits from such processes as education. Expected extra earnings could figure among the benefits. As with the weak version of human capital models it would not be necessary to enter into the question whether these earnings expectations are productivity related or not. The aim would not primarily be to explain earnings; the domain of evaluation of expectational human capital models would be the formation of expectations and adaptation.

However, the result of research of the type mentioned above may be that more radical alternatives should be considered. It may, for instance, suggest that only certain elements of the human capital models should be selected:

- that expectations about earnings and other benefits are important for the choice of education;
- that earnings capacity can be improved by spending resources on schooling, informal training, etc.

and put into different theoretical frameworks: e.g., fully dynamic models and models with heterogeneous human capital, disequilibrium, uncertainty, etc. The fully dynamic models could be strong version human capital models with a more elaborate description of the rest of the economy especially the supply of education in public schools financed by taxes. In partial models for instance, heterogeneous human capital concepts and adaptational processes in disequilibrium could be tried out.

As regards risk and uncertainty, some modelling efforts have been undertaken.[1)]

[1)] G.S. Becker, op.cit., 1964.
M. Nerlove, "On Tuition and Costs of Higher Education: Prolegomona to a Conceptual Framework." Journal of Political Economy, Supplement, 1972.
A. Razin, "Investment in Human Capital under Uncertainty." Discussion Paper No. 19, The Center for Economic Research, University of Minnesota, 1972.
Y. Weiss, "The Risk Element in Occupational and Educational Choices." Journal of Political Economy, 1972, No.6.

Footnote continued on next page.

They have primarily been directed, however, at augmenting the individualistic planning models - the weak version of the human capital models - with elements of uncertainty, for instance, as to the likelihood of being able to complete one's education and of obtaining certain earnings. These aspects deserve further attention, though they would be more difficult to introduce in the strong version of the human capital models.

Footnote continued from previous page.

J.S. Hause, "The Risk Element in Occupational and Education Choices: Comment." Journal of Political Economy, 1974, No.4.

D. Levhari, Y.Weiss, "The Effect of Risk on the Investment in Human Capital." The American Economic Review, December, 1974.

A. Razin, "Lifetime Uncertainty, Human Capital and Physical Capital." Economic Inquiry, September, 1976.

L. Olson, H.White, H.M. Shefrin, "Optimal Investment in Schooling when Incomes are Risky." Journal of Political Economy, 1978, No.3.

B. Eden. "Stockastic Dominance in Human Capital." Journal of Political Economy, 1980, No.1.

Appendix A The Homogeneity Assumption

Though the homogeneity assumption is very common in human capital literature it is seldom discussed.[1] Within this research project it was considered very important and it seemed necessary to analyze this assumption in more detail.[2] The question is: under what conditions can we have a homogeneous human capital concept, a "physical" concept - independent of income distribution and relative factor prices - and not merely a value concept?

To make this discussion clear, we shall first indicate the general implications of assuming human capital to be homogeneous. Later we discuss the situation when the economy remains in a given semi-stationary state.

Let us assume a finite number of different qualities of individuals. The homogeneous human capital concept then implies that we can measure the productive capacity of each individual in terms of a given unit. These units can also be summed up for the whole economy and would represent its human resources.

If the production possibilities of the economy are represented by a transformation function

$$T(c,q,h) = 0$$

where c = output vector

q = a vector of physical capital

h = a vector of human resources where the elements h_i $(i=1...n)$ represent individuals of different quality equalling IN where I is the unit matrix and N a vector where the elements N_i $(i=1...n)$ represent the number of individuals in each quality class.

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[1] There is a brief discussion in P. Taubman, Sources of Inequality of Earnings. Amsterdam, 1975, Chapter 1. These problems are also alluded to in A.S. Blinder, Toward an Economic Theory of Income Distribution. Cambridge, Mass., 1974, pp. 15-16.

[2] Some of the analyses were published by B.C. Ysander, "Homogeneity in Education." Learning and Earning. Stockholm, 1978.

The assumption of homogeneous human capital means there exists a scalar-valued function

$$\phi: H = \phi(h)$$

such that:

a) $T(c,q,h) = 0$ if and only if $T'(c,q,H) = 0$

and

b) $\phi(h) = \sum_{i=1}^{n} \alpha_i h_i$.

The meaning of this could be clarified in two steps.

Without assuming that the stock of human capital of different individuals can be measured in the same units, a necessary condition for forming such an aggregate measure of the stock of human capital (H) is that *the relative productivity capacity of each individual is independent of the configuration of the rest of the economy - i.e., independent of c and q.*[1] This H can then without loss of information, be used to represent the human capacities in the description of the production possibilities of the economy.[2]

The relative productivity or wages of different individuals (which in competitive equilibrium equals relative productivity) are, thus, not influenced by what they produce, what material they work with, what machines are used in production, etc. This is a very strong assumption. If, e.g., equal individuals taking course A and B respectively were always known to be perfect substitutes (could replace each other in given proportions, for example 1:2, and if the same held for all other courses) this requirement would be satisfied.

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[1] Assuming differentiable functions, a necessary condition for $T(c,q,h) = 0$ if and only if $T'(c,q,H) = 0$ is that:

$$\frac{\frac{\partial T}{\partial h_i}}{\frac{\partial T}{\partial h_j}} = \frac{\frac{\partial T'}{\partial H}\frac{\partial \phi}{\partial h_i}}{\frac{\partial T'}{\partial H}\frac{\partial \phi}{\partial h_j}} \equiv \frac{\frac{\partial \phi}{\partial h_i}}{\frac{\partial \phi}{\partial h_j}} \quad \text{which is independent of c and q.}$$

In competitive equilibrium we then also have

$$\frac{\partial \phi}{\partial h_i} \Big/ \frac{\partial \phi}{\partial h_j} = \frac{W_i}{W_j}$$

W_i, W_j = rental prices for the services of the human capital of individual i and j respectively.

[2] B.C. Ysander, op.cit., pp.262,263.

Another instance would be if they were always perfectly complementary (e.g., if one with course A always worked together with three with course B). Excluded are all situations when, for instance, course A is more or less valuable in relation to course B, depending on the (physical) capital equipment.

However, as the homogeneous human capital concept is used in the models, it follows that it is not only possible to form the aggregate H, but also that this can be done in a simple way - i.e., the same measuring rod can be used to measure the human capital of different individuals. This restricts the relations mentioned above between such stocks to perfect substitutes.[1] Relative productive capacities are independent of c, q and h.

Let us illustrate this and the previous point. The figure below depicts a situation where either 8 individuals of type h_i, or 12 individuals of type h_j, could achieve a given output ($\bar{Q}$). h_i might represent individuals with higher education and h_j individuals with only high school. One individual with higher education is indicated to be always substitutable for 1.5 individuals with only high school education. Thus the same output $\bar{Q}$ could be produced with 4 h_i and 6 h_j. Our first point means that we can always draw lines indicating the marginal rate of substitution among human capacities irrespective of the rest of the economy (c and q). The second means that these lines are straight lines independent of c, q and h.

In the example we can say that from the view of production 2 h_is always equal 3 h_js. Two h_is would receive as much pay as three h_js. However, this is 50% more per h_i, because one h_i is 50% more productive then one h_j. This gives us a simple measure of productive capacity or human capital. If we say that one h_j represents one unit of human capital, then one h_i represents 1.5 units of human capital. As long as such conditions are satisfied for all types of human capital they can all be measured in the same units and

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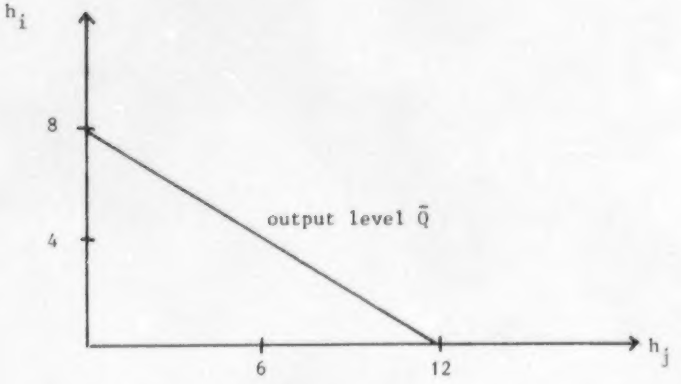
[1] We require the function ϕ to be of the form $\phi(h) = \sum_i \alpha_i h_i$. From that we derive that

$$\frac{\frac{\partial T}{\partial h_i}}{\frac{\partial T}{\partial h_j}} = \frac{\frac{\partial \phi}{\partial h_i}}{\frac{\partial \phi}{\partial h_j}} = \frac{\alpha_i}{\alpha_j}$$

, which is independent of c, q and h.

Figure 1 Possible combinations of production factors h_i and h_j to produce the output $\bar{Q}$



and the aggregate H is a simple sum of the individual elements. Relative wages would depend on relative human capital stocks.

The preceding statements referred to the aggregation of productive capacity of given individuals whose qualities differ at a given date. It must then be recognized that these qualities could stem from different sources. The two main types are innate qualities and qualities produced through investments. Despite the difference in origin of these types, they are assumed to be comparable according to the rules of aggregation mentioned above. For example, the capacity of an individual whose qualities only consist of innate resources are assumed to be a perfect substitute of the capacity of an individual whose resources derive from both sources. Moreover, all human resources created in different investment processes - at firms specializing in training (schooling) or also producing other goods and services (on-the-job training) - follow the same rules of aggregation.

However, in our models the human capital concept also implies that the measuring rod does not change over time. This means that we require:

c) $\phi(h)_t = \alpha' I N_t \mid \forall t$

where α is the vector of weights with elements $\alpha_i (i=1\ldots n)$.

No matter when individuals move from one quality class to another (e.g., by taking a given course) this will represent an equally great increase in their stock of human capital. Relative wages remain unchanged over time.

In fact, disregarding depreciation, in the models we write

$$H_{it} = \bar{H}_{i0} + \sum_{n=0}^{t-1} \Delta H_{in}$$

$H_{it}, \bar{H}_{i0}$ = human capital stock of individual i at date t and 0, respectively,

ΔH_{in} = addition to the human capital stock of individual i in period n.

A given ΔH_{in} is assumed to be equally valuable in all periods.

However, to better understand this investment process - the movement from one quality class to another - it is desirable to reduce it to manageable proportions. A radical approach would be to assume it to be a function of a homogeneous input. We could then write

$$H_{it} = \bar{H}_{i0} + \sum_{i=0}^{t-1} f_i \, (D_{it}),$$

where D_{it} = human investment inputs for individual i in period t.

Or, as a special case,

$$H_{it} = H_{i0} + g_i \, (R_{it})$$

R_{it} = the sum of human investment inputs for individual i up to date t.

We use different versions of such human capital production functions in our models. In the Simple Schooling Model, for example, this results in a definite relation between total resource input, stock of human capital and earnings. In other models the existence of more than one input factor and variations in labor market participation complicated these relations. However, the actual human capital stock of the individual is always derivable from the successive inputs in the human capital production process which in turn determines relative earnings from full-time work.

This is also true when we relate individual "ability" differences to the production of human capital.1) Even after introducing "ability" into the models we remain in the homogeneous human capital world. This means that

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1) It may by the way be interesting to note that the ability factor - in our models as in most other models - is only supposed to affect the production of human capital and thus not the use of human capital.

ability does not make people sprint off in various directions as regards productive capacities that could not be related by way of perfect substitutes.

The ability factor in the expression above would be found in the special functional form for each individual, - $f_i(\ )$ or $g_i(\ )$.

However, as the ability factor is used in the models, and provided that we accept the previous simplification, we could also write

$$H_{it} = H_{i0} + \sum_{i=0}^{t-1} \cdot f(D_{it}, \alpha_i)$$

where α_i = ability index.

Or, as a special case;

$$H_{it} = H_{i0} + g(R_{it}, \alpha_i).$$

This unidimensionality of ability is, of course, a restrictive assumption.

Human capital is regularly given the restrictive interpretation described above. Sometimes homogeneous depreciation and appreciation are added as in our models. It is interesting to note that we thereby achieve a simple and unified explanation of the potential earnings of individuals in terms of

- unidimensional ability
- homogeneous original human capital
- homogeneous human capital investments
- homogeneous depreciation and appreciation.

A direct way of using this approach and investigating the plausibility of the concept of homogeneous human capital would be to try to construct measures of human capital stocks (productive capacity) and relate them to such factors as earnings. Little work of this kind has been done. Theorists of the human capital school have, instead, looked for and tested other types of implications of the theory.[1] However, such a direct application of the human capital theory is more revealing as it is closer to the foundations of the theory.

Of course, for *a priori* theoretical reasons, we may be skeptical about the success of such an endeavor. One reason is that there are so many possibilities that might be tried. Nonetheless, they ought to be investigated as long as human capital theory of this kind continues to be used. We then have to decide how to

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[1] For an exception see: S.A. Horowitz, A. Sherman, "A Direct Measure of the Relation between Human Capital and Productivity." *Journal of Human Resources*, 1980, No. 1.

- define different aspects of initial endowments, aggregate them and give each individual an index number;
- define different aspects of ability, aggregate them and give each individual an index number;
- define different aspects of inputs to the investment process, aggregate them and give each individual an index number totally or for each period;
- define the production function for human capital production;
- define different aspects of depreciation and appreciation, aggregate them and give each individual an index number.

It should also be remembered that according to human capital theory the stock of human capital is closely related to potential earnings, but potential earnings do not clearly indicate the actual earnings of specific individuals during a given period. Due attention should therefore also be given to leisure, differences in non-pecuniary aspects between jobs and current human capital production. Given these precisions, a model like the one below could then be estimated. Statistical methods are available for estimating "latent variables" from measurable indicators of these variables and the relations (even interdependent) between them.2)

From the point of view of the concept of homogeneous human capital the interest lies in the possibilities in different situations to have the same weights and relations between indicators and the "latent" variables. One might at least be able to map out areas where the approach is more or less applicable, markets in disequilibrium, where barriers exist and monopoly reigns. The more of the latter kinds of markets there are, the less suitable the homogeneous human capital approach becomes.

However, there is another interesting aspect of this question of identifying and measuring human capital that deserves attention. If we could measure human capital and the process by which it was created we could, for example, investigate the rent elements in earnings, distinguish how education and ability separately contribute to the creation of human capital, measure the

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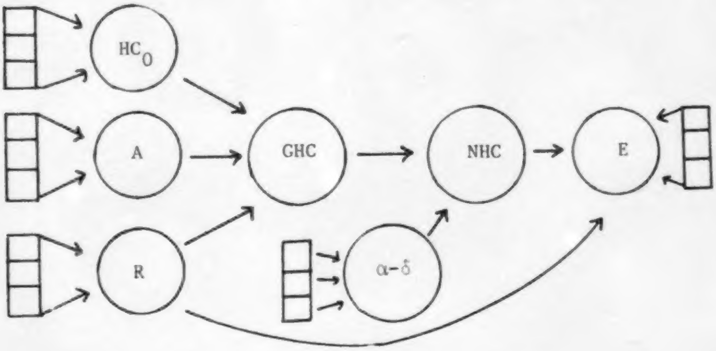
1) See, e.g., H. Wold, "Ways and Means of Multidisciplinary Studies." Department of Statistics, University of Uppsala. Research Report 78-3.

Figure 2 Scheme for a statistical model for estimation of "latent variables" in human capital theory



○ "latent variables" HC_0 = initial human capital endowment; A = ability; R = resources devoted to human capital production; GHC = gross human capital stock; (α-δ) = net rate of appreciation; NHC = net human capital stock; and E = potential earnings at date t.

▤ indicators of latent variables

initial stock of human capital and what part of human capital that results from cost-free appreciation, to such processes as biological maturing.

A few comments should also be made about the semi-stationary economy that remains on its given growth path. In this context the homogeneity assumption is quite congenial. All factors of production, produced and non-produced, would be growing at the same rate. They would always be used in the same proportions, and marginal rates of substitution remain the same. The economy behaves as if the conditions for an aggregate H as a simple sum measured in units of H were satisfied.[1] In this situation the assumption about homogeneous human capital is applicable.¹⁾

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[1] Cf. B.C. Ysander, op.cit., p.264.
However, this is only true as long as the economy remains on its given growth path. Therefore, situations including switches between different stationary growth paths, belong to those covered by the previous discussion.

The proportions of human capital possessed by different individuals correspond to the proportions of their "net rentals" (earnings).¹⁾ Thus, we could use "net earnings" as a measure of human capital. In a steady state economy the conditions necessary for a homogeneous human capital concept exist. However, two things should be observed in this connection. First of all, we must make certain that we really are in a stationary growth economy if we are to apply this type of measurement. Secondly, as was indicated above, the theoretical "net earnings" concept does not, for a number of reasons, coincide with observed earnings.

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1) Let $H_1^x(0)$ be the base of measurement and define

$$H_i(t) \quad \text{as} \quad \frac{p_i^x H_i^x(t)}{p_1^x}$$

where p_i^x/p_1^x represents, for example, the substitution relation in Figure 1.

We then have

$$\frac{rp_i^x(t)\ H_i^x(t)}{rp_j^x(t)\ H_j^x(t)} = \frac{rp_1^x\ H_i(t)}{rp_1^x\ H_j(t)} = \frac{w_i}{w_j} = \frac{H_i(t)}{H_j(t)} .$$

By way of example, let us assume an economy in competitive equilibrium on a stationary growth path where the rate of growth is γ. Let us also assume that the schooling model including ability applies to this economy, and that the population and each ability category grows at rate γ. The physical human capital of individual i after leaving school (H_i^x) remains unchanged, so that

$$H_i^{\text{[illegible]}}(t+1) = H_i^x(t)$$

and its price $p_i^x(t) = p_i^x$.

The relative "net rentals" of different individuals also remain constant

$$\frac{w_i}{w_j} = \frac{r\ p_i^x(t)}{r\ p_j^x(t)} \frac{H_i^x(t)}{H_j^x(t)} = \frac{rp_i^x\ H_i^x}{rp_j^x\ H_j^x} .$$

Cf. B.C. Ysander, op.cit., p. 264.

Appendix B Homogeneous Human Capital and Aggregate Production Functions

To enable more concrete statements to be made about education and growth, or education and income distribution, broad relations among macro variables have often been used.[1] Macro production functions describing national input-output relations are one example. They have been used for growth accounting and the formualtion of growth policies. They have also been applied to studies of the income distribution between factors of production. Supplemented with knowledge about ownership of factors of production, the distribution analysis could be extended to cover personal income distribution.

Of course, with all the required micro data available, one could construct complete micro models of the economy. Macro relations would not be necessary, though they could be practical to summarize results whenever possible. However, that stage has not yet been reached. To construct macro relations is therefore risky. The general nature of the problems encountered in this connection has been discussed by many authors. Problems arise, for example, when a macro production function is used not just as a device of empirical description, but is to be given a theoretical foundation so that the results become interpretable and can be used in explanatory schemes.[2] When used for the latter purpose it should be a relevant aggregation of, for example, the production possibilities of the micro units, their production functions and the behavior of microeconomic units. [1)]

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[1] See for example:
J. Tinbergen, Income Distribution. Analysis and Policies. Amsterdam, 1975.
J. Aarrestad, "On the Optimal Allocation of Labor to the Educational Sector." The Swedish Journal of Economics, 1975, No.3.
Z. Griliches, "Notes on the role of education in production functions and growth accounting." in L. Hansen (ed.), Education, Income and Human Capital. New York, 1970.
L.C. Thurow, Poverty and Discrimination. Washington, 1969.
J. Bernard, "Modèle d'optimation génèrale économie-éducation." Modèles mathémathiques pour la planification de l'enseignement. OECD, Paris, 1969.
R.R. Nelson, E.S. Phelps, "Investments in Humans, Technological Diffusion and Economic Growth." American Economic Review. Papers and Proceedings, May, 1966.
R. Solow, "Technical Progress, Capital Formation, and Economic Growth." American Economic Review. Papers and Proceedings, May, 1962.
C.H. Sheng, "Education and Economic Growth." Review of Economic Studies, 1976, pp. 509-518.

[2] D.L. Bosworth, Production Functions. A Theoretical and Empirical Study. Lexington, 1976.
F.M. Fisher, "The Existence of Aggregate Production Functions." Econometrica, 1969, No.4.

However, since with our human capital models we assume that human capital is homogeneous and can be aggregated why should we not continue to the macroeconomic level? From such a micro basis would it not be possible to return to the study of economic growth and renew the analysis of technical choice and relative prices for physical capital and human resources?

Let us here just briefly investigate what a macro production function would look like in relation to the Simple Schooling Model and then briefly discuss its possible applications.

Let us assume that there is no physical capital (or that physical capital can be measured and aggregated together with human capital).

According to the Simple Schooling Model we have at least approximately

$$Y^s = a\bar{H}_0 e^{rs}$$

Total labor earnings or the productive results of labor would then be:

$$Y = Y^{s_0} N_0 + Y^{s_1} N_1 + \ldots + Y^{s_s} N_s = a \bar{H}_0 (N_0 + e^{rs_1} N_1 + \ldots + e^{rs_s} N_s \,.$$

Y = total labor production

N_i = number of workers with schooling level s_i, where $i = 0 \ldots s$

Y^{s_i} = production per worker with schooling level s_i, $i = 0 \ldots s$.

If earnings are differentiated with respect to schooling we have:

$$\frac{\partial Y}{\partial s_i} = a \bar{H}_0 r e^{rs_i} > 0$$

$$\frac{\partial^2 Y}{(\partial s_i)^2} = a \bar{H}_0 r^2 e^{rs_i} > 0,$$

which means that more schooling leads to different increases in production, depending on how much schooling the person in question already has. Moreover, the second derivative with respect to education is also positive. This means that the marginal producitivity of school years increases and we may not reach an equilibrium in the factor market where educated labor is demanded

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1) K. Sato, Production Functions and Aggregation. Amsterdam, 1975.

by firms and supplied by the individuals. For both these reasons we would thus get a very "unusual" production function with schooling as a factor of production.

However, every additional year of schooling does not necessarily add the same amount of human capital. Therefore, it would be more satisfactory to have the production function expressed in terms of human capital. According to the human capital approach it is rather human capital that is the proper labor factor of production, not, e.g., labor and education. Human capital is embodied in the individual and cannot be bought and sold separately from labor. Human capital can rather be viewed as a factor increasing labor productivity. In an aggregated production function labor would therefore properly be represented by:

$$Q = f(H)$$

where $$H = \sum_{i=0}^{n} N_i \, H(s_i).$$

And in particular for the Simple Schooling Model, we find at least approximately

$$Y^{s_i} = a \bar{H}_0 \, e^{rs_i} = a \, H(s_i).$$

Thus

$$Y = a \sum_i N_i \, H(s_i)$$

$$\frac{\partial Y}{\partial H} = a > 0$$

$$\frac{\partial^2 Y}{\partial H^2} = 0.$$

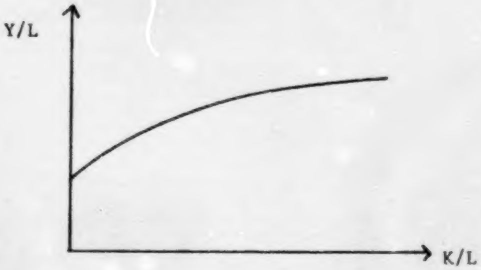
The marginal productivity equals a and is independent of who gets the schooling. The marginal productivity is constant. In the special case of the Simple Schooling Model it does not decline. We can also note that the distribution of human capital among individuals has no influence on the productive result.

The question of the utility of such a production function could now be raised. As with the conditions for aggregation, we have to distinguish between economies moving from one stationary state to another and economies that remain in one and the same stationary state.

In the former case the use of a production function is problematic. Fundamentally the difficulties are the same whether we want to construct the function in terms of output (Y), labor (L) and physical capital (K) $Y = [f(L,K)]$, or output and human capital (H) $[Y = f(H)]$. Therefore, only a few brief remarks on this subject are called for.

The general conditions for forming aggregates of output (Y) and physical capital (K) are the same as those discussed for human capital in Appendix A.[1] It may perhaps be argued at a conceptual level the use of human capital instead of labor more clearly indicates the difficulties involved in aggregating labor and physical capital. Human capital - as a productivity index - must be some kind of quality-adjusted measure of labor. As such it may be considered more relevant than labor (counting persons or hours) with disregard for the differences in human capacities. But exactly these differences in human capacities raise the question whether it can be argued that it is possible for them to materialize irrespective of the existing capital goods. And as regards physical capital, would not the contribtuion of different types of physical capital depend on the quality of the labor with which it is used?

However, aggregating is one thing and the construction of macro production functions another. Even if the perspective is restricted to economies in a stationary state (i.e., without growth) and with only one consumption good and one type of labor, there is no guarantee that we can construct a production function of the type illustrated below.[2]



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1) C.J. Bliss, Capital Theory and the Distribution of Income. Amsterdam, 1975, chapter 7.

2) C.J. Bliss, ibid., chapter 8.

If we could do so, economies with more capital could also be associated with a lower rate of interest. But for what purposes should the macro production be used? Not for growth accounting, as there is no growth. If we are generally not able to construct production functions in this restricted case, the possibilities of doing so in more complicated situations when growth is occurring must indeed be regarded as small.

What about distributional matters? Can the production function be used to establish a one-to-one correspondence between distribution of earnings, on the one hand, and techniques of production and the supply of physical and human capital on the other?

Let us consider a situation when the figure above is applicable. Moreover, we interpret physical capital per worker as skilled workers per unskilled worker or human capital per worker. The figure could now be read as follows: in an economy where such techniques are chosen that use more human capital/worker or where the supply of human capital/worker is large you have more production/worker and a lower rate of interest. In this case it appears as if the difference in output per worker or human capital per worker between two neighboring techniques consists entirely of the difference in their technical coefficients of production. "The ratio of these differences... is thus a technical datum independent of the rate of profit. Since it is also equal to the rate of profit it would appear in comparing the two economies, as if a rate of profit existed for technical reasons. Furthermore, since the profit share is equal to the elasticity of the wage-profit frontier, the distribution of income would appear to be uniquely related to the technology." 1) Or with the other interpretation: the distribution of income between skilled and unskilled labor would appear to be determined by the technology.

However, that is only the way things look in this special case. Generally, it would not look like that. And the situation is never such that it is the technology or capital intensiveness that determines the rate of interest.

If the economy stays within a given semi-stationary growth state the situation is different. Then a macro economic description of that development can always be constructed. All types of output and input are growing at the same rate. We may construct an aggregate output growing at rate γ. One ray on any constant-returns-to-scale production function could be used to relate the inputs and the output. This would, however, be more of bookkeeping than an explanatory theory.

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1) D.J. Harris, "Capital, Distribution and the Aggregate Production Function." American Economic Review, March, 1973.

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